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No. 11

BEARINGS

CONTENTS

The Design of Bearings, by FORREST E. CARDULLO - 3
Causes of Hot Bearings, by E. KISTINGER - - 15
Alloys for Bearings - - - - - - - - - 21
Ball Bearings - - - - - - - - - 26
Friction of Roller Bearings, by C. H. BENJAMIN - - 39

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CHAPTER I.

THE DESIGN OF BEARINGS.

The design of journals, pins, and bearings of all kinds is one of the most important problems connected with machine construction. It is a subject upon which we have a large amount of data, but, unfortunately, they are very conflicting. The results obtained from the rules given by different mechanical writers will be found to differ by 60 per cent or more. Many of our best modern engines have been designed in defiance of the generally accepted rules on this subject, and many other engines, when provided with what were thought to be very liberal bearing surface, have proved unsatisfactory. This confusion has largely been the result of a misconception of the actual running conditions of a bearing.

Friction of Journals.

A journal should be designed of such a size and form that it will run cool, and with practically no wear. The question both of heating and wear is one of friction, and in order for us to understand the principles upon which the design of bearings should be based, we must first understand the underlying principles of friction. Friction is defined as that force acting between two bodies at their surface of contact, when they are pressed together, which tends to prevent their sliding one upon the other. The energy used in overcoming this force of friction, appears at the rubbing surfaces as heat, and is ordinarily dissipated by conduction through the two bodies. The force of friction, and hence the amount of heat generated under any given circumstances, can be greatly reduced by the introduction of an oily or greasy substance between the rubbing surfaces. The oil or grease seems to act in the same way that a great number of minute balls would, reducing the friction and wear, and thus preventing the overheating and consequent destruction of the parts. On this account, bearings of all kinds are always lubricated. Thus the question of journal friction involves the further question of lubrication.

For the purpose of understanding as far as possible what goes on in a bearing, and the amount and nature of the forces acting under different conditions, several machines have been designed to investigate the matter. In general they are so arranged that a journal may be rotated at any desired speed, with a known load upon the boxes. Suitable means are provided for measuring the force of friction, and also the temperature of the bearing. Provided with such an apparatus, we find that the laws of friction of lubricated journals differ very materially from those commonly stated in the textbooks as the laws of friction. A comparison of the two will prove interesting.

Frictional Resistance in Lubricated and Unlubricated Bearings.

It is generally stated in the textbooks that the force of friction is proportional to the force with which the rubbing surfaces are pressed together, doubling, or trebling, as the case may be, with the normal pressure. This law is perfectly true for all cases of unlubricated bearings; or for bearings lubricated with solid substances, such as graphite, soapstone, tallow, etc. When, however, the bearing is properly lubricated with any fluid, it is found that doubling the pressure does not by any means double the friction, and when the lubricant is supplied in large quantities by means of an oil bath or a force pump, the friction will scarcely increase at all, even when the pressure is greatly increased. From the experiments of Prof. Thurston, and also of Mr. Tower, it appears that the friction of a journal per square inch of bearing surface, for any given speed, is equal to

$$f = kp^n \qquad (1)$$

where f is the force of friction acting on every square inch of bearing surface, p is the normal pressure in pounds per square inch on that surface, and k is a constant. The exponent n depends on the manner of oiling, and varies from 1 in the case of dry surfaces, to 0.50 in the case of drop-feed lubrication, 0.40 or thereabouts in the case of ring- and chain-oilers and pad lubrication, and becomes zero in case the oil is forced into the bearing under sufficient pressure to float the shaft.

The second law of friction, as generally stated, is that the force of friction is independent of the velocity of rubbing. This law also is true for unlubricated surfaces, and for surfaces lubricated by solids. In the case of bearings lubricated by oil we find that the friction increases with the speed of rubbing, but not at the same rate. If we express the law as an equation, we have

$$f = kv^m, \qquad (2)$$

where f is the force of friction at the rubbing surfaces in pounds per square inch, k is a constant, v is the velocity of rubbing in feet per second, and the exponent m varies from zero in the case of dry surfaces to 0.20 in the case of drop feed, and 0.50 in the case of an oil bath.

The third law of friction, as it generally appears in the textbooks, is that the friction depends, among other things, on the composition of the surfaces rubbed together. This, again, while true for unlubricated surfaces, is not true for other conditions. It matters nothing whether the surfaces be steel, brass, babbitt, or cast iron, so long as they are perfectly smooth and true, they will have the same friction when thoroughly lubricated. The friction will depend upon the oil used, not on the materials of journal or boxes, when the other conditions of speed and pressure remain constant. Many people think that babbitt has a less friction than iron or brass, under the same circumstances, but this is not true. The reason for the great success of babbitt as an "anti-friction" metal depends upon an entirely different property, as will appear later.

Combining into one equation the different laws of the friction of lubricated surfaces, as we actually find them to be, we have

$$f = kp^n v^m \qquad (3)$$

where f is the force of friction at the rubbing surface in pounds per square inch, k is a constant which varies with the excellence of the lubricant from 0.02 to 0.04, and the other quantities are as before. From this expression, we see that the friction increases with the load on the bearing, and also with the velocity of rubbing, although much more slowly than either.

Generation of Heat in Bearings.

The quantity of heat generated per square inch of bearing area, per second, is equal to the force of friction, times the velocity of rubbing. All of this heat must be conducted away through the boxes as fast as it is generated, in order that the bearing shall not attain a temperature high enough to destroy the lubricating qualities of the oil. The hotter the boxes become, the more heat they will radiate in a given time. When the bearing is running under ordinary working conditions, it will warm up until the heat radiated equals the heat generated, and the temperature so attained will remain constant as long as the conditions of lubrication, load, and speed do not change. This rise in temperature above that of the surrounding air, varies from less than 10 to nearly 100 degrees Fahrenheit, and is commonly about 30 degrees. We must keep either the force of friction or the velocity of rubbing, or both, down to that point where the temperature shall not attain dangerous values. As has been shown in the preceding paragraph, it was formerly believed that the force of friction was equal to a constant times the bearing pressure, and therefore, that the work of friction was equal to this constant times the pressure, times the velocity of rubbing. Now, since it is the work of friction that we are obliged to limit to a certain definite value per square inch of bearing area, it was concluded that a bearing would not reach a dangerous temperature if the product of the bearing pressure per square inch and the velocity of rubbing did not exceed a certain value. Accordingly, we find Prof. Thurston's formula for bearings to be

$$p\,v = C, \qquad (4)$$

where p is the bearing pressure in pounds per square inch, v is the velocity of rubbing in feet per second, and C has values varying from 800 foot-pounds per second in the case of iron shafts to 2,600 in the case of steel crankpins. This has long been the standard formula for designing bearings, and while it is not satisfactory in extreme cases, it is very satisfactory for bearings running at ordinary speeds.

Turning our attention again to the results obtained from the machines for testing bearings, we find that while the results are very even and regular for ordinary pressures and temperatures, when we begin to increase either of these to a high point, the friction and wear of our bearing suddenly increases enormously. The reason is that the oil has been squeezed out of the bearing by the great pressure. This squeezing out of the oil, and consequent great increase in the friction, has three effects. The absence of the lubricant causes the parts to scratch or score each other, thus rapidly destroying themselves, the great increase in friction results in a sudden very high temperature, in itself destructive to the materials of the bearing, and the heating is generally so rapid as to cause the pin and the interior parts of the box to expand more rapidly than the exterior parts, thus causing the box to grip the pin with enormous pressure. When the oil has been squeezed out in this manner, the bearing is said to seize.

Materials for Bearings.

It is evidently of advantage to make the bearing of such material that the injury resulting from seizing shall be a minimum. If the shaft and box are of nearly equal hardness, each will tend to scratch the other when seizing occurs, and the scoring is rapid and destructive. This action will be especially noticed in case the shaft has hard spots in it, while the rest is comparatively soft, as is the case in the poorer grades of wrought iron. If, however, the shaft is made of a hard and homogeneous material, like the better grades of medium steel, and the bearing is made of some soft material, like babbitt, the bearing will not roughen the journal, and so the journal cannot cut the bearing. This is the first reason why babbitt bearings are so successful.

A second reason for the success of babbitt bearing lies in the fact that they cannot be heated sufficiently to make the bearing grip the journal. They will rather soften and flow under the pressure without actually melting away, just as iron and steel soften at a welding heat. The harder bearing metals, such as brass and bronze, do not have these advantages, and have been almost entirely replaced by babbitt in bearings for heavy duty, especially when thorough lubrication is difficult.

Babbitt is a successful bearing metal for still a third reason. The unit pressure on any bearing is not the same at all points. The shaft is invariably made somewhat smaller in diameter than the box. If there is a high spot on the surface of the box, that spot will have a very large proportion of the total pressure acting on it, and as a result the film of lubricant will be broken down at that point, and local heating and consequent damage result. In the case of babbitt bearings, before the damage can become serious the metal is caused to flow away from that point under the combined influence of the heat and pressure, the oil film is again established, and normal conditions restored.

Influence of Quality of Oil.

The unit pressure which any bearing will stand without seizing depends upon its temperature and the kind of oils used. The lower the temperature of the bearings, the greater the allowable unit pressure. The reason for this is that oils become thinner and more free-flowing at the higher temperatures, consequently they are more easily squeezed out of the bearing, and it is more likely to seize. On this account, the higher the velocity of rubbing, the less the unit pressure that can be carried, but it does not follow that the allowable unit pressure varies inversely as the speed of rubbing, as was formerly thought.

The thicker and less free-flowing an oil is, the greater the unit pressure it will stand in a bearing without squeezing out. A watch oil, or a very light spindle oil, will only run under a very small unit pressure; sometimes they are squeezed out of the bearing when the pressure does not exceed 50 pounds per square inch. On the other hand, a cylinder oil of good body will stand a pressure of over 2,000 pounds to the square inch in the same bearing. There is a certain quality of oil which is best adapted to every bearing, and if possible it should be the one used.

A third cause influencing the pressure which may be carried is adhesiveness between the oil and the rubbing surfaces. Some oils are more certain to wet metal surfaces than are others, and in the same way, some metals are more readily wet by oil than are others. It is evident that when the surfaces repel, rather than attract, the oil, the film will be readily broken down, and when the opposite is the case the film is easily preserved.

Oil Grooving.

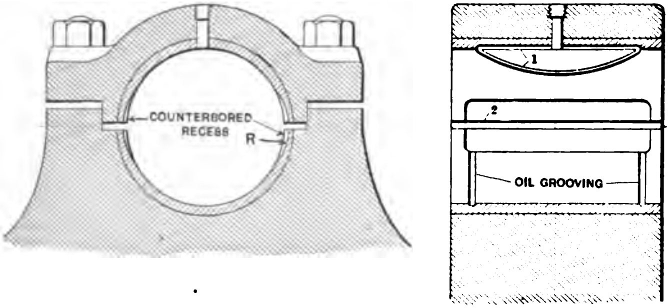
The mechanical arrangement of the box and journal may tend either to preserve or destroy the lubricating film. Both should be perfectly round and smooth, the box a trifle larger in diameter than the journal. The allowance commonly made for the "running fit" of the box and shaft is about 0.0005 $(D+1)$ inches, where D is the nominal diameter of the shaft in inches. Some manufacturers of fast running machinery make the diameter of the box exceed that of the shaft by nearly twice this amount. The oil should be introduced at that point where the forces acting tend to separate the shaft and box. At this point grooves must be cut in the surface of the box, so as to distribute the lubricant evenly over the entire length of the journal. Having been so introduced and distributed, the oil will adhere to the journal, and be carried around by it as it revolves to the point where it is pressed against the box with the greatest force, thus forming the lubricating film which separates the rubbing surfaces. The supply of lubricant thus continually furnished, and swept up to the spot where it is needed, must not be diverted from its course in any way. A sharp edge at the division point of the box will wipe it off the journal as fast as it is distributed, or a wrongly placed oil groove will drain it out before it has entirely accomplished its purpose.

An important matter in the design of bearings is the cutting of these oil grooves. They are a necessary evil, and should be treated as such, by using as few of them as possible. They serve, first, to distribute the lubricant uniformly over the surface of the journal, and second, to collect the oil which would otherwise run out at the ends of the bearing, and return it to some point where it may again be of use. As generally cut, oil grooves have two faults; first, they are so numerous as to cut down to a serious extent the area of the bearing, and second, they are so located as to allow the oil to drain out of the bearing. Let us take an ordinary two-part cap bearing such as the outboard bearing of a Corliss engine, and see how it is best to cut the grooves.

One of these bearings, as commonly made by good builders, is shown in Fig. 1. The oil is supplied, drop by drop, through a hole in the cap. If there were no oil grooves, only a narrow band of the shaft revolving immediately under this hole would be reached by the oil. If, now, we cut a shallow groove in the cap, lengthwise of the bearing, and reaching almost, but not quite, to the edges, the oil will be enabled to reach every part of the revolving surface. To this groove we sometimes add two, as shown by the dotted lines in Fig. 2, which show the inner surface of the cap as being unrolled, and lying flat on the paper. No series of grooves can be cut in the box which will distribute the oil as well or as thoroughly as those shown, and they should alway be used in the caps of such bearings in preference to any others.

Having distributed the oil over the revolving surface, our next care must be to see that it is not wiped off before it reaches the point for which it was intended. Accordingly, we should counterbore the box at the joint in such a way as to make a recess in which the surplus oil may gather, and which will further assist when necessary in distributing the lubricant. This counterbore should extend to within ¼ or ½ inch of the ends of the bearing, as shown in Fig. 1.

When the oil is supplied through the cap, grooves for the distribution of the oil should not be cut in the bottom half of the bearing, since they will only serve to drain the bearing of the film of oil formed there. The oil film is under great pressure at this point, and naturally tends to flow away when any opportunity is offered. If left to its own devices, part of it will squeeze out at the ends of the bearing and be lost. In order to save this oil, shallow grooves, parallel to the ends of the bearing may be cut in the lower box, as shown in Figs. 1 and 3. Their office is to intercept the oil which would flow out at the ends, and divert it to the counterbored recesses, where it can again be made of use. These are the only grooves that should ever be used in the lower half of a two-part bearing, and they should only be used in the larger sizes.



Machinery, N.Y.

Fig. 1. Section of Outboard Bearing, showing Oil Grooving and Counterbored Recess.

Two classes of bearings which may well be made without oil grooves are, first, the cross-head slippers of engines, and second, crank-pin boxes. The cross-head slipper should have a recess cut at each end, in the same way as the counterboring of the two-part box, as shown in Fig. 4. To this is sometimes added the semi-circular groove shown in dotted lines, which does no harm, although it is unnecessary. The best way to oil a crank-pin is through the pin itself. In the case of overhung pins, a hole is drilled lengthwise of the pin to its center. A second hole is drilled from the surface of the pin to meet the first one. A shallow groove should now be cut in the surface of the pin, parallel to its axis, and reaching almost to the ends of the bearing, as shown in Fig. 5.

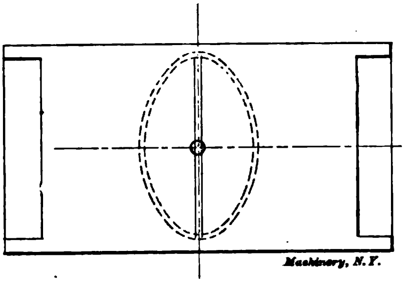


Fig. 2. Development of Cap, showing Oil Grooving and Counterboring.

No grooves should be cut in the boxes, but the edges where they come together should be counterbored.

As much care and attention should be given to the oil grooving as to the size of a bearing, yet it is a matter often left to the fancy of the mechanic who fits it. The purpose of the grooves, to distribute the oil evenly, should ever be kept in mind, and no groove should be cut which does not accomplish this purpose, except it be to return waste oil to a place where it may again be of use. Most commonly, bearings have too many grooves. So far from helping the lubricants, they generally drain the oil from where it is most needed. Use them *sparingly*.

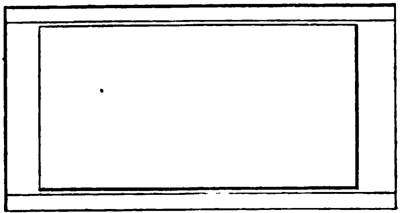


Fig. 3. Development of Lower Half of Outboard Bearing.

Calculating the Dimensions.

The durability of the lubricating film is affected in great measure by the character of the load that the bearing carries. When the load is unvarying in amount and direction, as in the case of a shaft carrying a heavy fly-wheel, the film is easily ruptured. In those cases where the pressure is variable in amount and direction, as in railway journals and crank-pins, the film is much more durable. When the journal only rotates through a small arc, as with the wrist-pin of a steam engine, the circumstances are most favorable. It has been found that when all other circumstances are exactly similar, a car journal, where the force varies continually in amount and direction, will stand about twice the unit pressure that a fly-wheel journal will, where the load is steady in amount and direction. A crank-pin, since the load completely reverses every revolution, will stand three times, and a wrist-pin, where the load only reverses, but does not make a complete revolution, will stand four times the unit pressure that the fly-wheel journal will.

The amount of pressure that commercial oils will endure at low speeds without breaking down varies from 500 to 1,000 pounds per square inch, where the load is steady. It is not safe, however, to load a bearing to this extent, since it is only under favorable circumstances that the film will stand this pressure without rupturing. On this account, journal bearings should not be required to stand more than two-thirds of this pressure at slow speeds, and the pressure should be reduced when the speed increases. The approximate unit pressure which a bearing will endure without seizing, is as follows:

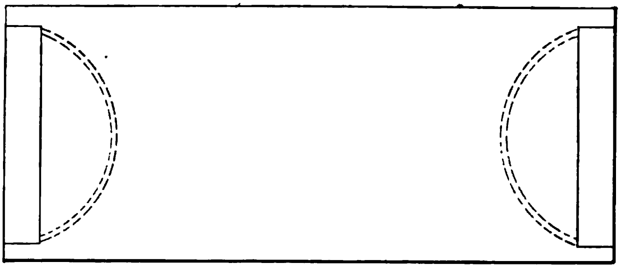

Machinery, N.Y.

Fig. 4. Face of Cross-head Slipper.

$$p = \frac{PK}{DN + K}, \qquad (5)$$

where p is the allowable pressure in pounds per square inch of projected area, D is the diameter of the bearing in inches, N is the number of revolutions of the journal per minute, and P and K depend upon the kind of oil, manner of lubrication, etc.

The quantity P is the maximum safe unit pressure for the given circumstances, at a very slow speed. In ordinary cases the value of this number will be 200 for collar thrust bearings, 400 for shaft bearings, 800 for car journals, 1,200 for crank-pins, and 1,600 for wrist-pins. In exceptional circumstances, these values may be increased by as much as 50 per cent, but only when the workmanship is of the best, the care the most skillful, the bearing readily accessible, and the oil of the best quality, and unusually viscous. It is only in the case of very large machinery, which will have the most expert supervision, that such values can be safely adopted. In the case of the great units built for the Subway power plant in New York by the Allis-Chalmers

Co. the value of *P* in the formula given above for the crank-pins was 2,000—as high a value as it is ever safe to use.

The factor *K* depends upon the method of oiling, the rapidity of cooling, and the care which the journal is likely to get. It will be found to have about the following values: Ordinary work, drop-feed lubrication, 700; first-class care, drop-feed lubrication, 1,000; force-feed lubrication or ring-oiling, 1,200 to 1,500; extreme limit for perfect lubrication and air-cooled bearings, 2,000. The value 2,000 is seldom used, except in locomotive work where the rapid circulation of the air cools the journals. Higher values than this may only be used in the case of water-cooled bearings.

The above formula is in convenient form for calculating journals. In case the bearing is some form of a sliding shoe, the quantity 240 *V* should be substituted for the quantity *D N* in the equation, *V* being the velocity of rubbing in feet per second. There are a few cases where a unit pressure sufficient to break down the oil film is allowable. Such cases are the pins of punching and shearing machines, pivots of swing bridges, and so on. The motion is so slow that heating cannot well result, and the effects of scoring cannot be serious. Sometimes bearing pressures up to the safe working stress of the material are used, but better practice is to use pressures not in excess of 4,000 pounds per square inch.

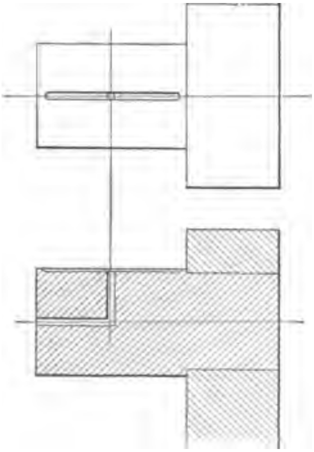


Fig. 5. Internally-oiled Crank-pin, showing Oil Passages and Grooves.

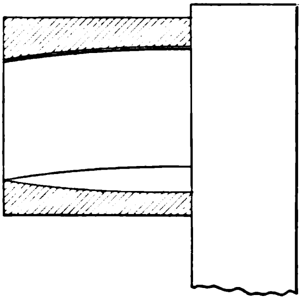


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Fig. 6. Section Showing the Bending of a Crank-pin and Consequent Unequal Wear of the Box.

In general, the diameter of a shaft or pin is fixed from considerations of strength or stiffness. Having obtained the proper diameter, we must next make the bearing long eough so that the unit pressure shall not exceed the required value. This length may be found directly by means of the equation:

$$L = \frac{W}{PK}\left(N + \frac{K}{D}\right) \qquad (6)$$

where L is the length of the bearing in inches, W the load upon it in pounds, and P, K, N, and D are as before.

A bearing may give poor satisfaction because it is too long, as well as because it is too short. Almost every bearing is in the condition of a loaded beam, and therefore it has some deflection. Let us take the case of an overhung crank-pin, in order to examine the phenomena occurring in a bearing under these circumstances. When the engine is first run, both the pin and box are, or should be, truly round and cylindrical. As the pin deflects under the action of the load, the pressure becomes greater on the side toward the crank throw, breaking down the oil film at that point, and causing heat. After a while the box becomes worn to a slightly larger diameter at the side toward the crank, in the manner shown in Fig. 6, which is a section showing an exaggerated view of the condition of affairs in the crank-pin box when under load.

It has already been noted that the box must be a trifle larger in diameter than the journal, and for successful working this difference is very strictly defined, and can vary only within narrow limits. Should the pin be too large, the oil film will be too thin, and easily ruptured. On the other hand, should the pin be too small the bearing surface becomes concentrated at a line, and the greater unit pressure at that point ruptures the film. This is exactly what happens when the pin is too long. The box rapidly wears large at the inner end, and the pressure becomes concentrated along a line as a consequence. The lubricating film then breaks down, and the pin heats and scores. The remedy is not to make the pin longer, so as to reduce the unit pressure, but to decrease its length and to increase its diameter, causing the pressure to be evenly distributed over the entire bearing surface.

The same principles apply to the design of shafts and center crankpins. They must not be made so long that they will allow the load to concentrate at any point. A very good rule for the length of a journal is to make the ratio of the length to the diameter about equal to one-eighth of the square root of the number of revolutions per minute. This quantity may be diminished by from 10 to 20 per cent in the case of crank-pins, and increased in the same proportion in the case of shaft bearings, but it is not wise to depart too far from it. In the case of an engine making 100 revolutions per minute, the bearings would be by this rule from one and a quarter to one and a half diameters in length. In the case of a motor running at 1,000 revolutions per minute, the bearings would be about four diameters long. While the above is not a hard and fast rule which must be adhered to on all occasions, it will be found to be an excellent guide in all cases of doubt.

The diameter of a shaft or pin must be such that it will be strong and stiff enough to do its work properly. In order to design it for strength and stiffness, it is first necessary to know its length. This may be assumed from the following equation:

$$L = \frac{20\,W\sqrt{N}}{P\,K}, \qquad (7)$$

where all the quantities are the same as in the preceding equations. Having found the approximate length by the use of the above equation, the diameter of the shaft or pin may be found by any of the standard equations given in the different works on machine design. It is next in order to recompute the length from formula No. 6, taking this new value if it does not differ materially from the one first assumed. If it does, and especially if it is greater than the assumed length, take the mean value of the assumed and computed lengths, and try again.

Examples of Calculating Dimensions for Bearings.

A few examples will serve to make plain the methods of designing bearings by means of these principles. Let us take as the first case the collar thrust bearings on a 10-inch propeller shaft, running at 150 revolutions per minute, and with a thrust of 60,000 pounds. Assuming that the thrust rings will be 2 inches wide, their mean diameter will be 12 inches. From equation No. 5 we will have for the allowable bearing pressure $\frac{200 \times 700}{12 \times 150 + 700}$, or 56 pounds per square inch. This will require a bearing of 60,000 ÷ 56, or 1,070 square inches area. Since each ring has an area of $0.7854\ (14^2 - 10^2)$, or about 75 square inches, the number of rings needed will be 1,070 ÷ 75, or 14. In case it was desirable to keep down the size of this bearing, the constant K might have had values as high as 1,000 instead of 700.

Next, we will take the main bearing of a horizontal engine. We will assume that the diameter of the shaft is 15 inches, that the weight of the shaft, fly-wheel, crank-pin, one-half the connecting-rod, and any other moving parts that may be supported by the bearings, is 120,000 pounds, and that two-thirds of this weight comes on the main bearing, the remainder coming on the outboard bearing. The engine runs at 100 revolutions per minute. In this case, $W = 80{,}000$ pounds, $P = 400$ pounds per square inch, and K depends on the care and method of lubrication. Assuming that the bearing will be flushed with oil by some gravity system, and that, since the engine is large, the care will be excellent, we will take $K = 1{,}500$. This gives us for the length of the bearing from formula No. 6:

$$L = \frac{80{,}000}{400 \times 1{,}500}\left(100 + \frac{1{,}500}{15}\right) = 26\tfrac{1}{2} \text{ inches (about).}$$

It is to be noted that, in computing the length of this bearing, the pressure of the steam on the piston does not enter in, since it is not a steady pressure, like the weight of the moving parts. The only matter to be noted in connection with the steam load is that the projected area of the main bearing of an engine shall be in excess of the projected area of the crank-pin.

For another example we will take the case of the bearings of a

100,000-pound hopper car, weighing 40,000 pounds, and with eight 33-inch wheels. The journals are 5½ inches diameter, and the car is to run at 30 miles per hour. The wheels will make 307 revolutions per minute when running at this speed, and the load on each journal will be 140,000 ÷ 8, or 17,500 pounds. Although the journal will be well lubricated by means of an oil pad, it will receive but indifferent care, so the value of *K* will be taken as 1,200. The length of the journal will then be

$$L = \frac{17{,}500}{800 \times 1{,}200}\left(307 + \frac{1{,}200}{5.5}\right) = 9\tfrac{5}{8} \text{ inches (about).}$$

As a last example, we will take the case of the crankpin of an engine with a 20-inch steam cylinder, running at 80 revolutions per minute, and having a maximum unbalanced steam pressure of 100 pounds per square inch. The maximum, and not the mean steam pressure should be taken in the case of crank- and wrist-pins. The total steam load on the piston is 31,400 pounds. *P* will be taken as 1,200, and *K* as 1,000. We will therefore obtain for our trial length:

$$L = \frac{20 \times 31{,}400 \times \sqrt{80}}{1{,}200 \times 1{,}000} = 4.7, \text{ or, say, } 4\tfrac{3}{4} \text{ inches.}$$

In order that the deflection of the pin shall not be sufficient to destroy the lubricating film we have

$$D = 0.09 \sqrt[4]{W L^3}$$

which limits the deflection to 0.003 inch. Substituting in this equation, we have for the diameter 3.85 or say 3⅞ inches. With this diameter we will obtain the length of the bearing, by using formula No. 6, and find

$$L = \frac{31{,}400}{1{,}200 \times 1{,}000}\left(80 + \frac{1{,}000}{3\tfrac{7}{8}}\right) = 8.85, \text{ say 9 inches.}$$

The mean of this value, and the one obtained before is about 7 inches. Substituting this in the equation for the diameter, we get 5¼ inches. Substituting this new diameter in equation No. 6 we have

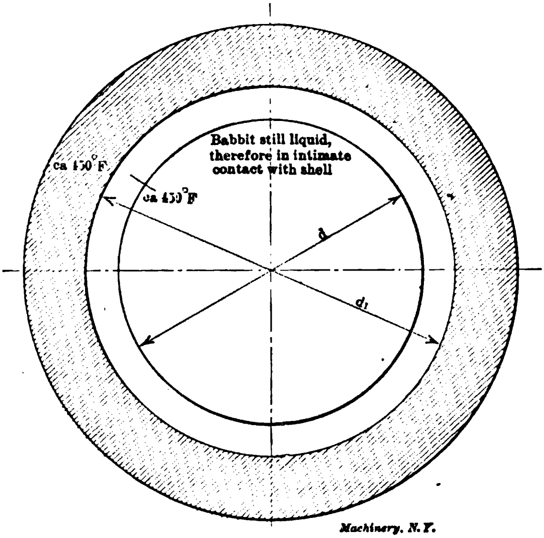
$$L = \frac{31{,}400}{1{,}200 \times 1{,}000}\left(80 + \frac{1{,}000}{5\tfrac{1}{4}}\right) == 7.1, \text{ say 7 inches.}$$

Probably most good designers would prefer to take about half an inch off the length of this pin, and add it to the diameter, making it 5¾ × 6½ inches, and this will be found to bring the ratio of the length to the diameter nearer to one-eighth of the square root of the number of revolutions.

CHAPTER II.

CAUSES OF HOT BEARINGS.

In our modern high speed steam and gas engines, turbines and the like, hot bearings are of more frequent occurrence than is generally supposed. Very often a new plant, just put into service, has to be shut down on this account. It not infrequently happens that the engine which has run "hot" is one of several, identical in design and construction, the bearings in the others having operated without trouble. Apparently there is no cause for this particular engine to give trouble, but in order to remove the difficulty, various makes of babbitt metals and bronzes are tried, sometimes with good results, sometimes without. Again, it occurs that a machine or engine operates at the beginning with perfect satisfaction, but after a time one or more of the bearings begin to run "warm," and finally "hot," so that relining becomes necessary. As a general rule it is then simply accepted as a fact that the bearings "ran hot"; seldom does any one think it worth while to seek out the fundamental causes for the trouble. That there is always the element of doubt, in regard to bearings, is evidenced by the fact that our modern engine builders usually deliver an extra set of bearings with the engine, so that, in the event of trouble, a new set is at hand. The following may be of some assistance towards discovering and


Babbit still liquid, therefore in intimate contact with shell
ca 450°F
ca 450°F
d
d_1

Machinery, N.Y.

Fig. 7. One-piece Bearing; Babbitt just Poured. Both Shell and Babbitt at the Solidifying Temperature.

eliminating, in a scientific manner, and along technical and metallurgical lines, the real causes of hot bearings.

Investigation will show that the main reasons for hot bearings are:

1.—Shrinkage or contraction of the babbitt.

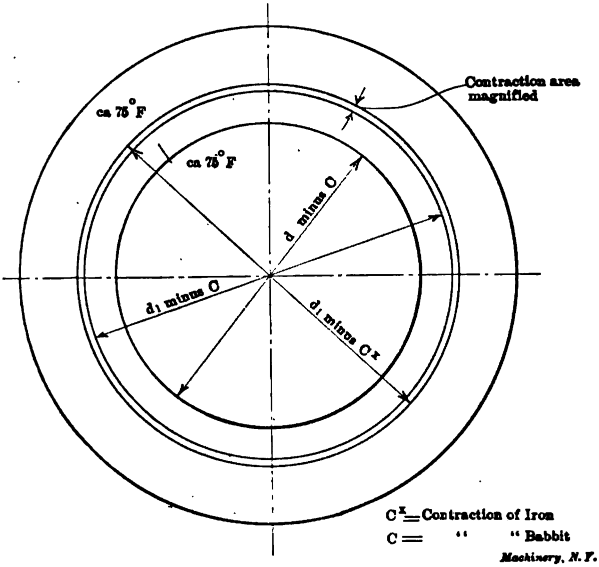
2.—Shrinkage strains set up in the babbitt metal liner by the unequal distribution of the babbitt metal over the shell.

3.—A lack of contact between the babbitt metal liner and the cast iron or cast steel shell.

4.—The lubricant becomes partially deflected into the wrong place.

Shrinkage or Contraction of the Babbitt.

a. Shrinkage in a diametral direction. As an illustration of this point, one may take the simple example of an iron ball and ring. If this ball, when cold, will just pass through an iron ring, it will not do so when somewhat heated and expanded. After cooling down, however, it will again pass through the ring. A similar action takes place in a bearing.



Machinery, N.Y.

Fig. 8. Same Bearing as shown in Fig. 7 Cooled Down to Normal Temperature.

In Fig. 7, of the accompanying illustrations, the babbitt liner may be considered to have been just poured in, and the metal to be still liquid. At the exact solidifying point the babbitt will have filled all the interstices and be in good contact with the cast iron or cast steel shell, provided the babbitt itself has sufficient fluidity to enable it to penetrate the smallest spaces. From this solidifying point on, the babbitt will contract according to its coefficient of contraction. Now, if the coefficient of contraction of the babbitt were the same as that of the material out of which the shell is made (usually cast iron or cast steel), and provided that the shell had acquired the same temperature as the babbitt, the shell and the babbitt liner would then contract equally, and a fairly good contact would result, and there would be nothing to set up counter strains during shrinkage. But, as the coefficient of contraction of almost all babbitt metals is approximately two or three times higher than that of cast iron or cast steel, a shrinkage or loosening of the babbitt liner from the shell must absolutely take place after the solidifying point of the babbitt is reached. Fig. 8 shows this contraction as it would appear if magnified. The fact that most bearings are "split" does not, of course, change this result. If the babbitt is secured in the shell by means of dove-tailed grooves, or other anchoring devices, so that the actual visible contraction from the shell is lessened or minimized, then an unavoidable consequence of these grooves or other devices is *shrinkage strains*, set up while the babbitt cools down, as explained further on.

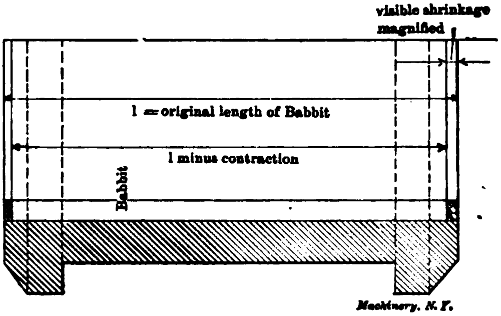


Fig. 9. Babbitt Bearing without Dove-tailed Grooves or other Retaining Device

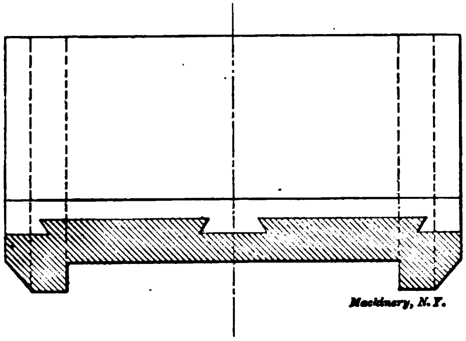


Fig. 10. Same Bearing as shown in Fig. 9, but with Dove-tailed Grooves. Visible Shrinkage Prevented, but Shrinkage Strains Produced.

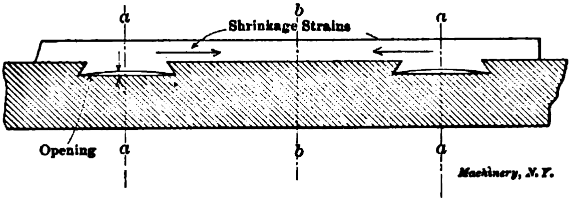
b. Shrinkage in an axial direction. With regard to shrinkage in the axial direction, it may be observed that the same results take place. Fig. 9 illustrates how the babbitt metal shrinks in a cast iron or cast steel shell in the axial direction, when there is no anchoring device employed. In Fig. 10 may be seen the old-fashioned dove-tailed groove construction, prohibiting an actual visible shrinkage, but causing shrinkage strains.

Shrinkage Strains Produced by an Unequal Distribution of Babbitt Metal Liner.

By referring to Fig. 11, it will be observed that the babbitt metal at *aa* is about twice as thick as at *bb*. The consequence is that, as the solidifying time of the greater mass *aa* is longer than that of the smaller mass *bb*, shrinkage strains are set up throughout the babbitt liner, which loosen it from the shell and have the tendency, in combination with the regular working pressures and shocks, to produce minute cracks in the liner.

Lack of Contact between Liner and Shell.

In a bearing shell, some parts of the liner are in close contact with the shell, as a result of careful pouring and the use of a properly made babbitt metal, while other parts of the liner will not be in good contact with the shell, by reason of shrinkage and the formation of air bubbles and oxide gases, which latter are especially liable to be formed in babbitts containing copper. With the idea of filling up the hollow spaces between liner and shell, it is a quite general American practice, and an English one also, to peen or hammer the babbitt liner. The advisability of this treatment is, however, very questionable. By the peening process the air will simply be driven from one point to another, and be forced into places where at first a good contact existed, thus destroying it. To secure a permanent and intimate contact between liner and shell by peening is impossible, on account of the elasticity of the liner material. When the hammer strikes the metal, a contact may be formed, but as soon as the force of the blow is gone, the metal will spring away more or less by reason of its elasticity. Furthermore, the babbitt metal becomes more brittle by peening, and its strength diminished; this has been proved beyond doubt by a number of tests. Peening, unless performed with the utmost precaution, also produces minute cracks in the structure of the babbitt, which will constantly be enlarged by the regular working pressures. For these reasons, European continental practice has now practically abandoned the peening of babbitt metal liners. Summing up, in spite of



Machinery, N.Y.

Fig. 11. Illustration of Shrinkage Strains produced by Unequal Distribution of Babbitt Metal.

good pouring, or peening, or dove-tailed grooves and other similar anchoring devices, the liners are in a greater or less degree loose in the shells.

The Lubricant Penetrating the Hollow Spaces.

When these loose bearings are in service, the hollow spaces between the liner and shell gradually become impregnated with an oil film, from the lubricant employed, as shown in Fig. 12. Now, the coefficient of heat-conductivity of oil is only about 1/200th of that of an ordinary babbitt metal, or of cast iron. Therefore, the heat created in the liner by the working friction will not be conducted away to the shell, and thence to the engine frame, as quickly as though an intimate contact existed between shell and liner. The result is that the bearing readily becomes hot, because the babbitt metal liner retains, instead of throwing off, the heat. The regular working pressure also sets up an hydraulic pressure in the oil film, between the shell and the liner, which tends to produce breakages and cracks in the liner, as may sometimes be observed when removing bearings from gas engines, pumping engines and the like, subject to high pressures and shocks. A consequence of shocks is also that a liner which is somewhat loose will become distorted and "work"; this "working" produces additional friction and increased temperatures. All the facts mentioned above tend toward the one result, *viz.*, the increasing of the temperature in the bearings, even to the extent of melting down the babbitt liner.

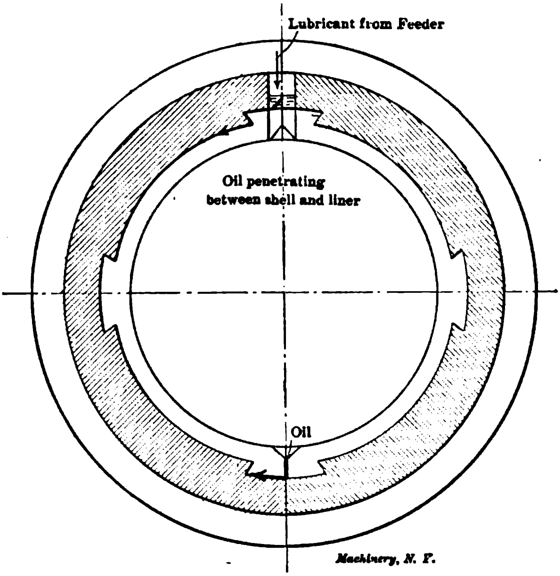


Fig. 12. Penetration of Oil between Shell and Liner.

From various tests which have been made, the results of one may be given here. A bearing with a perfect contact between liner and shell was tested under a constant load of 400 pounds per square inch and a constant sliding speed of 480 feet per minute. The same bearing was again tested under the same conditions, but with the liner not in intimate contact with the shell. As the tests were necessarily made under a slightly varying atmospheric temperature, the difference between the actual bearing temperature and the room temperature was taken as the basis of each, and in the former case, the result was 60 degrees F., while in the latter 85 degrees F. When such differences are obtained in a testing machine, under the best operating conditions, how much worse must be the influence of the slightest lack of contact under usual working conditions, such as we have them in steam engines, air compressors, pumps, gas engines, etc.!

Summing up the foregoing we may say that in most cases the direct causes of hot bearings are: A lack of contact between liner and shell, caused, first, by shrinkage and careless treatment of the babbitt, and second, by shrinkage strains produced by an unequal distribution of the liner masses over the shell; the formation of an isolating oil film, together with its consequences; cracks or breakages in the liner produced as explained. The means of avoiding these troubles, and the principles of a good and safe bearing construction, must consequently be an absolutely intimate and homogeneous contact between liner and shell; an equal distribution of the liner over the shell; and a strengthening of the liner against the shocks and working pressures. If these conditions are faithfully carried out, many troubles and much expense may be avoided.

CHAPTER III.

ALLOYS FOR BEARINGS.

In an important article, in the *Journal of the Franklin Institute* for July, 1903, Mr. G. H. Clamer discussed the advantages and disadvantages of various compositions and alloys for bearings, and especially alloys for railway journal brasses. He also quoted the results of many tests on various compositions made on an Olsen testing machine designed by Prof. Carpenter of Cornell University. The present chapter is devoted to an abstract of Mr. Clamer's article, and contains all the most important features of his discussion on a subject on which not so much is generally known as would be desirable.

Upon close examination we find that there are but few metals available for bearings. They are copper, tin, lead, zinc and antimony. While other metals may be introduced in greater or less proportions, the five mentioned must constitute the basis for the so-called antifriction alloys. The combinations of these metals now used may be grouped under the two heads of white metal and bronze. Bronze is the term which was originally applied to alloys of copper and tin as distinguished from alloys of copper and zinc; but gradually the term "bronze" has become applied to nearly all copper alloys containing not only tin, but lead, zinc, etc., and no sharp lines of demarcation exist between the two.

Principal Requirements of Bearing Metals.

White metals are made up of various combinations of lead, antimony, tin, copper and zinc, and may contain as few as two elements, or all five. Bronzes are made up of combinations of copper, tin, lead and zinc, all of them containing copper and one or more of the other elements. The essential characteristics to be considered in any alloy for bearings are composition, structure, friction, temperature of running, wear on bearing, wear on journal, compressive strength, and cost.

It is utterly impossible to have one alloy reach the pinnacle of perfection in all the above requirements, and so it is important to study the possible compositions and determine for what purpose each is adapted. It has been shown that a bearing should be made up of at least two structural elements, one hard constituent to support the load, and one soft constituent to act as a plastic support for the harder grains. Generally speaking, the harder the surfaces in contact, the lower the coefficient of friction and the higher the pressure under which "gripment" takes place. It would seem for this reason that the harder the alloy the better; and it was with this idea in mind that the alloys of copper and tin were so extensively used in the early days of railroading. A hard, unyielding alloy for successful operation must, however, be in perfect adjustment, a state of affairs unattainable in the operation of rolling stock. For this reason the lead-lined bearing was introduced and the practice of lining bearings has now become almost universal in this country.

General Comparison Between Hard and Soft Alloys for Bearings.

While the harder the metals in contact the less the friction, there will also be the greater liability of heating, because of the lack of plasticity, or ability to mold itself to conform to the shape of the journal. A hard, unyielding metal will cause the concentration of the load upon a few high spots, and so cause an abnormal pressure per square inch on such areas, and produce rapid abrasion and heating.

The bronzes will, generally speaking, operate with less heat than softer compositions, while the softer metals will wear longer than the harder metals. In the matter of wear of journal, however, the soft metals are more destructive. Particles of grit and steel seem to become imbedded in the softer metal, causing it to act upon the harder metal of the journal like a lap. High-priced compositions are being used that have but little resistance to wear compared with cheaper compositions, and low-priced alloys are in service that are not cheap at any price. It is generally conceded that soft metal bearings cause a marked decrease in the life of the journal, and yet they have many marked advantages, as we shall presently see.

Alloys Containing Antimony.

1. *Lead and Antimony:* These metals will alloy in any proportion. With increase in antimony the alloy becomes harder and more brittle. It has been determined that when it is made of 13 parts antimony and 87 parts lead, the composition will be of homogeneous structure. If there is a greater proportion of antimony, free crystals of antimony appear, imbedded in the composition; and if less than 13 per cent, there appear to be grains of the mixture itself imbedded in the lead as the body substance.

According to one writer, an anti-frictional alloy should consist of hard grains, to carry the load, which are imbedded in a matrix of plastic material, to enable it to mold itself to the journal without undue heating. Such a condition would be met in a lead and antimony alloy having above 13 per cent antimony; but it is not advisable to use in any case more than 25 per cent antimony, as the composition would be too brittle. The same writer claims that alloys having from 15 to 25 per cent antimony are the best adapted for bearings.

Mr. Clamer, however, does not agree with this, and says that alloys containing below 13 per cent antimony can likewise be said to consist of hard grains consisting of the composition itself, imbedded in the softer material, lead, as mentioned above. He says: "It has been my experience that, although the friction may be higher in such alloys, the wear is greatly diminished, and where pressures are light, causing no deformation, this is a great advantage. I have seen many instances in service where alloys between 15 per cent and 25 per cent were greatly inferior to alloys between 8 per cent and 12 per cent, owing to their frequent renewal due to wear. It will perhaps be interesting to hear that the Pennsylvania Railroad Company, at the suggestion of Dr. Dudley, their chemist, have adopted the 13 per cent antimonial lead alloy as a filling metal for bearings in order to obtain the best results. In a general way my own work in the subject has confirmed the opinion that lead is the best wear-resisting metal known, and that with increasing antimony, or increasing hardness and brittleness, the wear becomes more marked. This is due to the splitting up of the harder particles."

The friction, as we may naturally expect, becomes less with increase of antimony, and the temperature of running likewise diminished when running under normal conditions; but the harder the alloy, the more difficulty is experienced in bringing it primarily to a perfect bearing, and the greater the liability of heating through aggravated conditions. The wear on the journal one would naturally expect to be decreased with increasing hardness; but this journal wear is in all probability not due so much to the alloy directly, as it is to the fact that the softer metals collect grit, principally from the small particles of steel from the worn journal, and, acting as a lap, cause rapid wear. With the harder metals these particles are worked out without becoming imbedded.

The cost of the lead and antimony alloy is the least which can be produced. It can be used in many services where higher-priced alloys are being relied upon mainly for their high cost. It is one of the greatest extravagances of large industrial establishments to use materials that are too good for certain uses, and even perhaps unsuited, under the supposition that they must be good because they paid a good price for them. This fact has no greater exemplification than in the purchase of babbitt metal, and is due to the great uncertainty which exists, not only among consumers, but among manufacturers, many of whom carry on their business much the same as the patent-medicine man.

2. *Lead, Antimony, and Tin:* It should not be assumed that antimony-lead is the cheapest alloy to use under all circumstances; not so, for when high pressures are to be encountered, tin is a very desirable adjunct. Tin imparts to the lead-antimony alloy rigidity and hardness without increasing brittleness, and can produce alloys of sufficient compressive strength for nearly all uses. The structure of a triple alloy of this nature is quite complicated, and not yet sufficiently defined.

The cost of the alloy increases with increase of tin; but for certain uses, where sufficient compressive strength cannot be gotten by antimony, because of its accompanying brittleness, it is indispensable, and will answer in nearly every case where the tin basis babbitts are used.

3. *Tin and Antimony:* These are seldom used alone as bearing alloys, but are extensively used for so-called Brittania ware, and in equal proportions for valve seats, etc.

4. *Tin, Antimony, and Copper:* This combination is what is known as genuine babbitt, after its inventor, Isaac Babbitt, who presumably was the first man to conceive the idea of lining bearings with fusible

metal. The formula, which for no arbitrary reason he recommended, is as follows:

Tin	89.1
Antimony	7.4
Copper	3.7

This formula is still considered the standard of excellence in the trade, and has been adopted by many of the leading railroads, the United States Government, and many industrial establishments. It is used in the majority of cases where cheaper composition would do equally as well. It is the most costly of all bearing alloys because of the high content of tin.

5. *Tin-Antimony-Lead-Copper:* This quadruple combination of metals cannot be satisfactorily described, as it would no doubt take years of study to fathom the complicity of the metallic combinations here represented. Suffice it to say that lead, although of itself a soft metal, renders this alloy, when added in but small proportions, harder, stiffer, more easily melted and superior in every way to the alloy without it, and yet consumers will raise their hands in horror when a trifling percentage of lead is found in their genuine babbitt. This is one of the instances where cheapening of the product is beneficial.

The foregoing represents the more important combinations of alloys of tin and lead basis. These are of far more importance in the arts than the white metals, the main portion or basis of which is zinc.

At various times new combinations of zinc have been proposed, but, with very few exceptions, they have not come into popular use for two reasons: First, because of the great tendency of zinc to adhere to iron when even slightly heated. What is technically known as galvanizing the journal is effected under these conditions. Second, because of the brittleness produced under the effects of heat, such as is produced by friction when lubrication is interfered with, and consequent danger of breakage.

Bronzes.

Bronze is the term which originally was applied to alloys of copper and tin as distinguished from alloys of copper and zinc.

1. *Copper and Tin:* This, according to our general conception of the word, is a bronze only when the copper content exceeds that of the tin. According to the proportions in which the metals exist, it has widely different properties. In general, the alloy hardens when tin is present up to proportions of 30 per cent, or a little over, and when this limit is exceeded, it takes on more and more the nature of tin until pure tin is reached. From a scientific point of view this alloy is one of the most interesting, and has attracted the attention of many investigators, who have spent years of study on it, to learn its various properties and explain its constitution.

The alloys which interest us most, however, are those which are so constituted as to be adapted for bearing purposes. These would be said to contain from 3 to 15 per cent tin, and from 85 to 97 per cent copper. The alloy of tin containing a small percentage of copper is often used as a babbitt metal, but this comes under the class of white metals, which have already been discussed. Bronze containing above 15 per cent of tin has been recommended at various times for bearings, owing to its hardness, but very unwisely, for such a bearing demands mechanical perfection and perfect lubrication. It has no plasticity of its own, and as soon as the oil film is interrupted, rapid abrasion and "gripment" take place, with hot boxes as the result. The very erroneous idea is still held by many, that to resist wear and run with the least possible friction, a bearing alloy must be as hard as possible. It is true that hard bodies in contact move with less friction than soft ones; but the alloy which is the least liable to heat and cause trouble is the one which will stand the greatest amount of ill use; by this is meant an alloy which has sufficient plasticity to adapt itself to the irregularities of service without undue wear.

The alloys of copper and tin were used extensively some twenty or twenty-five years ago, and were considered the standard for railroad and machinery bearings. The old alloy, known as "Cannon Bronze," containing 7 parts copper and 1 part tin, is still being specified by some few unprogressive railroad men and machinery builders.

Copper and Tin, and Copper, Tin, and Lead Series.

	Copper.	Tin.	Lead.	Friction.	Temp. above Room.	Wear in Grams.
1	85.76	14.90	....	13	50	.2800
2	90.67	9.45	....	13	51	.1768
3	95.01	4.95	....	16	52	.0776
4	90.82	4.62	4.82	14	53	.0542
5	85.12	4.64	10.64	18½	56	.0380
6	81.27	5.17	14.14	18½	58	.0327
7	75?	5?	20?	18½	58	.0277
8	68.71	5.24	26.67	18	58	.0204
9	64.34	4.70	31.22	18	44	.0130

2. *Copper, Tin, and Lead:* This composition is now recognized the standard bearing bronze, its advantage over the bi-compound coming from the introduction of lead. The bronze containing lead is less liable to heat under the same state of lubrication, etc., and the rate of wear is much diminished. For these reasons and the additional fact that lead is cheaper than tin, it seems desirable to produce a bearing metal with as much lead and as little tin as possible. The metal known as Ex. B. composition (tin 7 per cent, lead 15 per cent, copper 78 per cent) is stated to be the best that can be devised. This alloy contains the smallest quantity of tin that will hold the lead alloyed with the copper. By adding a small per cent of nickel, however, to the extent of one-half to 1 per cent, a larger proportion of lead may be used, and successful bronzes have been made by this process, which contained as much as 30 per cent lead. Such bronzes, containing a large amount of lead, through the addition of nickel, are known in the trade as "Plastic Bronzes" and are a regular commercial article. The table above gives the results of tests on different compositions of bronzes.

CHAPTER IV.

BALL BEARINGS.

The following discussion on ball bearings consists of an abstract of a paper by Mr. Henry Hess, presented before the American Society of Mechanical Engineers. It is of particular interest and value, because, although the field of usefulness of the ball bearing is as wide as the domain of mechanical engineering, but little information on ball bearings can be found in the engineering hand books or text books. Generally, the subject is dismissed with occasional reference. Sometimes a formula for carrying capacity is given, but this is usually wrong. There is, of course, much matter scattered through the technical press giving isolated experiences with a few bearings that happen to come within some one's observation. But, undoubtedly, the insufficient information on the elements of ball bearings is responsible for the directly contradictory statements to be found, and the generally accepted opinion that ball bearings are suitable only for comparatively light loads. This was the situation found by Prof. Stribeck, a German investigator, who has been experimenting with ball bearings for the German Small Arms and Ammunition Factories, of Berlin. The present chapter is a résumé of Prof. Stribeck's report, together with a number of notes based on Mr. Hess's own experience.

The Wear of Ball Bearings.

Sliding bearings wear out by abrasion of the carrying surfaces, but ball bearings do not give out on account of wear. In fact, they do not wear. They may be ground out by admitting grit, but that is an abnormal condition for ball bearings, the same as it would be for sliding bearings. The only normal cause for the giving out of ball bearings is the stress on the material, when this stress exceeds certain limits. Lightly loaded bearings can be so designed as to eliminate this cause altogether, and to insure practical indestructibility. In heavily loaded bearings this condition may not be possible to realize within practical dimensions, but the proportions may be so chosen that the stress will not cause a break-down within the lifetime of any mechanism to which the ball bearing is applied. An important principle in the design of ball bearings is that the balls may be subjected to loads, increasing as the shape of the supporting surface more nearly becomes complemental to that of the ball. A ball running between races having a flat or straight line cross section will not support as great a load as if the section were curved, or in other words, if the balls were running in a groove. The groove, of course, must never have a curvature equaling that of the ball, as that would substitute sliding for rolling contact.

Formulas for Loads on Ball Bearings.

The frictional resistance of a ball bearing is lower the less the number of balls. Bearings should be designed to have between 10 and 20 balls. For this number of balls the equation

$$P_o = \frac{5}{z} P_b \qquad (1)$$

should be used. In this equation

P_b = total load on the bearing consisting of one row of balls, in kilograms.

P_o = greatest load on one ball, in kilograms.

z = number of balls.

The load carrying capacity of one ball is $P_o = k\,d^2$, in which

d = the ball diameter, in eighths of an inch,

k = a constant depending upon the material and shape of the ball supporting surface. This constant should vary between 3 and 5 for common materials for ball bearings, and between 5 and 7.5 for improved steel alloys.

As balls are usually made to English measurements, one-eighth inch has been selected as unit for the ball diameters.

From equation (1) we have

$$P_b = P_o \frac{z}{5}, \text{ and substituting } k\,d^2 \text{ for } P_o, \text{ we have}$$

$$P_b = k\,d^2 \frac{z}{5}.$$

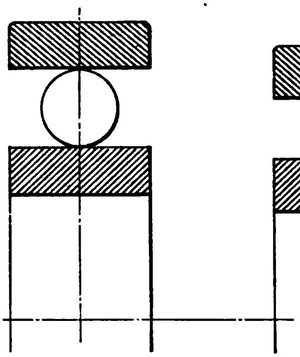
Speed rotation, in as far as it is uniform, does not affect the carrying capacity of a ball bearing. This applies, however, only to radial bearings, but not to thrust bearings of the collar type. In these, the carrying capacity decreases with the increase of speed. Variations in load reduce the carrying capacity, the effect increasing with the amount of load change and the rapidity of the change. Uniformity of ball diameter is very essential as the calculated carrying capacity can be realized only if each of the balls sustains its share of the load. High finish on both ball and ball sustaining surfaces is also essential. The presence of grinding scratches will very materially cut down the load carrying capacity.

Frictional Resistance of Ball Bearings.

The frictional resistance of ball bearings has, by actual measurement, been found to vary from 0.0011 to 0.0095. These are the coefficients of friction referred to the shaft diameter, thus permitting direct comparison with coefficients of sliding friction. Ball bearings having a coefficient of friction materially above 0.0015 under the greatest allowable load should not be recommended, because they are too short-lived. The high resistance indicates the presence of too large an element of sliding friction. The coefficient of 0.0015 for a good ball bearing under its greatest load, independent of the speed within limits, will, however, rise to approximately 0.0030 under a reduction of the load to about 1/10 of the maximum.

The Requirements of a Good Ball for Ball Be

The requirements for a good ball are, in the first
shape and size. The permissible limit of error wil
character of the material. It is evident that where
than the other balls in the bearing, it must be capa
tion sufficiently large so as to permit the others to
load, and for that reason, the smaller the deformati
rate to size must the ball be. In the second plac
surface finish is essential. What is usually cons
finish is in a ball bearing totally inadequate. Gri
marks must not only not be recognizable by the
tected with an ordinary pocket lens, the balls s
This, at least, is true of balls for bearings expec
and to carry heavy loads under high speed. T
balls is that the material out of which they ar



Figs. 13 and 14. Examples of Sim

limit as high as can be had. The unifo
the mass of the ball is also very esse
fact, is one of the most important factor
because some balls in a lot are better t
be based on the poorer ones. Such
better balls carrying more than their
heavy a stress, and possibly breakage
provided it is uniform, is better than
error in regard to the truth and siz
inch.

Radial Be

Other things being equal, it is
surfaces at right angles to the load
bearing such as shown in Fig. 13,
be had from the modification sh
races have curved cross section in
races have the advantage of great
been mentioned. Cutting a local
as in Fig. 15, for the purpose

races, is common practice, but it is not good practice. So long as the loads are low enough, a filling opening may be of no account, but at high speeds and loads this groove is objectionable, since then the catching of the balls at the junction of the filling groove results in damage to the balls, and through these to the race surface.

Thrust Bearings.

In thrust bearings, of the type shown in Fig. 16, the requirement that the sustaining surface should be at right angles to the direction of the load is provided for. These bearings are frequently made with the surfaces *A* and *B* parallel. Providing that these surfaces are made truly parallel, that design is good, but in practice it is seldom possible to get these surfaces truly parallel, because even if such parallelism were possible of attainment, it could not be maintained under the deflections due to the load. It must be remembered that initial errors or deflections of a thousandth of an inch will cause the balls on one side to carry the entire load. For a given case this would demand bearings of larger size than otherwise necessary. By seating the lower collar on a spherical surface as shown at *B*, Fig. 16, this collar can adjust itself in such a way as to distribute the load over the entire number of balls. The speed at which these bearings are run enters decidedly into the carrying capacity of this type of bearing. The utility of these bearings is greatly reduced when speeds exceed 1,500 revolutions per minute.

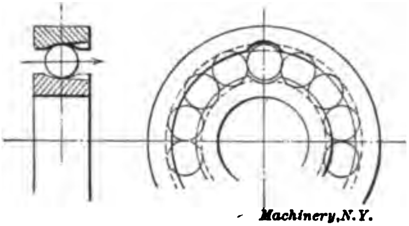

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Fig. 15. Objectionable Filling Groove in Ball Bearing.

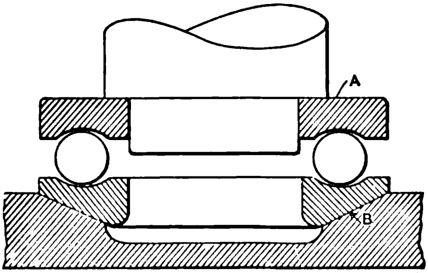

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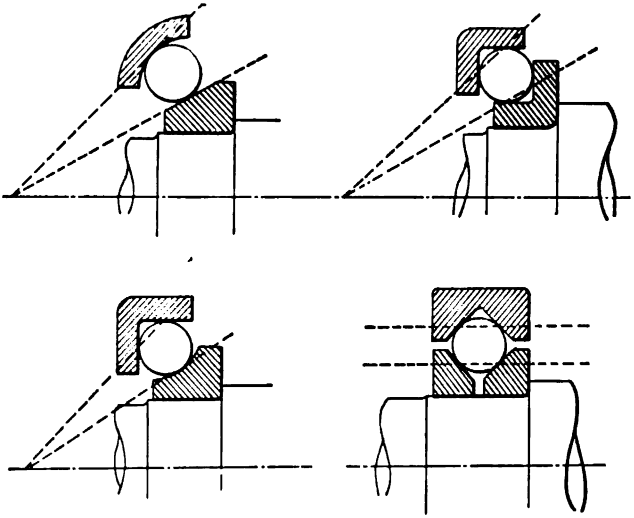
Fig. 16. Collar Thrust Bearing.

Angular Load Bearings.

The shapes and modifications of angular load bearings are innumerable. Figs. 17, 18, 19, and 20 may be taken as typical instances of these bearings, representing 2, 3, and 4-point contacts. In order to secure rolling contact, the contact points of the balls on the races should be points on a cone of rotation whose apex lies in the center line of the shaft, or they may be points on the surface of an imaginary cylindrical roller that would be parallel to the shaft. The defect in all these forms of bearings is their adjustable feature. This places them absolutely at the mercy of every one capable of handling a wrench. A bearing properly proportioned with reference to a certain load may be enormously overloaded by a little extra effort applied to the wrench, or, on the other hand, the bearing may be adjusted with too little pressure, so that the balls will rattle, and the results consequently be unsatisfactory. The prevalent idea that angular ball bearings can be adjusted to compensate for wear is erroneous. The wear will form a groove on the loaded side of the race, deepest at the point of maximum load, and adjusting the cone endwise will only cause the balls to be more tightly pinched between the sound portions of the races, which will most likely cause overload. The rough surface of the groove previously worn will attack the balls, and in due time the entire race and bearing will be destroyed.



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Figs. 17, 18, 19, and 20. Examples of Angular Bearings.

Theoretically, it would seem that a radial bearing would be incapable of carrying thrust load, owing to the wedging of the balls between the races. In Fig. 21, however, is shown the condition of a ball bearing where the ball does not entirely fill the space between the races, if the bearing is not under load, and which, when under load, will assume the position shown. The ball does not come in contact with the race grooves where these are deepest, but so that the tangents to the race curvature at the contact points form angles with the line of thrust.

A calculation of the amount of the wedging action in Fig. 21, with the radial freedom of ball permissible in these bearings, indicates an inadvisably large amount of wedging. Actual running tests, however, as well as a large fund of accumulated experience, have already proved that these bearings will carry much more thrust load than the calculation of the theoretical wedge angle indicates as possible. It is probable that the deformation which occurs at the point of ball contact, which results in small surface *areas* of contact, instead of mere *points*, has a mean tangent to the compression surface of greater inclination, and that the wedge is therefore more blunt. It has been experimentally determined that the thrust carrying capacity of the uninterrupted type of annular bearings is to the radial capacity as 0.1 is to 1, and may be as great as 0.25 to 1, the variation depending upon the ball diameter, race curvature, and number of balls. It has been experimentally found that speed has but a slight influence on the thrust carrying capacity of this class of ball bearings, and for speeds above 1,500 revolutions per minute, these radial bearings are more efficient thrust carriers than the collar type.

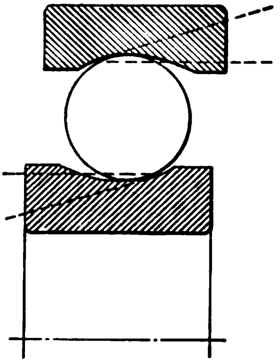

Fig. 21. Radial Bearing used as Thrust Bearing.

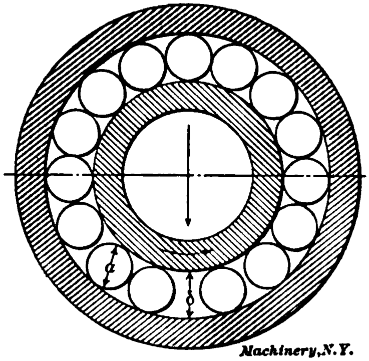

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Fig. 22. Diagram used in Analyzing Condition of Sliding Friction.

The Supposed Sliding Friction in Ball Bearings.

Many designers have supposed that in ball bearings adjacent balls press against one another with considerable force. With the inner race in Fig. 23 running as indicated, the balls will also rotate as shown. The surfaces of the balls run in opposite directions, and therefore rub against one another. This is assumed to be a serious defect by those who reason that these surfaces are in contact under pressure. The same general cure in innumerable forms, as shown in Fig. 24, has been proposed time and again. This cure consists in the provision of smaller balls interposed between the larger ones, so that all the contacting surfaces roll in the same direction relative to one another. This remedy is, however, fallacious, in that it brings about the very condition it seeks to avoid. If two large balls, Fig. 25, compress a smaller one between them, and the three have their centers connected by a straight line, they will retain their relative positions, but if the interposed ball has its center to one side, as in Fig. 26, then this ball will be forced outward. The resort to a cage for retaining the interposed roller or ball results in the latter being pressed against the sides of the cage, and keeps the ball in forcible sliding contact, the very thing that it was intended to avoid. In another design, Fig. 27, the interposed member is brought into contact with the race, and while the various balls have a rolling contact in relation to one another, the interposed member has a wrong direction with reference to the race against which it is forced, and thus a sliding contact is produced. All these designs are based on a failure to recognize that axiom in mechanics according to which a force whose direction is normal to the supporting surface has no component in any other direction.

Analyze the conditions in a ball bearing, and, referring to Fig. 22, suppose that the shaft is loading the inner race, and that the latter is fallaciously assumed to act as a wedge, forcing the balls at the bottom apart and consequently producing pressure between the balls at the top. In that case the space δ must be smaller than the ball diameter a. The rotation of the inner race, however, will carry the balls around the bearing, and the diameter a is therefore forced through the smaller space δ. To do this the ball must lift the inner race. The force to do this is imparted by the load and is equal to the rolling friction, and can, therefore, amount to but a fraction of that load. We would then have the absurd condition of this smaller force overcoming the larger original force. Were we to assume that the inner race is not raised by a ball in passing, but that the ball is compressed sufficiently to get through, it would mean the absurdity that the small force represented by the rolling friction would be sufficient to deform the ball.

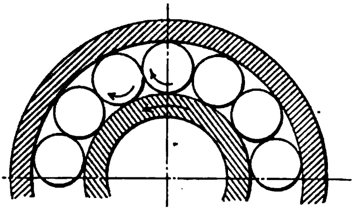

Fig. 23. Diagram showing Direction of Rotation of Balls, indicating Sliding Friction.

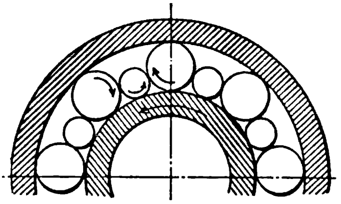

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Fig. 24. Fallacious Means for avoiding Sliding Friction between Balls.

Correct Ball Bearing Mounting.

The following requirements are placed on correctly mounted ball bearings:

a. The proper size must be selected for the load and conditions in question. Rated capacities are usually for steady loads and speeds, but variations from these conditions demand a cutting down of the listed capacity.

b. Bearings must be lubricated. The often repeated statement that ball bearings can run without a lubricant is not correct.

c. Bearings must be kept free from grit, moisture, and acid. No lubricants developing free acids should be permitted.

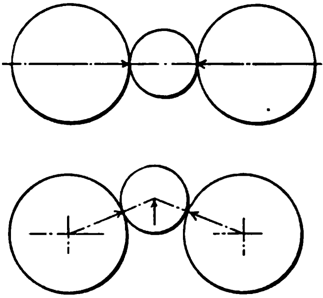
d. The inner race of the bearing should be firmly secured to the shaft. This can be done by a light driving fit, reinforced by binding the race between a substantial shoulder and a nut.

e. The outer race must have a sliding fit in its seat.

These conditions should under all circumstances be adhered to, and a failure to do so will result in very unsatisfactory bearings. The two following conditions are frequently disregarded, and while the disregard of these conditions is not so serious as of those mentioned before, it is safe engineering to follow them, and a disregard of them is a standing invitation to trouble.

f. Thrusts should always be taken up whether in the same or opposite directions by the same bearing.

g. Bearings should never be dismembered, or at least never more than one at a time. That will avoid all danger of mixing the balls from different bearings; such balls from different bearings are very apt to vary more than is permissible for the individual bearing.



Figs. 25 and 26. Analysis of Sliding Friction.

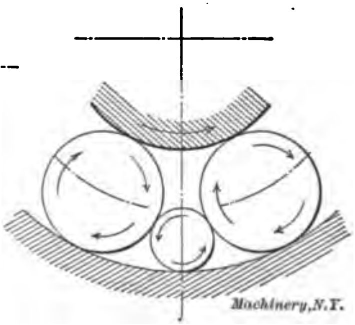


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Fig. 27. Fallacious Means for Avoiding Alleged Sliding Friction.

Illustrations of Correct Mounting.

The development of ball bearings being so recent is probably the cause that so little information as to correct mountings for various conditions is available. The experience of those best informed has been that faulty mountings are very general, and for this reason illustrations have been given of correct mountings, going more into details than would be considered necessary for a more familiar mechanical subject.

Fig. 28 shows a bearing in which the inner race has a light driving fit on the shaft, and is securely clamped between a shoulder on the shaft and a nut. The shoulder on the shaft should be high enough to get a firm grip on the surface of the side of the race. It is good practice to make this shoulder about one-half as high as the race thickness, perhaps a little less for large bearings and a little more for small bearings. The outer race has a tight sliding fit in the housing, so that the bearing as a whole may be able to respond to relative shifting of the shaft and housing without being subjected to end thrust through the balls.

Fig. 29 shows a radially loaded shaft held against endwise motion in either direction. This bearing is capable of carrying thrust load in either direction, but never more than one bearing on the same shaft should be held in this manner. This bearing differs from the preceding mounting only therein that it has the outer race secured between shoulders in the housing. This arrangement and the preceding one are usually found combined on the same shaft, which is then held endwise at one point only, so that temperature changes, or deflections of the shaft, can cause no cramping.

Fig. 30 shows separate radial bearings for radial and thrust loads. This type of bearing is used when it is desirable to take thrust load on bearings of the radial type, although the space available does not permit of a single radial bearing of sufficient diameter to take both loads. One bearing is then mounted entirely free circumferentially so as to take the radial load, while the other bearing is mounted between shoulders, and takes the thrust load.

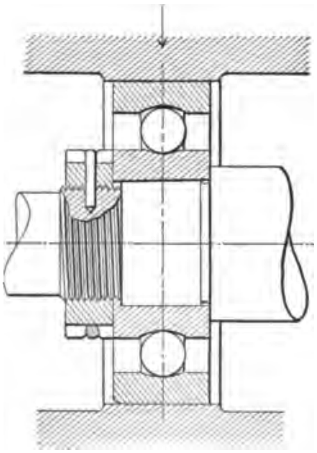

Fig. 28. Free Mounting for Radially Loaded Bearing.

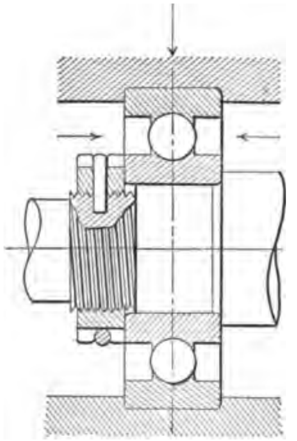

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Fig. 29. Radially Loaded Bearing held against Longitudinal Motion.

Figs. 31 and 32 show thrust loads on a collar bearing in one direction only. Here the stationary race is provided with a spherical seat so that it will distribute the load over the complete circle of balls. In order to permit compensating shifting, the fixed seat must be radially free of the shaft and of the housing. The shoulder on the shaft should be large enough so as not to permit any bending strains on the rotating race. When it is inconvenient to provide a large enough shoulder on the shaft, a washer can be inserted between the shoulder and the race, as shown in Fig. 32. In Fig. 33 is simply shown a modification of the bearings in Figs. 31 and 32. This bearing takes thrust loads in two directions on two collar bearings. Fig. 34 shows an arrangement by which thrust load in two directions may be taken up by a single collar bearing. This arrangement is one which economizes space, cost of bearings, and number of parts. Fig. 35 shows an arrangement where a radial bearing is used for taking the radial thrust, and a collar bearing is used for taking the end thrust. Fig. 36 shows an arrangement for taking up radial load as well as thrust load in two directions, these loads being carried on one radial bearing and two collar bearings. In this design attention may be called to the distance piece inserted for binding the inner race of the radial bearing against the shoulder of the shaft.

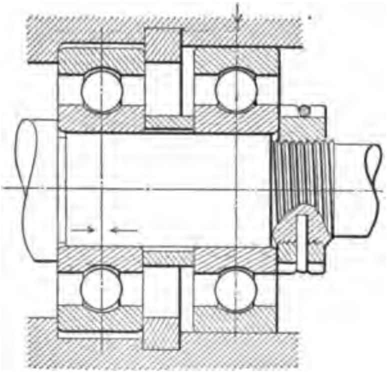

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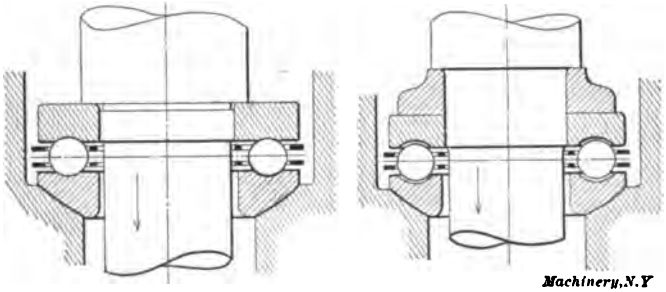
Fig. 30. Separate Radial Bearings for Radial and Thrust Loads.

It is occasionally inconvenient to arrange the bearing so that the parts of the inner races can be clamped to the shaft, or it may be desirable to have a shaft sliding through the bearing. In such cases a sleeve may be introduced on which the inner race of the bearing is firmly clamped endwise, the shaft simply resting in this sleeve. This gives a long bearing to the shaft, which would not be possible if the shaft was directly mounted in the ball race, because the peening effect of the vibrating loads would, even if the race itself was prolonged, be concentrated on a narrow zone of the shaft. A bearing of this kind is shown in Fig. 37.



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Figs. 31 and 32. Thrust Load in One Direction on Collar Bearings.

In Fig. 38 is shown a bearing which is intended for shafting which may not be fully to standard size. The inside of the ball race is tapered, and a split bushing, tapered on the outside as shown, will, when tightened, bind the shaft as well as the race to it, and will compensate for all variations in size. The nut should not be used to draw the bushing in, but should merely act as a lock to hold it in place after it has been driven home with a soft hammer.

Ball bearings should always be enclosed so that lubricant will not be lost by leakage, and so that foreign matter will be excluded. Fig. 39 shows an efficient way of enclosing a ball bearing without using any packing. At the end where the shaft passes out of the enclosure, a flange should be bored out about 0.020 inch larger in diameter than the shaft. This flange should be separated into two lips by an angular groove, either cored or bored, as shown at *A*. These lips should not be less than ⅛ inch wide and should have sharp edges. The groove should be provided with a hole or narrow slot *B* at its lowest point to communicate with the bearing oil space. The groove itself should have a width of not less than 3/16 inch, and a depth of about 5/16 to ⅜ inch, and should not be filled with packing material. Fig. 40 shows

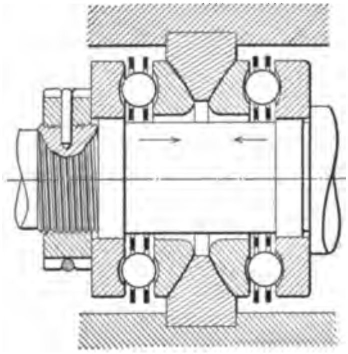

Fig. 33. Thrust Load in Two Directions on Two Collar Bearings.

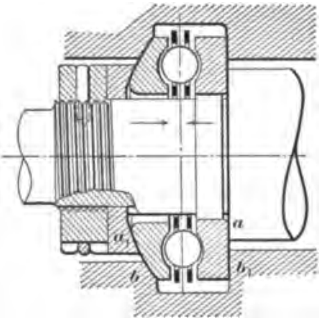

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Fig. 34. Thrust Load in Two Directions on One Collar Bearing.

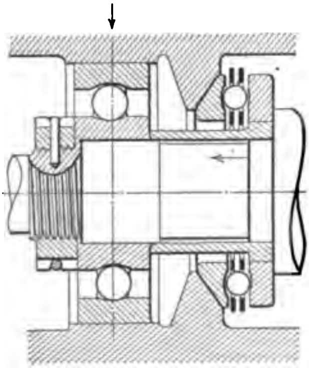

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Fig. 35. Radial Load, and Thrust Load in One Direction.

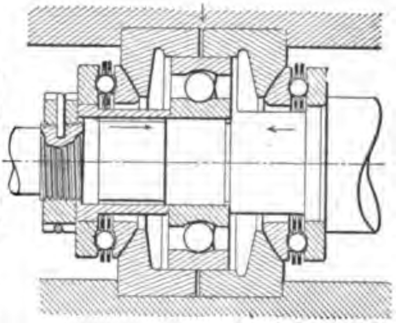

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Fig. 36. Radial Load, and Thrust Load in Two Directions.

an arrangement of a similar kind, excepting that here is introduced a second groove and a third lip. This arrangement is employed where water may be occasionally encountered, and will prevent its entrance. What little may find its way past the outer lip into the outer groove is soon drained out again through the holes provided.

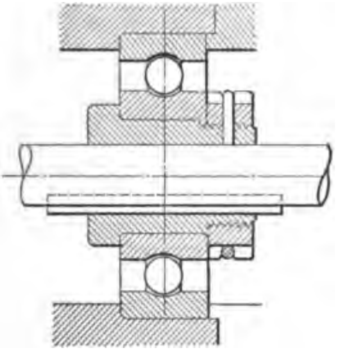

Fig. 37. Shaft Free in Longitudinal Direction in Inner Race.

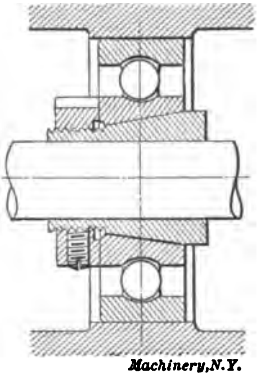

Fig. 38. Adapter Bearing for Shafting varying from Standard Size.

Where much grit is encountered, as in grinding machinery, a packing may be necessary, and filling the outer groove with a fairly consistent grease will provide such a packing without introducing friction. A bearing of this kind is shown in Fig. 41. A grease cup of the spring loaded piston type will automatically maintain the integrity of this packing. In some cases felt ring packings may be used, but these ought to be soaked in good soft paraffine, and a spring wire ring should be placed around the felt washer so as to force the outer edges of the washer outward, which will cause the felt to come into more intimate contact on the sides. Felt washers may also be applied as shown in Fig. 43. Here the washer is tapered on one or on both sides, and the

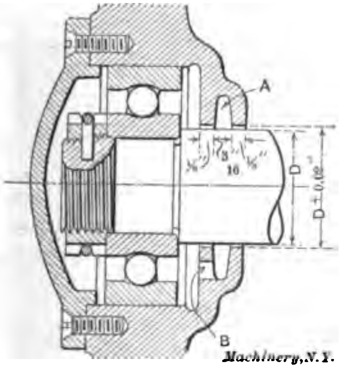

Fig. 39. Example of Enclosed Bearing.

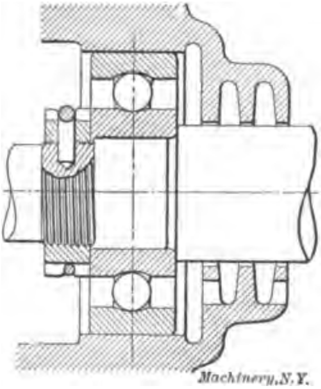

Fig. 40. Bearing Enclosure with Double Groove.

sealing of the enclosure is made entirely against the sides of the surfaces against which the felt washer bears. The felt washer is pressed inwards by means of springs on the outside. The two modifications shown in Figs. 42 and 43 are intended to be inserted between the faces of a stationary boss and a rotating hub. The modification in Fig. 44, however, is enclosed entirely within one or the other. A felt ring is set into a counterbore and held in place by a light metal cap sprung into position.

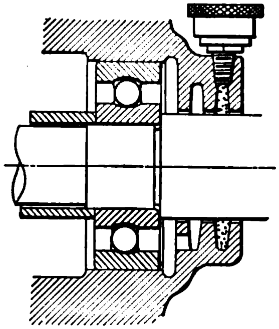

Fig. 41. Enclosed Bearing with Grease Packing.

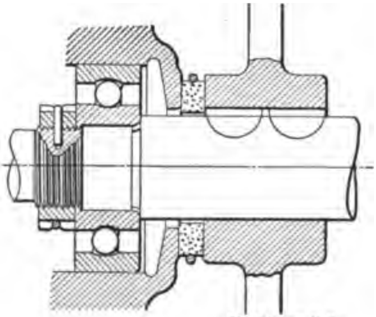

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Fig. 42. Felt Ring Packing.

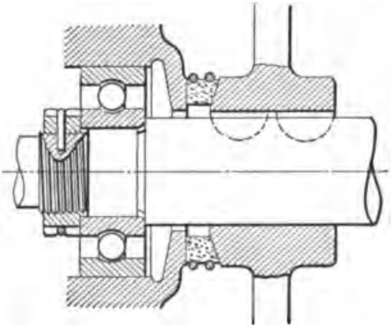

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Fig. 43. Angular Felt Ring Packing.

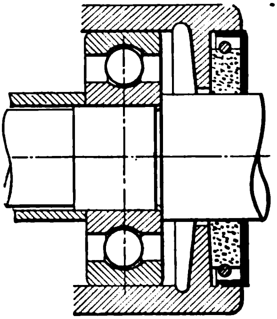

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Fig. 44. Modification of Felt Ring Packing in Fig. 42.

CHAPTER V.

FRICTION OF ROLLER BEARINGS.

During the years 1904-05 a series of tests on roller bearings was conducted at the Case School of Applied Science, Cleveland. A complete report of these tests was published by Professor C. H. Benjamin in the October, 1905, issue of MACHINERY, of which the following is an abstract. An attempt was made in these experiments to compare roller bearings with plain cast iron bearings and with babbitted bearings under similar conditions. Four sizes of bearings were used in the tests, measuring respectively 1 15-16, 2 3-16, 2 7-16 and 2 15-16 inches in diameter. The lengths of journals were four times the diameters.

TABLE I.

Journal 1 15-16 inches in Diameter. 480 Revolutions per minute.

Total Load, pounds.	FRICTION.		VALUES OF f.	
	Hyatt	Plain.	Hyatt.	Plain.
64.2	2.34	10.24	.036	.160
114.2	3.27	12.10	.029	.106
164.2	4.21	19.10	.026	.116
214.2	4.78	22.35	.022	.104
264.2	5.15	26.10	.019	.099
Average	....	....	.026	.117

In the first set of experiments Hyatt roller bearings were compared with plain cast iron sleeves at a uniform speed of 480 revolutions per minute, and under loads varying from 64 to 264 pounds. The cast iron bearings were thoroughly and copiously oiled, the lubrication being rather better than would be the case in ordinary practice. Table I shows the results of the test on one bearing in detail, and from this it is seen that the value of f, the coefficient of friction, diminishes as the load increases.

TABLE II.

Values of Coefficient of Friction f. Speed 480 Revolutions per minute.

Diameter of Journal.	HYATT BEARING.			PLAIN BEARING.		
	Max.	Min.	Ave.	Max.	Min.	Ave.
$1\frac{15}{16}$	.036	.019	.026	.160	.099	.117
$2\frac{3}{16}$	.052	.034	.040	.129	.071	.094
$2\frac{7}{16}$	.041	.025	.030	.143	.076	.104
$2\frac{15}{16}$	.053	.049	.051	.133	.091	.104

Table II gives a summary of this series of experiments for the different sizes of journals, the different loads being the same as in Table I. The relatively high values of *f* in the 2 3-16 and 2 15-16 roller bearings were due to the snugness of the fit between the journal and the bearing, and show the advisability of as easy a fit as in ordinary bearings.

TABLE III.

Journal 1 15-16 inches in Diameter. Speed of 560 Revolutions per minute.

Total Load.	Friction.			Value of *f*.		
	Hyatt.	McKeel.	Babbitt.	Hyatt.	McKeel.	Babbitt.
113.3	3.64	3.77	8.38	.032	.033	.074
162.3	3.77	4.24	8.97	.023	.026	.055
211.3	4.04	5.24	8.97	.019	.025	.042
260.3	4.31	5.37	8.97	.016	.021	.034
309.3	4.57	6.46	10.15	.015	.021	.033
358.3	4.71	6.73	10.75	.013	.019	.030
407.3	4.84	7.27	11.98	.012	.018	.029
456.3	37.70	7.81	20.90	....	.017	.046
Averages	....	....	....	.0186	.0225	.043

The same Hyatt bearings were used in the second set of experiments, but were compared with the McKeel solid roller bearings and with plain babbitted bearings freely oiled. Table III shows the detailed results of experiments on one size of journal, and is similar to Table I.

Under a load of 358.3 pounds the solid roller bearing showed an end thrust of about 20 pounds, which would account for the difference in friction between that and the Hyatt. Table IV gives a summary of the tests in this series and may be compared with Table II. The relatively high values for the Hyatt 2 7-16 bearing must be due to a slight cramping of the rolls due to too close a fit, as was noted in some of the former experiments. Under a load of 470 pounds, the Hyatt bearings developed an end thrust of 13.5 pounds and the McKeel one of 11 pounds. This end thrust is due to a slight skewing of the rolls and would vary, sometimes even reversing in direction.

TABLE IV.

Values of Coefficient of Friction f. Speed 560 Revolutions per minute.

Diameter of Journal.	Hyatt Bearing.			McKeel Bearing.			Babbitt Bearing.		
	Max.	Min.	Ave.	Max.	Min	Ave.	Max.	Min.	Ave.
$1\frac{15}{16}$	.032	.012	.018	.033	.017	.022	.074	.029	.043
$2\frac{3}{16}$	.019	.011	.014	......	......	......	.088	.078	.082
$2\frac{7}{16}$	.042	.025	.032	.028	.015	.021	.114	.088	.096
$2\frac{15}{16}$	.029	.022	.025	.039	.019	.027	.125	.089	.107

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No. 13

BLANKING DIES

CONTENTS

Introduction - - - - - - - - - - 3

Method of Making Blanking Dies, by C. F. EMERSON - 5

Blanking and Piercing Dies for Washers, by C. F. EMERSON - - - - - - - 12

Making Blanking Dies to Cut Stock Economically, by C. F. EMERSON - - - - - - - - 18

Construction of Split Dies, by C. F. EMERSON - - 26

Novel Ideas in Die Making - - - - - - 29

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INTRODUCTION.

It is rather difficult to classify and give proper definitions of the many varying kinds and types of dies used on the power press for rapid production of duplicate work. While there are, of course, some general classes into which all tools of this description may be divided, the various types overlap, so to say, and one is sometimes in doubt as to the proper classification of tools which combine the features of different types. In the following, however, the distinctions between the main types have been pointed out in general outlines, the definitions being broad enough to permit of adjustment according to special conditions.

All dies may, in the first place, be divided up into two general classes: *cutting dies* and *shaping dies*. Cutting dies include all dies which simply cut or punch out required pieces of work from the stock fed into the press, without changing the condition or form of the stock in the plane in which it was located in the material from which it is cut. Shaping dies include all dies which change the form of the material from its original flat condition, producing objects in which the various surfaces are not in the same plane. The last mentioned main subdivision often includes also the characteristics of the first, that is, some shaping dies are, in fact, a combination of cutting and shaping dies, the blank for the work to be shaped or formed being first cut out to the required outline from the stock, and then shaped to the desired form.

The main classes of dies, as will be recognized, are based on the *use* of the dies. The first of the classes mentioned, cutting dies, may, however, be further subdivided according to the *construction* of the various types of dies in this class. We then distinguish between four distinct types, *plain blanking dies*, *follow dies*, *gang dies*, and *compound dies*.

Plain blanking dies are the simplest of all types of dies, and are used to cut out plain, flat pieces of stock having, in general, no perforations, the work being turned out complete at one stroke of the press.

Follow dies, not infrequently also termed tandem dies, are used for work which must be cut out from the stock to required shape, and at the same time be provided with holes or perforations of any kind. The principle of the follow die is that while one part of the die punches the hole in the stock, another part punches out the work at a place where at a former stroke a hole has already been punched, so that a completed article results from each stroke of the press, but, in reality, two operations have been performed on the work before completion. The follow die cannot be depended upon to turn out very accurate work, because it depends largely on the skill and care of the operator for the production of duplicate work. In both the plain blanking and the follow dies, the punch, or upper member, and the die, or lower member, of the complete tool, are distinct elements, the work being cut out or perforated by the entering of the punch into the holes provided for it in the die.

Gang dies are used when several blanks are punched out simultaneously from the stock. The advantage of the gang die over the plain blanking die is the saving of time.

Compound dies differ from plain blanking and gang dies in that the simple punch and die elements are not separated, one in the upper and one in the lower half of the complete tool, but these elements are combined so that both the upper and the lower part contain each a punch and a die. The faces of both punches, dies and strippers are normally held at the same level, and the strippers are spring supported so as to give way when the stock is inserted between the faces, and the press is in action. The springs are so adjusted that they are strong enough to overcome the cutting resistance of the stock, after which they will be compressed until the ram reaches the end of its stroke. A compound die produces more accurate work than the three types previously referred to, for the reason that all operations are carried out simultaneously at one stroke, while the stock is firmly held between the spring-supported opposing die faces. The disadvantage of the ordinary compound die is the difficulty encountered in "setting up," and the complexity of the design, which usually requires more or less frequent repairs.

The second main division of dies, the shaping dies, cannot be subdivided according to the construction of the dies in the same manner as the cutting dies. Shaping dies are usually designed more or less on the compound principle, outlined above, but owing to the great variety of work performed in shaping dies, the designs vary too greatly for a classification on the basis of constructional features. They may, however, be divided into sub-classes according to the general use to which they are put. We would then distinguish these four main subdivisions: *bending dies*, *forming dies*, *drawing dies*, and *curling dies*.

Bending dies are used when part of the surface of a piece of work is pushed from its original plane into a new shape in such a manner that the bent work does not form a closed curve.

Forming dies are used when the blank is required to be formed into a hollow shape, by being pushed into a cavity in the die.

Drawing dies are used for the same purpose as forming dies, but the process differs therein that an outer portion of the flat blank to be formed is confined between two rigid flat surfaces, so that, when drawn radially inwards from between them, no wrinkles can form.

Curling dies are used for bending over the ends or edges of the work into a circular cross section, like the turning over of the edges of hollow objects of sheet metal, etc.

Finally, we must mention the sub-press die, which, however, cannot be defined as a special class of die, but merely as a principle on which all the different classes of dies, cutting as well as shaping dies, may be worked. The sub-press principle is simply that the upper and lower portion of the die, the punch and die, are combined into one unit by guide rods fastened into the lower part of the die and extending through holes in the upper part, or by some other provision for guiding. This construction permits of a high degree of accuracy, eliminates the necessity of lining up the punch and the die each time they are set up on the press, and thus saves a great deal of time and cost.

In the following, we shall, however, deal only with the simpler forms of cutting dies, plain blanking and gang dies, except in Chapter V, where reference will also be made to some of the more complicated types of dies.

CHAPTER I.

METHOD OF MAKING BLANKING DIES.

From a mechanical standpoint it can truthfully be said that we are living in an age of dies. Never before has the industrial world made use of the punch and die as it is doing to-day. And no wonder; for this useful tool in all its different phases has proved beyond all reasonable doubt that it can turn out more work in less time than the combined efforts of a room full of milling machines, shapers, and drill presses. To those who are unfamiliar with the die and its work the above may not appear feasible; but one has only to visit a modern

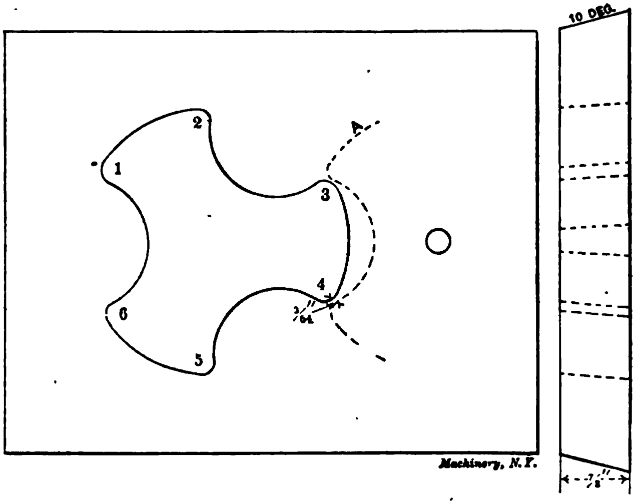

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Fig. 1. Die used as Example in Illustrating Principles of Making Blanking Dies.

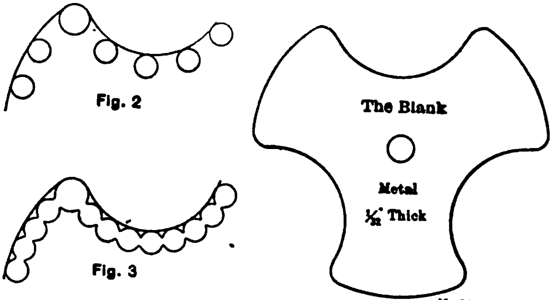
sheet metal factory to be convinced of the surprising rapidity with which the power press with its punches and dies will turn out not only work of all kinds of shapes and sizes, but accurate work as well.

Of the many different kinds of dies in use, the blanking die is probably the most widely employed. The reason for this is that almost all work that requires the use of various other kinds of dies has its beginning with the blanking die; for it is this die that cuts the work from the flat stock before it is completed by the other dies. In making the blanking die there are a few essential points to be taken into consideration, among which are the following:

1. Use good tool steel of a sufficient length, width, and thickness to enable the die to hold its own.

2. In laying out the die, care should be taken that as little of the stock as possible is left over, as waste, in cutting out the blanks.

3. Be sure not only that the die has the proper amount of clearance (which should be no more than two degrees and no less than one degree); but also that the clearance is filed *straight*, so as to enable the blanks to readily drop through.



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Figs. 2 and 3. Method of Removing Surplus Stock or "Core."

4. In working out the die, machine out as much as possible; don't let the file do it all.

5. In hardening the die, do not overheat the same, as the cutting edge of a die that has been overheated will not stand up to the work, and requires so much sharpening in order to produce perfect blanks, that at its best it is nothing more than a nuisance.

In laying out the blanking die, the face of the die is first polished smooth and drawn to a blue color by heating. This gives better satisfaction by far than using coloring acid, for it gives a clear white line on a dark surface to work to, and is easier on the eyes, particularly when working by artificial light as is often necessary. When the die to be laid out is a blanking and piercing die, allowance of 3/64 inch must be made for the "bridge," *i. e.*, the narrow strip of metal that separates the holes in the stock from which blanks have already been cut. Fig. 1 shows how this is done; the dotted line *A* is drawn merely to show how the die is laid out.

After the die is laid out it is ready to be worked out. Now there are several different ways of working out the surplus stock in a die of this kind. One is to drill say a half-inch hole at a safe distance from the line, and then fasten the die in a diemaker's milling machine and mill out the stock close to the line with a taper milling cutter, which gives the die the necessary clearance, thereby saving considerable time when filing out the die.

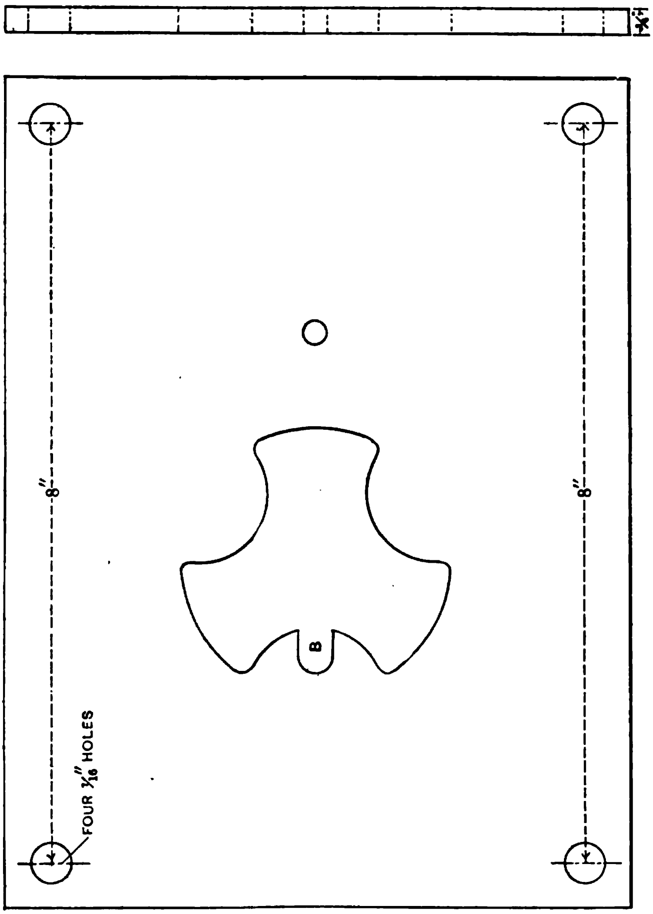

Fig. 4. Stripper Plate.

Another method, which is most commonly used, is to drill out the surplus stock on a drill press, after the manner shown in Figs. 2 and 3, which is done as follows: The six holes for the corners numbered 1, 2, 3, 4, 5, 6, Fig. 1, are first drilled and reamed taper, after which the other holes are drilled. These holes are drilled an even distance apart, and must therefore be spaced off, and then spotted with a prick punch before they are drilled. The best way to do this is to first scribe an inside line at a distance from the outside line equal to one-half the diameter of the holes to be drilled, then space off, and spot. In spacing off, do not use dividers, but use a double prick punch. Using a pair of dividers requires too much time, besides the points get dull quickly enough without using them when it is unnecessary.

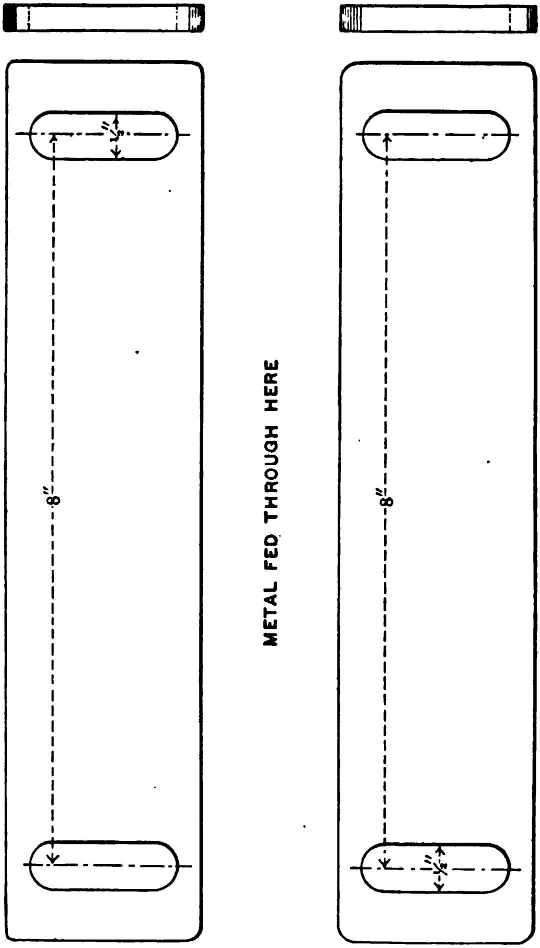

Fig. 5. Gage Plates.

After the centers have been lightly spotted with the double prick punch, use an ordinary prick punch and make the spots a trifle deeper, so that the drill will more easily take hold.

In drilling, use the method shown in Figs. 2 and 3, for in this way the holes can be drilled closer together, thereby making it easier to get rid of the surplus stock and saving the time of broaching out the webs. The die blank should be slightly tipped by placing a narrow strip of flat stock under the edge of same, as shown in Fig. 6, when the die is being drilled. This is done to give the necessary clearance, and does away with that time-killing operation of reaming the holes with a taper reamer from the back after they are drilled. After the surplus stock is gotten rid of, the die is finished up by filing, using a coarse file to begin with, and finishing with a smooth one.

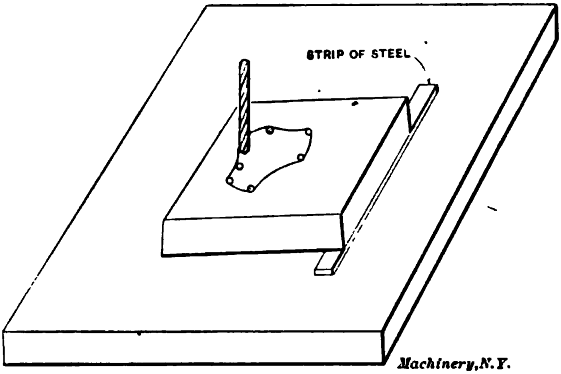

Fig. 6. Method of Obtaining Clearance when Drilling out the "Core."

Usually the die is made to fit a sample blank or a templet. This is done by entering the templet from the back as far as it will go after the die has been filed to the inside of the line. A lead pencil is then used to mark those parts of the die where the templet bears.

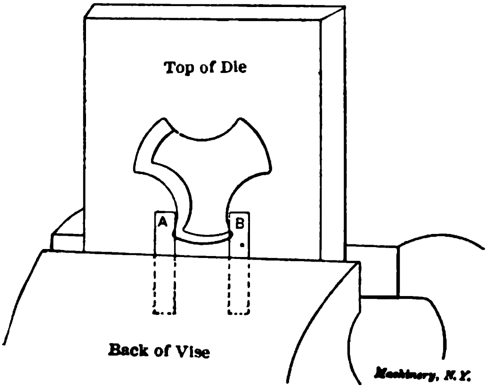

Fig. 7. Guarding the Corners in Filing the Die.

The templet is then removed, the pencil marks filed out, the templet again entered and so on, until it is worked through the die. In filing out a die of this kind, where there is any danger of injuring that part of the die which has already been finished, use two strips of sheet steel, *A* and *B*, in the manner shown in Fig. 7, the round corners which are already finished being thus protected from the edges of the file.

In hardening the die, heat it to a cherry red, preferably in a gas furnace or a clean charcoal fire, and dip endwise into the solution used for hardening. When the die is sufficiently cold so that it can be taken hold of by the hands, withdraw it quickly and place it on the fire until it has become so warm that it will make water sizzle when dropped thereon; then immerse once more until cold. This is done to relieve the internal strains caused by hardening, and acts as a preventive to cracking. The face of the die is now polished, and the temper drawn to a light straw color, after which the die is allowed to cool of its own accord in oil. When cool, the die is ground on a surface grinder on the top and bottom, and if it is required it is lapped to size, which completes the operations.

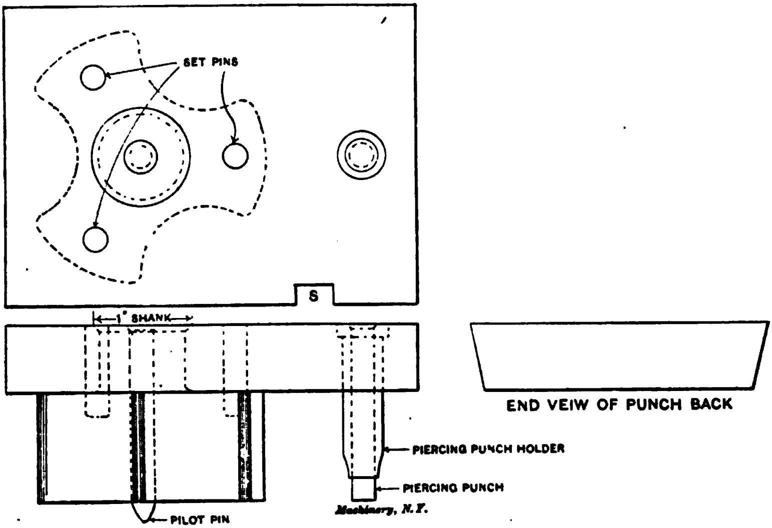

Fig. 8. Punch used with Die in Fig. 1.

The punch is made after the manner shown in Fig. 8, and needs very little explanation. The dovetail punch back shown holds the punches in position, and is securely held in the press by the aid of a key. The slot *S* forms a position stop by engaging in a stud in the dovetail channel in the ram of the press, thereby eliminating the necessity of again resetting the tools in case the punch requires sharpening. The blanking punch is made from a tool steel forging, and is machined and sheared through the die in the usual manner. The one-inch shank is made a good driving fit in the punch back, and is upset as shown after the punch is driven in. The three set pins help to more securely hold the punch in position, and prevent it from turning.

The piercing punch is held in position by the piercing punch holder, which is driven tight in the punch back. The piercing punch is lightly driven in, and is made of drill rod, and can be very readily replaced

in case it is broken. The pilot pin is also made of drill rod, and can be very easily and quickly taken out when the punch requires sharpening.

The stripper and gage plates for this die are shown in Figs. 4 and 5. They are fastened by four 7/16 cap screws to the die bed, used for holding the die in position when in use, and form, without doubt, not only the best, but by far the cheapest of the various methods employed for this purpose. While this method cannot be used on all kinds of blanking dies, it can, however, be used with the best of results on dies similar to the one described, and eliminates the unnecessary operation of drilling and tapping holes in the die itself to hold the stripper and gage plates in position. Not only that, but the gage plates as shown are used in connection with many other dies of a similar nature, thereby doing away with the necessity of having a set of gage plates for every die, as would otherwise be the case.

As the illustrations speak for themselves, no more explanation seems necessary, except perhaps that the slot *B* shown in Fig. 4 is to allow for an automatic finger to act as a position stop for the metal when it is run through.

CHAPTER II.

BLANKING AND PIERCING DIES FOR WASHERS.

One of the simplest dies to make, coming under the head of blanking and piercing dies, is perhaps the die for blanking and piercing brass washers. The reason for this is that in making this die, the file and vise are not used; the construction and shape of this die are such as to allow it to be made by machinery. To lay out a single washer die is a very easy matter, but to lay out a die for cutting two or more washers at one time, so as to cut the greatest amount of blanks from the least amount of stock, is not understood as it should be. One of the reasons for this is that it is the custom in some shops to have the foreman, or some one else appointed by him, lay out all the dies before they are given to the die-maker to work out.

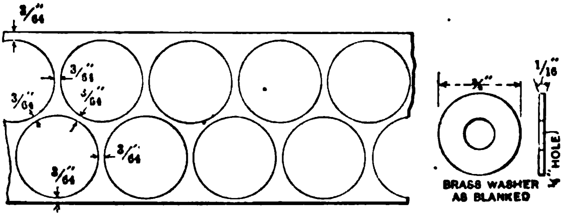

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Fig. 9. Stock after having been run through the Die in Fig. 10, and Washer to be made.

In laying out a washer die for blanking two or more washers at one time, one of the main points to be remembered is that all the holes from which the blanking and piercing are done must be laid out in an exact relation to each other, so as to eliminate the possibility of "running in" (*i. e.*, cutting imperfect, or half blanks, by cutting into that part of the metal from which blanks have already been cut). The required amount of blanks must also be considered, for it sometimes happens that the amount wanted does not warrant the making of a die that will cut more than one at a time.

Fig. 10 shows how a die is laid out for blanking and piercing two washers at one time, so as to use up as much of the metal as possible. As shown, the ¾-inch holes marked *C* and *D* are the blanking part of the die, while the ¼-inch holes *A* and *B* are the piercing part. The distance between the center of *C* and *A* is 51/64 inch, as is also the distance between *D* and *B*. By referring to Fig. 9, which shows a section of the stock after it has been run through this die, it will be seen that there is a narrow margin of 3/64 inch of metal, known as "the bridge," between the holes. In laying out the die this margin must be taken into consideration, which is done in this manner: diameter of washer to be cut plus bridge equals distance from center to center, *viz.*, $3/4 + 3/64 = 51/64$. The dotted circle shows that the die is laid out so that one washer is skipped in running the metal through at the start. This is done in order to make the die a substantial and strong one. It can be very readily seen that if the circle *E* was the blanking part instead of *D*, the die would be a frail one, and would not be strong enough for the work for which it is intended.

Another important point in laying out a die of this kind is to lay out the die "central," *i. e.*, laying out the die so that when it is keyed in position ready for use in the center of the die bed, it will not have to be shifted to the right or left side in order to make it line up with the punch. It may not be amiss to say in connection with the above that the punch back which holds the blanking and piercing punches in position should also be laid out "central"; this will be more fully described later on.

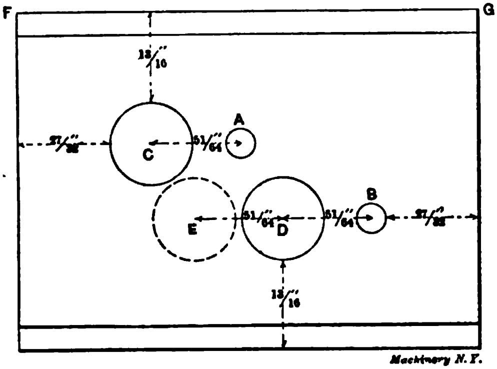

Fig. 10. Plan View of Die for Punching two Washers Simultaneously.

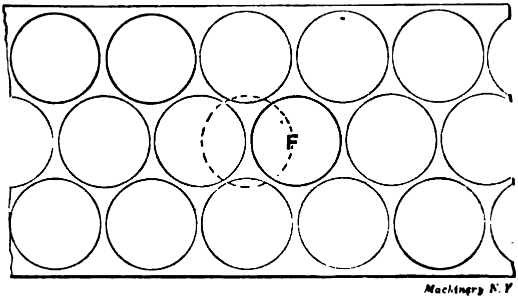

Fig. 11. Stock after having been run through Die in Fig. 12.

Fig. 12 shows the layout for blanking and piercing three washers at one time, and hardly needs any explanation; the explanation given in connection with Fig. 10 sufficiently explains Fig. 12.

Fig. 11 shows a section of the stock after it has been run through this die. It can be seen that the holes match in very closely together, and that very little stock is left. It is also seen that the three holes punched are not in a straight line, in so far as the width of the metal is concerned. This is done in order to save metal; the dotted circle *F* is merely drawn to show that wider metal would have to be used if the holes were in a straight line.

Fig. 13 shows the plan of a die for blanking and piercing eight washers at one time. The parts which are numbered are the blanking parts, while the parts that are lettered are the piercing parts of the die. This die is laid out similarly to Fig. 12, with the exception that there is provision for eight blanks instead of for three. Fig. 14 shows a section of stock after it has been run through this die. To give a better idea as to how the blanks are punched out in the manner shown, the sixteen holes in the metal from which blanks have been cut are numbered and lettered the same as the die. It should be understood that the metal is fed through in the usual way, which is from right to left, and that the ¼-inch holes are first pierced out, before the ¾-inch blanks are cut.

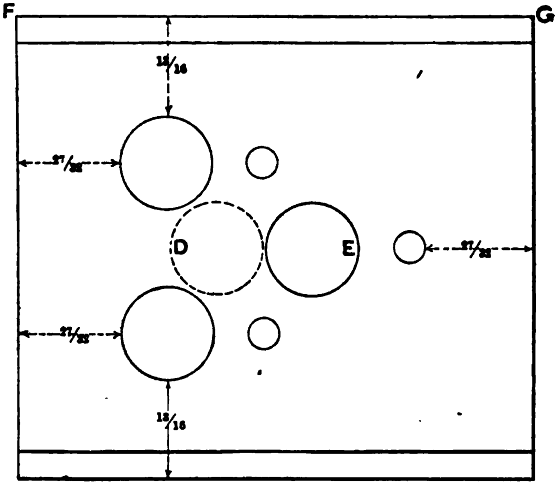

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Fig. 12. Plan View of Die for Punching Three Washers Simultaneously.

By referring again to Fig. 13, the layout for cutting two, three, four, five, six and seven blanks can be determined. The parts numbered and lettered 1—*A* and 5—*E* are the layout for two blanks. For three blanks: 1—*A*, 2—*B*, and 5—*E*. For four blanks: 1—*A*, 2—*B*, 5—*E*, and 6—*F*. For five blanks: 1—*A*, 2—*B*, 3—*C*, 5—*E*, and 6—*F*. For six blanks: 1—*A*, 2—*B*, 3—*C*, 5—*E*, 6—*F* and 7—*G*. For seven blanks: 1—*A*, 2—*B*, 3—*C*, 4—*D*, 5—*E*, 6—*F*, and 7—*G*.

The die bed used for holding the die in Fig. 13 in position when in use should have its dovetail channel running in the direction *KL*, while the dovetail channel for the dies shown in Fig. 10 and 12 should run in the direction *FG*. The reason for this is the longer bearing surface for the dovetail obtainable by such arrangement.

It should be remembered that all holes in dies of this kind are lapped or ground to size after hardening; they should be perfectly round and have 1 degree clearance. In some shops the holes are left straight for ¼ inch, and then tapered off 2 degrees.

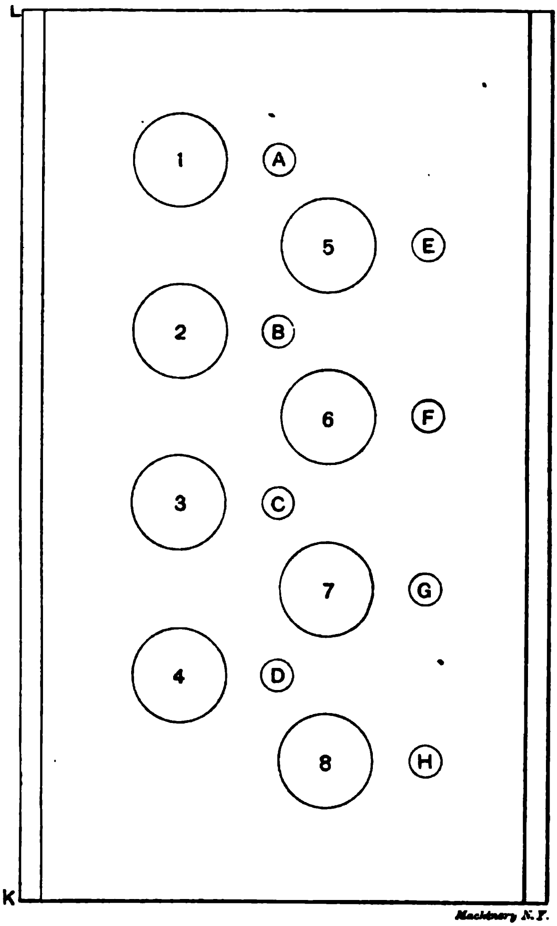

Fig. 13. Plan View of Die for Punching Eight Washers Simultaneously.

An important point to bear in mind in making the punch is to have a perfect "line up." It may not be generally known, but it is nevertheless a fact, that blanking tools that blank, or that pierce and blank two or more blanks at one time, will run longer without sharpening, cut cleaner blanks, and, in fact, give all around better results, if the punches are a perfect "line up" with the die, than if they are lined up in the so-called "near enough" way. A perfect line up, as referred to in the above, is a line up that will allow a punch that consists of two or more punches to enter the die the same as if the punch consisted of just one punch. The advantage of the perfect line up over the other is that when in use the punches do not come in too close contact with the edges of the die. They enter the die, but do not bear against the edges in such a way as to dull the die, or round over the sharp cutting edge of the punch.

A punch that is almost a perfect line up will enter the die, but it requires more force to make it enter. Why? Because in entering, one of the punches, for instance, rubs hard against the side of the die, and if set up in the press and allowed to run, that punch, no matter how small, will dull the edges of the die as well as the edge of the punch itself. The result is that the press must stand idle while the tools are being sharpened, and if the real cause of the trouble is not remedied, it is "the same old thing" over and over again.

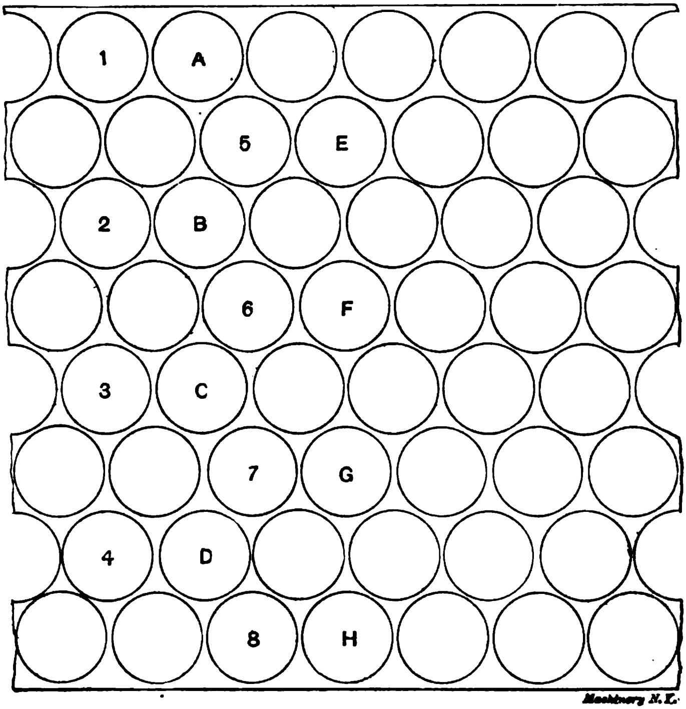

Fig. 14. Stock after having been run through the Die in Fig. 13.

Just a few words in regard to making the punch. In making the punch, the punches must be made so that they will fit the die not too loose, nor too tight. The blanking punches are hardened and ground to size. The taper shank is finished to size after hardening, so that when the punches are driven into the punch back they will stand straight and not lean to one side.

In laying out the dovetail punch back, first clamp the back central on the face of the die. This is done so that when the punches are driven in position in the punch back, and set central in the ram of the press, ready to be used, no shifting is required in order to make the punch line up with the die, which is keyed in the center of the die bed. After clamping the punch back in this position, the blanking part of the die nearest the end is scribed on the face of the punch back. Do not scribe all the holes and rely upon finding the center of each circle thus scribed with a pair of dividers, and then true up these centers on a faceplate in order to get a perfect "line up"; this method increases the chances of error, especially when there are six or eight punches to be set in position. A better way is to scribe one circle as stated above, and remove the punch back from the face of the die; find the center of the circle scribed; true up this center, and drill and bore out the hole to fit the taper shank of the blanking punch.

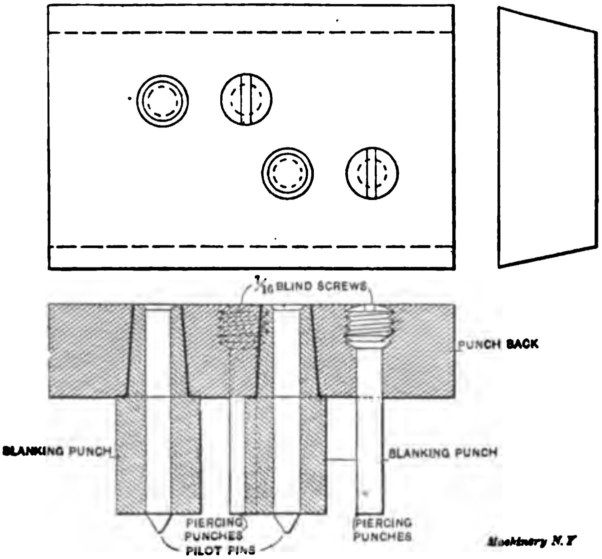

Fig. 15. Punch Back, with Punches inserted.

Fig. 15 shows how a punch of this kind is made. The punch as shown is used with the die shown in Fig. 10. After the hole is bored to size, the already finished blanking punch is driven in tight in the manner shown. Two narrow parallels say ½ × ¾ inch are now laid on the face of the punch back, and the blanking part of the die that corresponds with the punch driven in is slipped over the same, until the face of the blanking die rests upon these parallels, after which the die is clamped tightly thereon. The next hole is now trued up with a test indicator until the hole runs dead true. The die is then removed, and the hole for the taper shank is worked out, and the punch driven in. Where there are more punches to be set in, the same method is used until they are all in position. This insures a perfect line up, providing that ordinary care and precaution has been used in doing the work. In boring out these holes it is best to use a bolster having a dovetail channel, and to hold the punch back in position with a key. This is better than using straps to fasten the punch back to the faceplate, as the straps are likely to interfere with the parallels and the die, when locating the exact position for the holes to be bored.

In locating the position for the piercing punches, it sometimes happens that the holes are so small that they cannot be bored. The holes are then transferred by a drill that runs true and is the same size as the holes in the piercing die, the die being used, so to speak, as a drill jig.

Fig. 15 shows how the piercing punches are held in position. The punches are made of drill rod, and are prevented from pushing back by hardened blind screws as shown. If thin, soft metal is used, the method for holding the two pilot pins in position shown in the previous chapter may be employed. When the piercing punches are made and held in position as shown in Fig. 15, a spring stripper is sometimes used, and is fastened to the punch back, and the holes for the piercing punches in this stripper are made a sliding fit, in order to prevent the punches from springing or shearing. When the ordinary form of stripper is used, the piercing holes are also made a good sliding fit.

CHAPTER III.

MAKING BLANKING DIES TO CUT STOCK ECONOMICALLY.

A most important point for the die-maker to bear in mind in making blanking dies for odd shapes is to lay them out so that the minimum amount of metal will be converted into scrap. In fact, hardly too much stress can be laid upon this one point alone. It is an easy matter to waste a considerable percentage of the stock by lay-outs which may *appear* to be fairly economical. The die-maker should make a careful study of the most economical relation of blanking cuts to one another and to the stock. It is the object of the present chapter to point out by actual examples how stock can be saved which may be converted into scrap if the die-maker is not constantly watching out for possible economies. As an illustration, it sometimes happens that by laying out the dies so that the blanks are cut from the strip at an angle of 45 degrees, as shown in Fig. 17, a considerable economy of metal can be effected over a right-angle arrangement, that is, one to which the dies are set so as to cut the blanks straight across the strip. The angular location permits the use of narrower stock and materially reduces the amount of scrap metal. Fig. 16 shows the plan of the die, and needs little or no explanation, as the manner in which it is laid out is obvious; the plan of the strip shown in Fig. 17 also clearly shows how the die is laid out.

Another method that is often used to save metal is that shown in Figs. 19 and 20. This method is used where the required amount of blanks does not warrant the making of a double blanking die; also when, unavoidably, there is a considerable amount of stock between the blanks after the strip has been run through as shown at *A* in Fig. 19. To save this metal the strip is again run through in a reverse order after the manner shown in Fig. 20, thereby using up as much of the metal as it is possible to do. Besides blanking and piercing the blank when running the metal through the first time, the holes numbered 4, 5, and 6, Fig. 18, are also pierced. This is done for the reason that when the metal is run through the second time it prevents cutting of "half blanks" by "running in," or, in other words, the liability of cutting imperfect blanks by cutting into that part of the metal from which blanks have already been cut. This guiding action is effected by three pilot pins in the blanking punch (not shown) which engages in the three pierced holes, made when the strip was

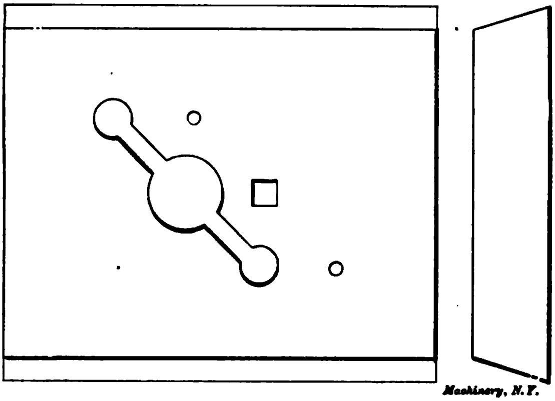

Fig. 16. Example of Blanking Die.

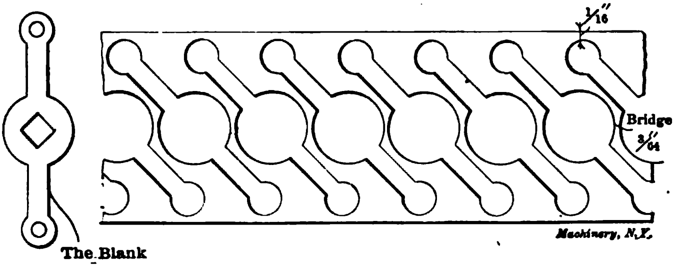

Fig. 17. Section of Stock after having been run through Die in Fig. 16.

run through the first time. The pilot pins engaging with the pierced holes cause the second lot of blanks to be cut centrally with the holes, and also to be accurately centered between the portions of stock from which the blanks have already been cut. When this die is in use, the metal is run through in the usual way from right to left until half of the required amount of blanks are cut, after which the piercing punches for the holes are taken out and the metal is run through again and the other half of the required amount of blanks is cut.

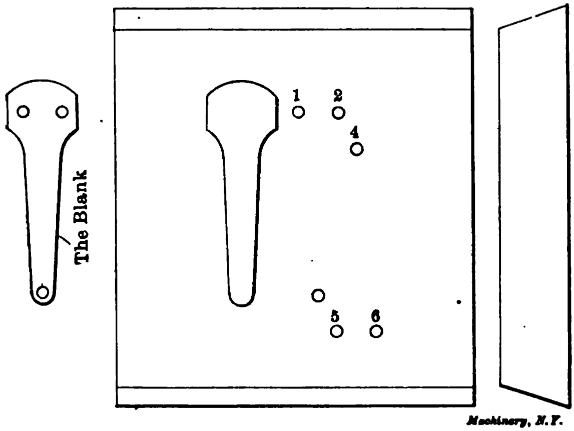

Fig. 18. Another Example of Blanking Die.

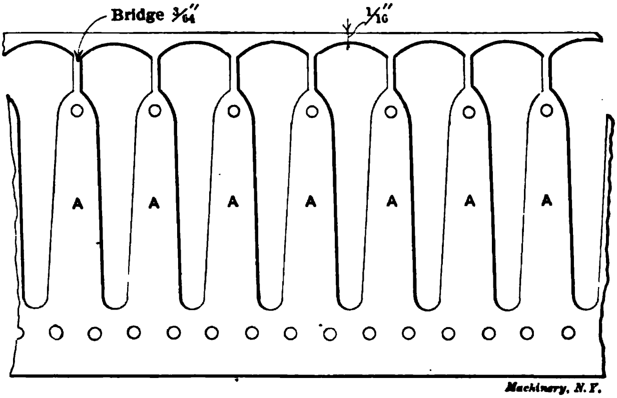

Fig. 19. Stock after having once been run through Die in Fig. 18.

In laying out this die, which is done after the manner shown in Fig. 28, the line *A* is used as the center line for the piercing holes numbered 1 and 2 in Fig. 18, and the line *B* is the center line of the blanking part of the die. The line *C* is the center line that shows the center of the next blank to be cut and is laid out 53/64 inch from the line *B*. This dimension is fixed by the fact that the widest part of the blank is 25/32 inch, and the bridge between the blanks is 3/64 inch, the sum of which equals the distance from center to center of adjacent blanks. The line *D* is the center line for the blank *C*, Fig. 20, which is cut when the metal is run through the second time, and is made at 0.414 inch or one-half of 53/64 from the line *C*, Fig. 28, inasmuch as the blank is cut centrally between that part of the metal from which the blanks *A* and *B*, Fig. 20, are cut.

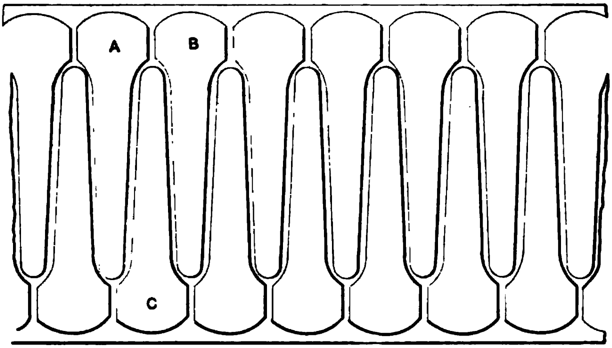

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Fig. 20. Stock after having been run through Die in Fig. 18 twice.

Fig. 21 shows a double die for blanking and piercing brass, producing the shape shown in the sketch at the left; it is laid out so as to save as much of the metal as is practically possible without added expense in so far as the operation of blanking and piercing is concerned. By referring to Figs. 22 and 23 it can be seen that the strip of metal from which the blanks are cut is run through a second time for reasons that will be given. One reason is that wider metal can

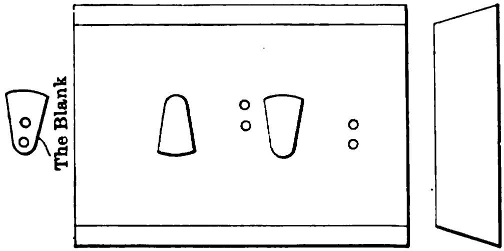

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Fig. 21. A Third Example of Blanking Die.

be used by doing so, which in itself is a saving in so far as the cost of metal is concerned. Wide brass can be bought at a lower price per pound than narrow brass; the other reason is that a strip of metal 1/16 inch wide and as long as the entire length of the strip is saved on every strip that is run through. If narrow metal were used there would be waste of ⅛ inch of metal (*i. e.*, 1/16 inch on each side) of every strip run through, and on two strips from which no more blanks can be cut than from the wider strip shown in Fig. 23 there would be a waste of ¼ inch of metal. On the other hand, by using wide metal the waste would be only 3/16 inch, as indicated in the cut. Fig. 29 shows how this die is laid out, and should be sufficiently clear to explain itself.

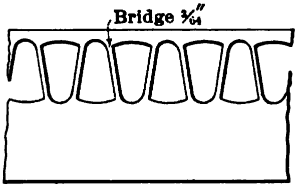

Fig. 22. Stock after having been run through Die in Fig. 21 once.

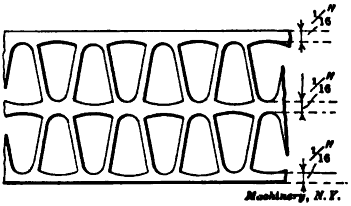

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Fig. 23. Stock having twice been run through Die in Fig. 21.

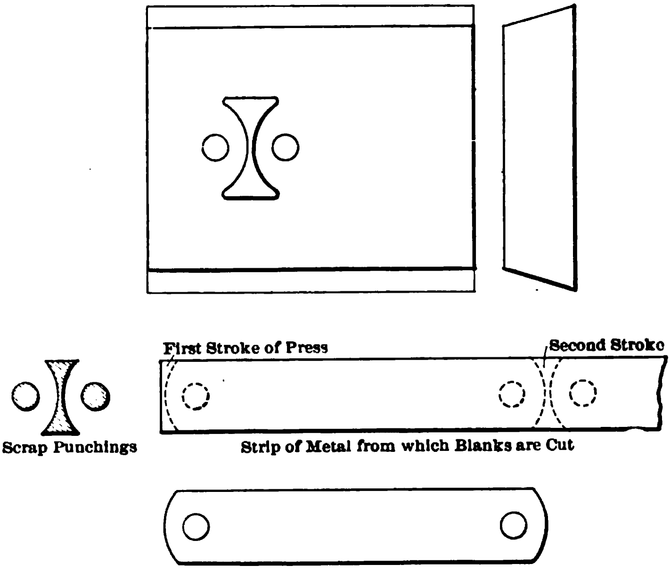

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Fig. 24. Blanking Die for Producing Links.

To fully understand the manner in which the metal is gradually worked up after each stroke of the press, short sections are shown in Fig. 25. At the first stroke four holes are pierced and two plain blanks —with no holes—*A A* are cut out. At the second stroke there are also four holes pierced and the two blanks *B B* are cut that have the holes pierced at the previous stroke. At the third and fourth strokes

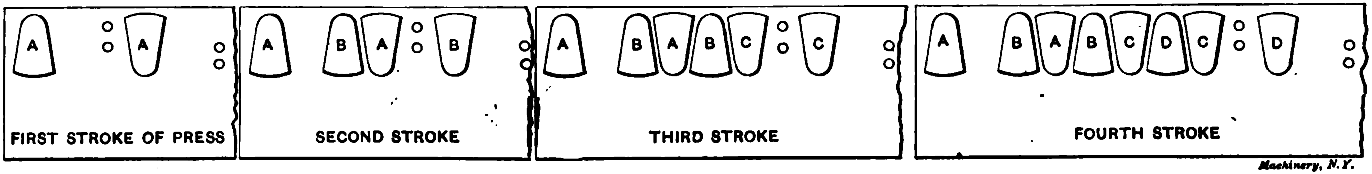

Fig. 25. Appearance of Stock after each successive Stroke of the Press.

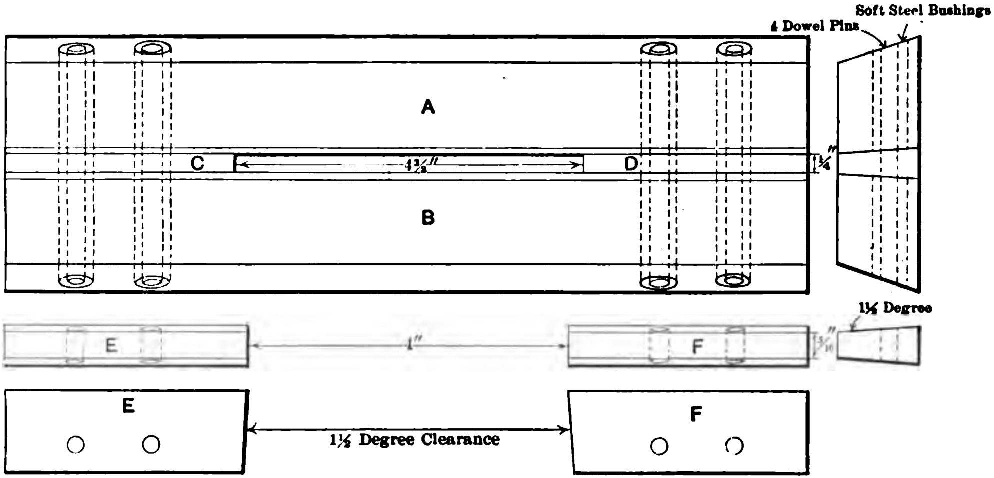

Fig. 26. Die with Interchangeable Parts, permitting Two Sizes of Blanks to be Punched by Changing the Center-Pieces only.

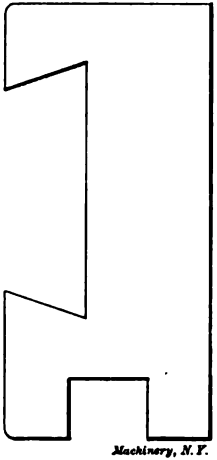

Fig. 27. Gage for Planing Die Blanks.

the holes begin to match in with each other, as shown, so that when the metal is run through it will look like the strip shown in Fig. 22. It should be borne in mind that four holes are pierced and two blanks are cut at each stroke of the press; also that the metal is fed after each stroke a distance equal to the distance from the center of *A* to the center of *B*, as indicated in the strip marked "second stroke," Fig. 25, and which is 25/32 inch (see Fig. 29). By way of further explanation it may not be amiss to state that the distances from the center of *A* to *B*, *B* to *C*, *C* to *D*, and *D* to *C*, as shown in the strip marked "fourth stroke" are each 25/64 inch, or half of 25/32 inch.

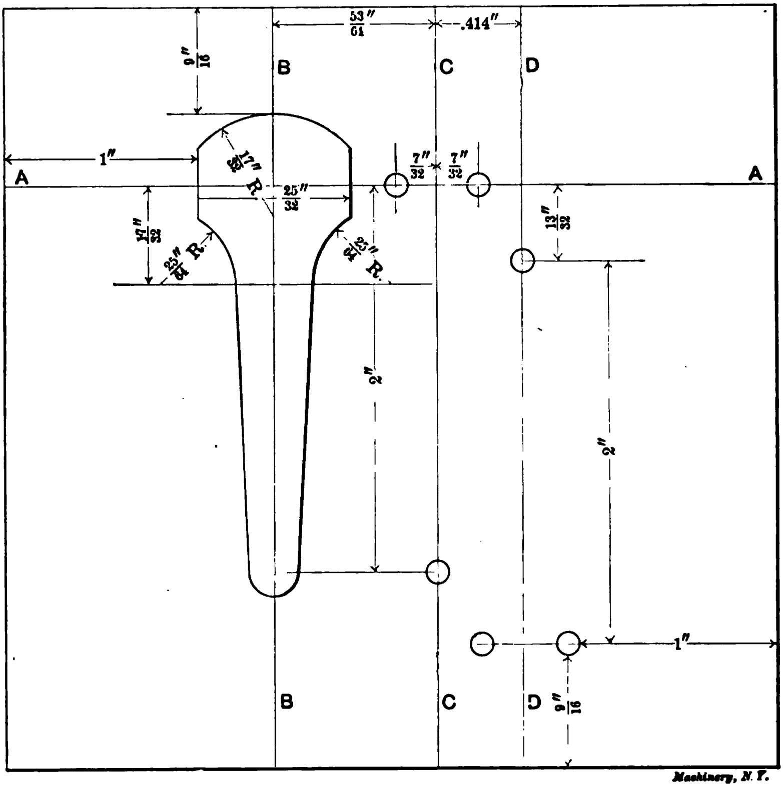

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Fig. 28. Layout of Die shown in Fig. 18.

While the dies shown in Figs. 24 and 30 are commonly known, it may not be out of place to say a few words with reference to them, as they form an important part in the economical production of sheet metal goods. The first or Fig. 24 shows a die that is used to produce from narrow ribbon a long blank with rounded ends and with a hole pierced in each end. The principal feature of this style of die is that there is very little waste of material in the production of the blanks, as will be noted from the sketch of the scrap punchings shown at the left, and another feature is that by the aid of an adjustable stop, not shown, almost any length of blank can be made without altering or resetting the tools after they have been set up in the press. The working part of the die is laid out a little to the left of the center so as to give sufficient length for the gage plates which are fastened to the die by ¼-inch cap-screws. These gage plates are used to keep the metal in position while it is being fed from right to left as the blanks are cut from the strip.

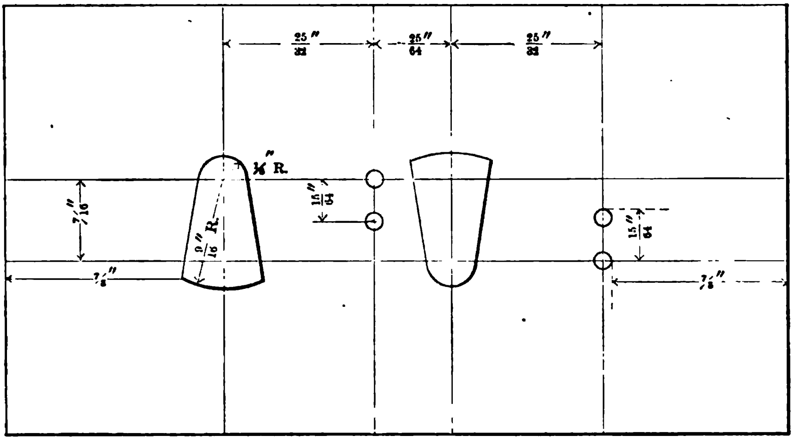

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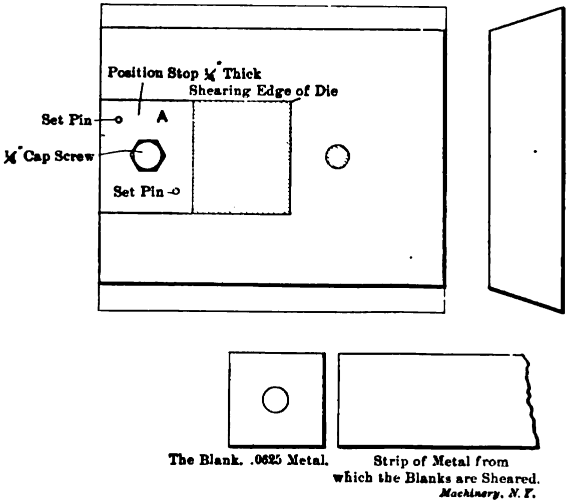
Fig. 29. Layout of Die shown in Fig. 21.



The Blank. .0625 Metal.

Strip of Metal from which the Blanks are Sheared.

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Fig. 30. Blanking Die for Square Washers. Shaded Portions in Die indicate Parts punched out from Stock.

Fig. 30 is a combination piercing and shearing die and is used for producing the 1-inch square washer shown in the cut. The principal feature of this die is that there is no waste of metal in producing the blank, only, of course, the ¼-inch round punching taken from the center. The strip of metal in this case can be fed from right to left or front to back, as preferred.

CHAPTER IV.

CONSTRUCTION OF SPLIT DIES.

A die of great importance in the production of sheet metal parts is the split die. There are two principal reasons for using the split die. One is that it sometimes happens that the blanks to be cut are of such a shape that the die can be more quickly and cheaply made by making a split die than by making a solid or one-piece die. The other reason is that when the required blank must be of accurate dimensions, and there is a chance of the solid die warping out of shape in hardening, the split die is preferred because it can be much more easily ground or lapped to shape.

Fig. 31 shows the manner in which the ordinary split die is usually made. After the die is worked out, it is hardened and ground on the top and bottom. The two sides *A* are then ground at right angles with the bottom.

The cutting parts of the die, *B*, are next ground at an angle of 1¼ degrees with the bottom, so as to give the necessary clearance in order that the blanks may readily drop through. The key *D* is now set in place, and the die is keyed in the die bed by the aid of a taper key. The key *D* prevents the die from shifting endwise; the keyway should have rounded corners as shown, which not only give added strength, but also act as a preventative to cracking in hardening. The last operation is to grind the two circular holes. This is done by first lightly driving two pieces of brass or steel rod into the holes until they are flush with the face of the die. The exact centers are then laid out and spotted with a prick punch, care being taken so as to get the centers central with the sides *B*. The die is now fastened to the faceplate of a universal grinder, and the center mark is trued up with a test indicator until it runs exactly true. The brass rod piece is then driven out, and the hole ground to size, with 1½ degree taper for clearance. The other hole is next ground out in a similar manner, which completes the operations in so far as the die is concerned. It often happens with a die of this kind that when it is placed in the die bed and the key driven in place, it will "close in." To overcome this, the die is relieved after the manner shown at *C*, which does not in any way prevent it from being securely held in place when in use.

Fig. 26 shows a rather novel form of a split die; this die with a slight change practically takes the place of two dies. It is used for piercing slots in brass plates. The size of the slot for one style of plate is 4⅝ inches long by ¼ inch wide; for the other plate the slot is 4 inches long by 5-16 inch wide. The cutting part of the die, shown in Fig. 26, is made in four sections, *A*, *B*, *C*, *D*. The cut fully explains itself and therefore needs no detailed explanation. It may not be out of place, however, to say that the soft steel bushings, as shown, are used to allow for the contortion of the parts *A* and *B* in hardening. It may be added that the four bushings shown in the piece *A* were driven in first; then solid pieces were driven in the part *B;* then the holes were drilled in these latter pieces, being transferred from the bushings in the part *A*. In Fig. 26 are also shown the parts used in connection with this die for piercing the 4 x 5-16 inch slot. These parts are made as shown, and are hardened only at the cutting ends. Outside of the fact that this style of die practically takes the place of two dies, there is still another feature in connection with it that will bear mentioning; there is no special or extra die bed required for this die when in use.

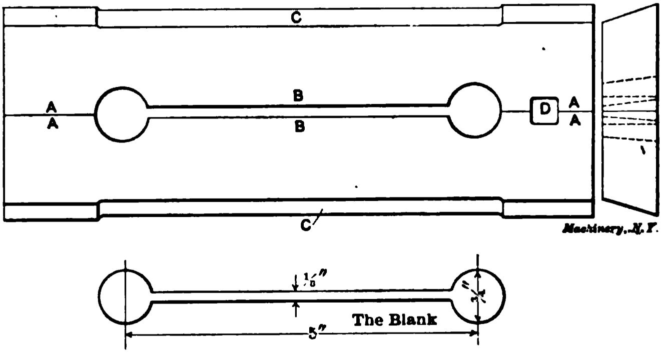


Fig. 31. Example of Split Die.

It may not be amiss at this time to say a few words with reference to die beds. (In some shops this part is called bolster, die block or die holder.) Perhaps the most commonly used and the best die bed for general use in the press room is the style of bed shown in Fig. 32. This die bed is principally used for the reason that the screws that fasten the die bed to the bed of the press do not have to be screwed entirely out, either in placing the die bed in the press or in taking it out, as the slots *C* and *D* are made at right angles with each other for just this reason.

The dovetail channel is planed so that when the die is keyed in position the center of the die is central with the slot *C*. The side of the die bed marked *A* is planed at an angle of 10 degrees, and is parallel with the slot *C*. The side marked *B* is planed at an angle of 13 degrees and is at an angle of 1 degree with the centerline. The reason for planing this side to an angle of 13 degrees instead of ten is that the increased angle causes the die to lie flat, and prevents it from raising or tilting up in any way when the key is driven in.

In speaking of the key, it may well be added here that the taper-key method of holding blanking dies in the die bed is the best of the various methods which are generally used. The set-screw method is doubtless the poorest of all. The key as shown in Fig. 32 is driven in on the front side of the die bed. This is optional, however, as the practice differs. In some shops the key is driven in on the front side while in others it is driven in on the back.

Of late years there has been a tendency among large concerns to have all their die beds for the power press made from semi-steel castings, or of machine steel for certain classes of heavy work, instead of from gray iron as heretofore. This is being done because a gray iron bed that is used day after day for holding dies for cutting heavy metal will not stand up during long and hard usage as it should. Past experience has proven that gray iron die beds in time become out of square; then, again, they sometimes crack. With the semi-steel, or the soft steel die bed, this does not happen. It has been found that semi-steel and machine steel die beds pay for themselves many times over.

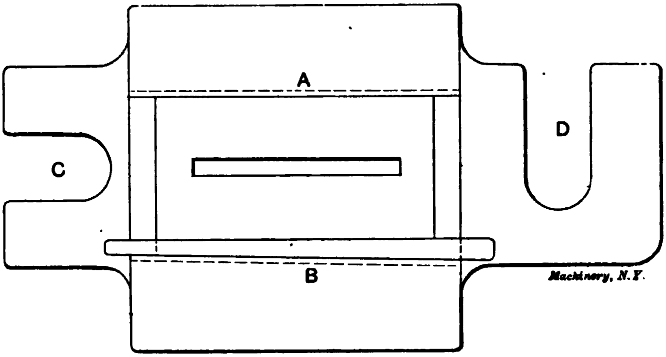


Fig. 32. Example of Die Bed.

In planing up the stock from which the blanking dies are sawed off before they are worked out, a gage similar to the one shown in Fig. 27 should be used for planing up the different widths of dies. In this way the dies will be of a uniform width and thickness, which makes it possible to have them interchangeable with the respective die beds for which they are used.

CHAPTER V.

NOVEL IDEAS IN DIE MAKING.

A few years ago, what is now the Providence Mfg. & Tool Co., of Providence, R. I., began the manufacture of a mechanical accountant, the invention of Mr. Turck, the present superintendent of the shop. Mr. Turck's experience, so far as shop work and tool design is concerned, had not been in the direction of die-making, so that in equipping the new plant for the manufacture of the accounting machine he was at first hampered by his lack of knowledge on this subject. The die work required was of a high order. The construction of machines of this type is often such that errors are cumulative. Several similar parts are used, attached to each other in series, for instance, in such a way that if the holes by which they are riveted to each other are slightly wrong in their dimensions, the error will be multiplied by the number of parts. The machine depends for its operation quite largely on the action of pawls upon fine ratchet teeth, and on the meshing of fine pitched gears and toothed segments with each other. The effect of cumulative errors in such circumstances would be to throw these fine pitched ratchets and gears out of step, and make the operation of the machine impossible. Long leverages are also a disturbing factor. When a long, slender member is located by two rivet holes close together, it takes careful work in punching those rivet holes to bring the parts into alignment. In the following some very interesting tools, used mainly for blanking purposes, but also for bending and other operations necessary to complete the product, are shown.

In the halftone in Fig. 33 are shown a number of press-made parts. Some of these are interesting in themselves, while others are remarkable principally for the methods used in producing them. Part No. 12, for instance, is a very simple piece, but the punch and die used in piercing the holes, while not unusual so far as surface appearances go, will serve well to illustrate some of the original practices of this shop. This punch and die, shown in Fig. 35, perform the simple operation of punching the nineteen small holes in the blank, which is located over die *A* by the carefully fitted aperture in jacket *B*. The punch is composed of a body *C*, a cast-iron holding plate *D* in which the small punches *E* are driven, a stripping plate *F*, held as shown, and forced outward by the compressed rectangular ring *G* of rubber behind it.

The Construction of a Piercing Punch with a Novel Stripper Plate.

The making of this punch and die follows, in general, the order given below. Stripper *F* is first made of tool steel. The holes for the dowels *H* are next drilled. Then the holes through which punches *E* pass are laid out from model or drawing, as the case may require, and drilled to a *larger* diameter than the punches which are to pass through

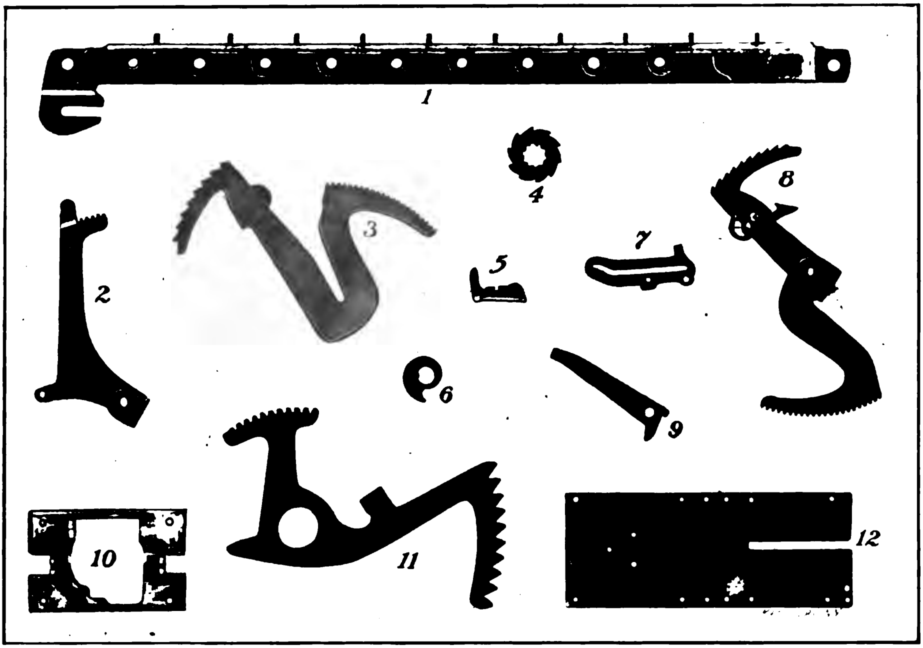


Fig. 33. Some Examples of Good Press Work.

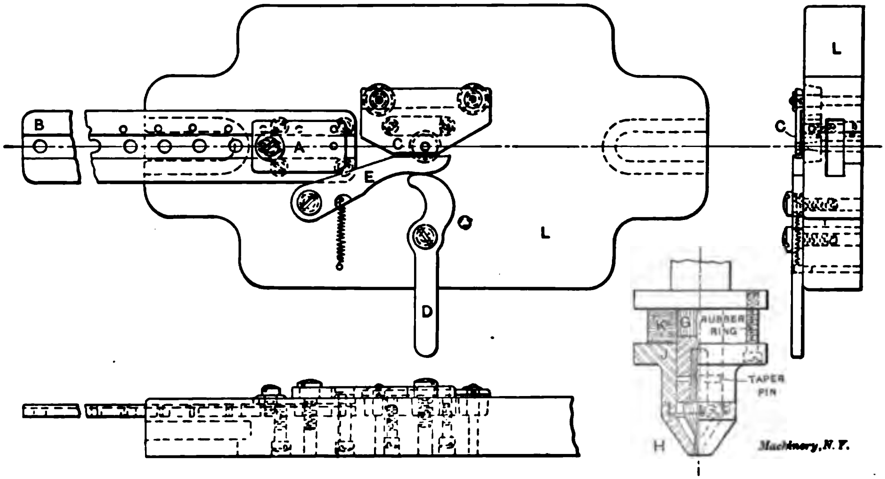


Fig. 34. Construction of Die for Double Punching.

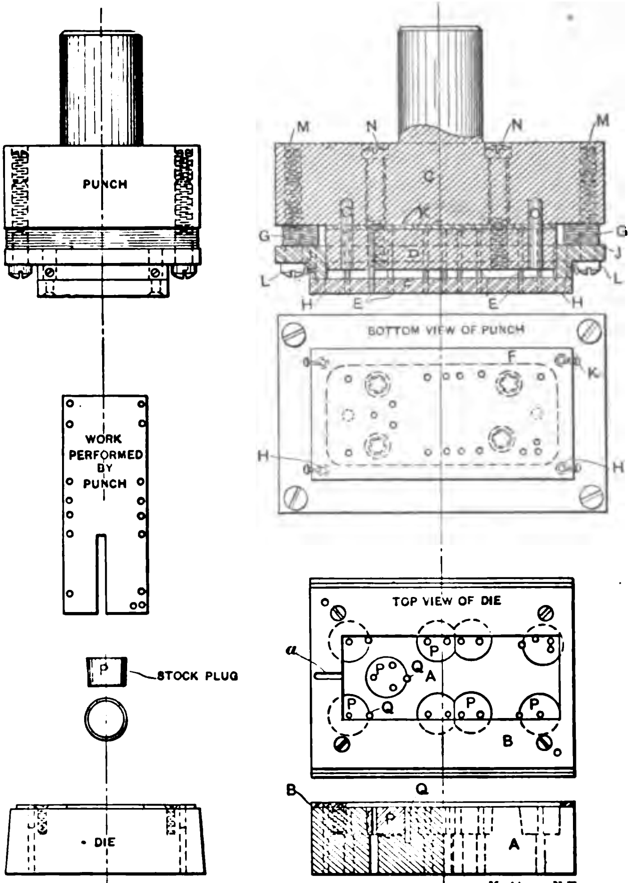
them. After these holes have been drilled, the plate is hardened and ground, and the holes for the punches are filled up again by driving into them plugs of tool steel wire, of suitable size. The location of these holes is now laid out again on plate *F*, and this time very carefully; then they are finished to the exact size, or slightly below, if they are to be lapped. Since the body of the plate is hard, it cannot cave in or wear as it would if left soft. A full bearing on the stock to be blanked is absolutely necessary if the work is to be well done. The plugs allow the plunger holes to be located after the hardening of plate *F*, thereby preventing displacement from the heat treatment. To the stripper plate are now riveted the four dowels *H*, which enter holes in the stripper rim or "collet" *J*, and locate the plate. Small round-headed setscrews bear on pins *H* and hold *F* and *J* together. Punch holder *D*, of cast iron, is machined to fit closely in collet *J*, and the holes for the punches are transferred to it from stripper plate *F*. The punches *E*, made of tool steel wire, are now driven into the holder, headed over at the back side, and ground flush. The punches may then be hardened in the usual manner. Before being assembled on the punch body *C* with the rubber spring *G*, a hardened steel backing *K* is inserted between *D* and *C* to take the thrust of the hardened punches.

The rubber spring *G* is cut from sheet stock and may be made either from separate strips built up on each of the four sides of the punch, or from rectangular rings, if that can be done without wasting the stock. Screws *L* are adjusted to bring the face of the stripper flush with the faces of the punches, after which headless setscrews *M* are screwed in to make the adjustment permanent. Screws *L* may then be taken out and replaced without losing the adjustment. The punch holder *D* and pad *K* are held to the holder by screws *N* and dowels *O*.

A Piercing Die with Inserted Tool Steel Plugs for Cutting Edges.

The body *A* of the die is made of soft steel or cast iron. In this body are driven standard taper plugs of tool steel of suitable size, and so arranged as to be in position to furnish a tool steel material for all the actual cutting surfaces of the die. In the case shown in Fig. 35, nine of these plugs are used, carrying from one to three holes each. In making the recesses for these plugs standard tools are used. The seats are first drilled nearly to size, and then finished with a tapered end mill or counterbore, which is kept carefully ground to the proper dimensions, so that when the plug is driven in until it binds tightly on the taper, it will also seat on the bottom. These various plugs *P* are prevented from turning in the holes by dowel pins *Q*, in most cases, or, where the plugs run into each other (as shown in two cases in the die here described), by the interlocking of the flat abutting surfaces. These precautions make it possible to remove the plugs at any time and return them accurately to their original positions.

The die plate *A* having been fitted with its plugs as described, the holes in stripper plate *F* are now transferred to it by any suitable means, all these holes being received in the tool steel plugs as explained. The plugs may now be removed, to be hardened and lapped separately. The clearance holes for the scrap are drilled, and the plugs are returned to their proper places. The jacket *B*, which locates the blank on the die, may, if desired, be punched from stock of suitable thickness by the blanking die used for making the blank



Machinery,N.Y.

Fig. 35. A Piercing Punch and Die involving some Original Ideas.

to be operated on in this piercing die. The edges of the opening are then merely filed enough to allow the work to enter and be withdrawn easily. A slanting groove, as shown at *a*, is cut with a round file into the jacket at one end to permit the insertion of a pick or awl to remove the work.

The points of interest in this die are: The rubber-backed stripper plate; the use of a soft stripper plate bushed in the manner described with hardened tool steel; and the insertion of plugs of tool steel in a soft die block to form the cutting edges of the die.

The rubber spring has proven very satisfactory. It will last for a number of years in dies having ordinary use, if it is not exposed to oil and other deteriorating influences. Being in the upper member, there is little likelihood of its being spoiled in this way. The use of this stiffly spring-supported stripper plate gives a punch and die of the design shown all the advantages of a sub-press, so far as concerns the ability to punch small holes in thick material and leave thin walls of metal between open spaces in the punching. As evidence of the ability to do work of this kind with a punch and die of the style just described, parts 7 and 10 in Fig. 33 may be particularly noted. Here the holes are considerably smaller in diameter than the thickness of the stock, and the internal spaces have been punched so close to the edge, in places, that the remaining section is narrower than it is thick.

The method of bushing the stripper plate by drilling the holes large originally, plugging them with tool steel wire after hardening, and redrilling them to the proper size, makes it possible to harden the surfaces in contact with the work, without distortion of the dimensions between the holes. Plates of large size, even, are made in this way.

The advantage claimed for the method by which the stripper plate is made may also be claimed for the use of hardened plugs in a soft die body, since it is possible to harden these parts individually without changing their location with reference to each other. In addition, both of these schemes allow changes to be made in the dies with a minimum of trouble and expense. If it is desired to change the location of a hole in the die, the old plug may be removed and a new one inserted. In the same manner, new holes may be drilled in the stripper plate in which new tool steel wire plugs may be driven for new guiding holes for the punches, although the change is limited by the size of the plugs. This consideration is of considerable importance if the parts manufactured are subject to improvement from time to time. This provision reduces the expense of spoiled work as well, since it is not necessary to throw away an expensive press tool if one or two of the holes are wrongly located.

Rubber-backed vs. Sub-press Dies.

It will be noted that part No. 12 in Fig. 33 (for which the punch and die just described were designed) is made in three operations. Under ordinary conditions, experience seems to indicate that this procedure is preferable to the use of the sub-press. The rubber spring supported stripper plate, as just described, gives all the advantages of the sub-press, so far as ability to do fine work on thick stock is concerned. Slender punches are supported by the stripper in the same way as in the sub-press; the rubber spring holds the stripper so firmly onto the work that the distortion of thin stock is prevented. The sub-press certainly has the advantage of ease of setting in the machine, since it is not necessary to carefully line up and punch and die.

which are in permanent alignment. It is possible, however, that the high initial cost of the sub-press would in many cases more than pay for the extra wages of an experienced and careful man in setting up tools during the lifetime of the punch and die. It must also be admitted that work cannot be done as rapidly with the three sets of tools necessary for making the piece in the manner here described, as would be possible if a sub-press were used. The saving in first cost, however, and in the cost of subsequent operations, is believed to be sufficient in the case of the Providence Mfg. & Tool Co. to show a balance on the right side of the sheet for the simpler form of press tool. It should be said in this connection that this firm freely makes and uses the sub-press die.

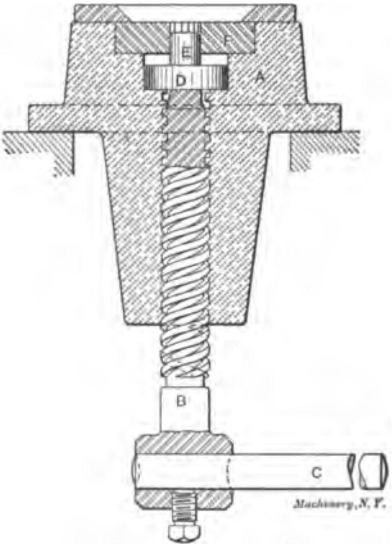


Fig. 36. Die for Bringing Up Drawn-down Corners.

The Thickening of Corners Drawn Out in Blanking.

An operation of particularly great interest is a coining process used for reshaping the points of gears, ratchets, etc.—such parts, for instance, as are shown in samples 4 and 6. In such a piece as No. 4, whatever the design of the die, the blank produced will be found to have the points drawn down thinner than the stock thickness. To bring the part back to uniform thickness with sharp points, the device shown in Fig. 36 is used. Here we have an attachment to a hand screw press. The body *A* is fastened to the bed of the press. The screw *B* projects through the bed and carries at its lower end a handle *C*, which is adjusted to one side or the other to bring it in position to be swung by the foot of the operator. In a counterbore in body *A* is seated the plug *D* and the ejector *E*. *D* and *E* are forced upward by the action of screw *B*. At *F* is a die, given the shape desired for the outline of the finished part; it is slightly enlarged, however, for a short distance at its upper end. The part as it leaves the blanking press is purposely made a little large in outline at the points where the thinning occurs, due to the drawing out of the stock. When the piece is inserted by the operator in the upper end of this tapering die, the extra metal thus provided is forced inward to thicken the points to the required amount as the punch is brought down upon the work by the hand of the operator. When the piece has been forced to the bottom, it is clamped between the plane surfaces of ejector *E* and the punch above it (not shown), and the metal is forced to flow to that part of the blank where it is most needed. The result is a flat ratchet with plane faces and uniform thickness. It will be understood, of course, that during this coining operation ejector *E* and plug *D* seat in the counterbore in body *A*, screw *B* being lowered out of contact. A push of the operator's foot on handle *C* brings the ejector up again until the piece is forced out of the die. The thread of the screw is of such a steep pitch that the screw will return again by its own weight.

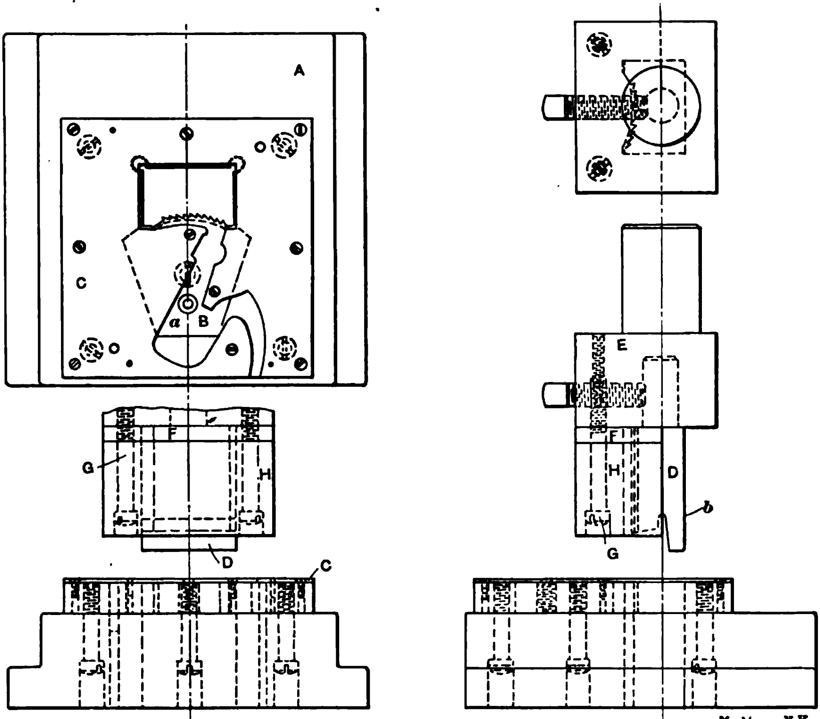


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Fig. 37. Example of Type of Die used for Shaving.

The comparative slowness of operation resulting from the use of a hand and foot power press and hand feeding is, in a measure, characteristic of this shop. It is the belief of the superintendent that better results can be obtained at times by methods like that shown than by more "modern" ones. The aim is, through careful workmanship and careful inspection, to have the parts so nearly right when assembling time comes, that no fitting will need to be done in the assembled machines. No fitting is, in fact, allowed. Certainly the method described for striking up the corners of these ratchets is a much less dangerous one than would be the case if a power press were used, so the idea has its advantages, so far as safety is concerned, at least.

A Typical Shaving Die.

In such parts as are shown at 3 and 11 in Fig. 33, the ratchet teeth and gear teeth are only roughed out in the blanking die, being finished by a second cut or "shaving" process. A typical die and punch for this operation are shown in Fig. 37. Here, as in Fig. 35, the work is held by a rubber spring backing while the punch is at work. The die is made of a soft body *A*, in which is inserted the hardened piece *B* carrying the cutting edges which are to form the ratchet teeth on the work. This piece *B* has its teeth cut on it in the milling machine, the hole at *a* serving to center the piece for this operation. This gives assurance that the teeth will be properly spaced, and cut accurately to the proper radius. A rectangular opening with carefully machined sides is made through the die block *A*. Into this opening the toothed cutting edges of piece *B* project. As in Fig. 35, a "jacket" *C* is provided for locating the work over the cutting die. The punch *D* is set into a holder *E*, which in turn is fastened in the ram of the machine. A projecting guiding surface, *b*, on the punch, enters the rectangular opening in the die and bears against it on the back and sides. This keeps the cutting surface of the punch up to its work against the cutting edge of the die. As shown, the cutting edge of the punch is beveled. This gives a slight top rake to the edge, and produces a shearing cut as well, the outer corners coming into action before the

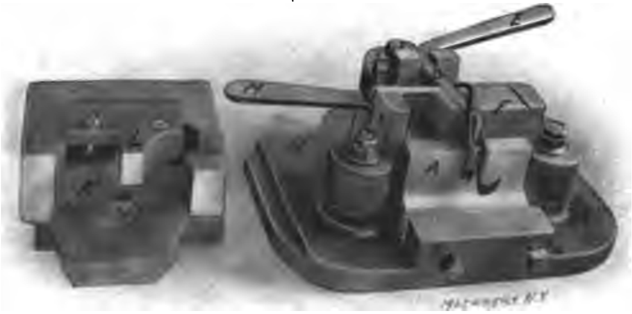

Fig. 38. Bending Attachment with Removable Die. Operation Completed.

center of the outline reaches the stock. The rubber spring backing at *F* is held by screw *G* between the pressure block *H* and the punch holder *E*. It performs the same functions at the stripper plate in the other die.

Bending Punchings to Provide Double Bearings.

It will be noticed that samples 1, 5 and 8 in Fig. 33 have been made on the principle of bending the punchings to give a double bearing at pivotal points, the long bearing insuring lateral steadiness of the part without making it necessary to resort to the use of castings with long hubs. This principle is carried out throughout the calculating machine which is this firm's principal product. In some cases, especially where the pivot holes are punched previous to bending, as is the case in sample 8, very accurate work must be done in the bending to bring the part to exactly the right form. In the sample referred to, for instance, the ratchet teeth on one side and the gear teeth on the other must bear a definite relation to each other, and to the axis about which the part rotates. The bending tools by which the forming operation is performed for this part are shown in the halftones in Figs. 38 and 39 and the line cut Fig. 40. Referring to Fig. 40, the blank for part 8 (shown at No. 3 in Fig. 33 before the piercing of the pivot holes) is laid on top of former *A*, where it is located by the pins *B B* which enter the pivot holes. In this position the part lies between the fixed jaw *C* and the movable jaw *D*, which are then clamped together on the blank by bringing handle *E* to the position shown, where its wedge-shaped cam surface *b* has entered between the long ends of the jaws *D* and *C*, and brought the outer ends together.

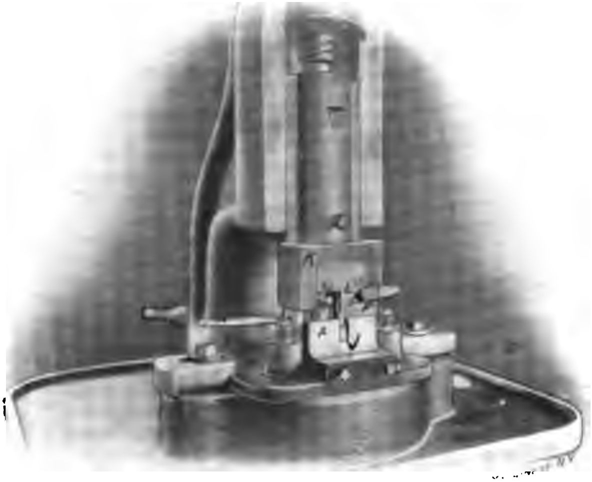

Fig. 39. Bending Device in use in Screw Press.

The jaws *D* and *C* and lever *E* are all attached to the holder *F*, which is a sliding fit on three vertical posts *G*, fast to the base *H* of the fixture. Slide *F* is held to the upper extreme of its travel against the lock nuts and washers at the top of posts *G* by spiral springs *J* at each post. These parts are shown to good advantage in the halftone, Fig. 38. *K* is a plunger mounted in the ram of the press. It bears on finished projections on slide *F* at three points as shown, while the hardened part *L* bears on the top of lever *D*, directly over the work. When *K* and *L* strike slide *F* and lever *D* in their descent, they carry with it

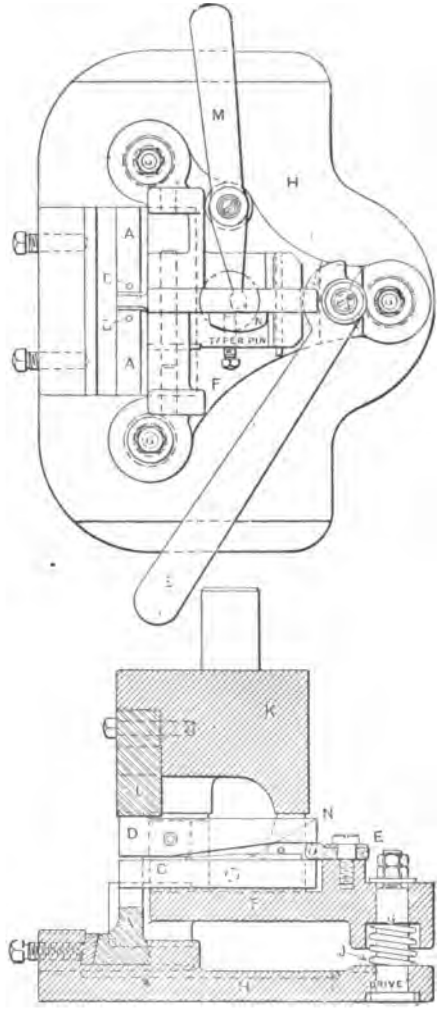

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Fig. 40. Construction of Bending Attachment.

the slide and its attached levers, and the work as well, against the slight resistance of springs *J*. The work grasped between the levers is thus carried down through the opening in die *A*. This action serves to bend the part to the form desired. Fig. 39 shows the operation completed. As shown, this work is done in a hand screw press. This is another example of manufacturing methods which at first sight seem rather crude, but which have proved, in the opinion of the superintendent of this shop, to be most satisfactory, his contention of greater accuracy and more uniform results from such methods applying particularly in the case of forming operations of this kind.

The piece is ejected from the tool at the completion of the bending by lever *M*, which thrusts forward the ejector *N*. This ejector is at its working end slightly less in thickness than the stock of the punching operated on, and is thus able to enter freely between the jaws

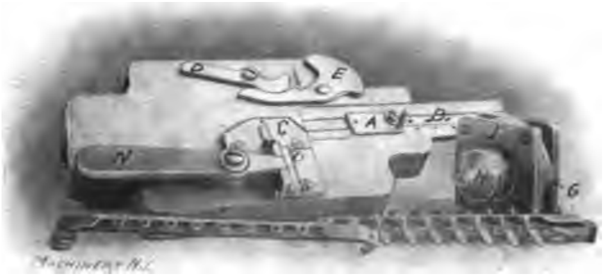

Fig. 41. Double Punching Die shown in Fig. 34.

and eject the work. In this tool, members *A*, *C* and *D* are changed for different parts, the rest of the structure being the same, and serving for a number of different operations.

A Die for Double Punching.

In the case just described, where double bearings occur, the holes are punched before bending. This is not always the case, however. In samples 1 and 5 in Fig. 33, the parts are first bent and then punched, the operation being performed in a very interesting way. The punch descends and makes the hole in the upper thickness of the stock. Continuing through an intermediate die, and carrying before it the punched-out stock, it arrives at the second or lower thickness of stock. The continued movement of the punch then presses the little plug of punched-out metal through the lower thickness of stock, and this forms the second hole. Strange to say, it has been found in practice that this second hole is generally the better one of the two, even though it is made with a soft plug of steel instead of with a hardened punch.

The line cut Fig. 34 and the halftone Fig. 41 show the double punching tools used in making the pivot holes in sample 1, Fig. 33. This, it will be seen, is a progressive operation, all the parts in the lot being punched for one of the holes, after which the die is altered and the next hole in order is punched in all parts—and so on. The piece to be operated on is located lengthwise by slipping it over a gage pin in sliding block *A*, which may be adjusted to any position on slide *B* to suit the hole it is desired to punch at the time. Being located on block *A* in the manner described, it is swung around until the intermediate die *C* enters the channel formed by the two sides of the work. Cam lever *D* is then swung to the position shown in the line cut, where it has brought clamp lever *E* against the stock, holding it firmly in position for the operation. The punch *F* is a simple turned piece of hardened steel, held by a taper pin in punch holder *G*. It is surrounded by a stripper *H* which is screwed to a holder *J*, backed by the usual rubber spring at *K*. This serves to hold the work firmly during the operation, and strip the work from the punch when it returns to its upward position. As before described, the punch in its descent breaks through the upper thickness of stock, carries the plug of soft metal thus formed before it until it comes in contact with the lower thickness, where it forces the plug through, and forms the lower hole. It will be noticed that intermediate die *C*, though held firmly so far as displacement horizontally in any direction is concerned, is yet provided with a rocking face where it bears on the body of the die *L*. This arrangement takes the strain of the punching from the slender intermediate die, which is thus bent downward until it is firmly supported by the stock of the part being worked on beneath it. For removing the work after the operation, an ejector *M* is provided, with a handle *N*, which operates in a way which will be easily understood from an inspection of Fig. 41. It is not shown in Fig. 34, having been added at a date later than that of the drawing from which this cut was made.

Practice in Hardening Punches, Etc.

Blanking punches are hardened in this shop in a way that is originated here and not practiced elsewhere, at least not to any great extent. After the blanking punch has been cut into the female portion of the die, and finished ready for hardening, it is placed in the fire and brought to a slightly lower heat than ordinarily used for hardening clear through. Cyanide is then deposited on the parts of the tool to be hardened—that is, on the periphery of the cutting edge. It is allowed to "soak in," it sometimes being necessary to apply cyanide two or three times, depending on the size and bulk of the punch. It is then again brought to the proper heat, which should be a little lower than is ordinarily used for hardening clear through. Then it is quenched in oil. With large and bulky pieces it is first necessary to immerse the work in water as a preliminary cooling operation. This immersion should merely be a dash into the water and out again, after which the piece is put into the oil until cooled.

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No. 14

DETAILS OF MACHINE TOOL DESIGN

CONTENTS

Elementary Principles of Cone Pulleys and Belts, by W. L. CHENEY - - - - - - - - - 3

Cone Pulley Radii, by JOHN J. HARMAN - - - - 7

Strength of Countershafts, by FRANK B. KLEINHANS - 15

Tumbler Gear Design, by JOHN EDGAR - - - - 19

Faults of Iron Castings, by FORREST E. CARDULLO - 26

Proportions of Machines Built in a Series of Sizes, by STANLEY H. MOORE - - - - - - 36

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CHAPTER I.

ELEMENTARY PRINCIPLES OF CONE PULLEYS AND BELTS.

Everyone knows that cone pulleys are usually made with regular steps; that is, if it is one inch from one step to the next, it is also one inch from the next to its next, and so on, the reason being that when the centers of the shafts on which the cones run are a fair distance apart, the belt will pass very nearly half way around that part of each cone on which it is running, and the length of the belt will consequently be approximately equal to twice the distance between the shafts, added to half the circumference of the grade of one of the cones on which it is running, and half the circumference of the grade of the other cone on which it is running. As the steps are even, the half circumference of any two grades of each cone will, when added together, produce the same result. For example, if we had two cones, the diameters of the several grades of which were 6, 8, 10 and 12 inches, it is evident that the sum of half the diameters taken anywhere along the cones, as they would be set up for work, would in every case be the same; and if the diameters are the same, it follows that the circumference must also be the same, and, of course, that half the circumference must be the same, so that when the centers of the shafts are a fair distance apart, and the difference between the largest and smallest step of the cone not too great, the same belt will run equally well anywhere on the cone, because it runs so near half way around each grade of the two cones on which it is running, that the slight difference is within the practical limit of the stretch of the belt.

But when the shafts are near together, and when the difference between the largest and smallest step of the cone is considerable, the belt is not elastic enough to make up this difference. Fig. 1 shows a three-step cone, the grades being 4, 18, and 32 inches diameter, respectively, each being 14 inches on the diameter different from the next, and the step being therefore half of 14 inches, that is, 7 inches in each case. Of course, it is not likely that such a cone as this would be made for practical use, but it is well to go to extremes when looking for a principle. Now, it is evident that two cones, even if like the one shown in the cut, were set up far enough apart, they would still allow the belt to run very nearly half way around each grade of the two cones, the angularity of the belt would be slight, and the length of belt would therefore still be as mentioned above.

But (again taking an extreme case) by reference to Fig. 2, which is intended to represent a belt running from the largest grade of one cone to the smallest grade of the other cone, we see that the belt runs three quarters of the way around the large pulley, and only one quarter of the way around the small one, the distance between the shafts in this case being 19¾ inches.

The length of this belt will evidently be equal to three quarters of the distance around the large pulley, plus one quarter the distance around the small pulley, plus the distances *A* and *B*, which we find to be each 14 inches. The circumference of a 32-inch diameter pulley is 100½ inches, and the circumference of a 4-inch diameter pulley is 12½ inches (near enough for our present purpose); three quarters of 100½ is 75⅜, and one quarter of 12½ is 3⅛; the length of a belt, then, to go around a 4-inch pulley and a 32-inch pulley, running at a distance of 19¾ inches apart, is 75⅜ plus 3⅛ plus 14 plus 14; total, 106½ inches.

Now, let us take the middle cone, when the belt is running on two pulleys, both 18 inches diameter (see Fig. 3), and, of course, the same distance apart as before. The circumference of an 18-inch pulley is 56½ inches, and half the circumference of two 18-inch pulleys is evidently the same as the whole circumference of one 18-inch pulley; the length of belt in this case will then evidently be 56½ plus 19¾ plus 19¾; total, 96 inches. It is therefore evident that a belt long enough to run on a 4- and 32-inch pulley, 19¾ inches apart, is 10½ inches too long to run on two 18-inch pulleys 19¾ inches apart, and, of course, it is therefore 10½ inches too long to run on the middle grades of such a cone as we have in consideration.

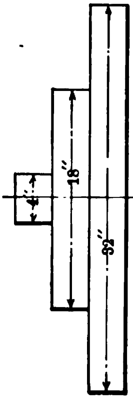
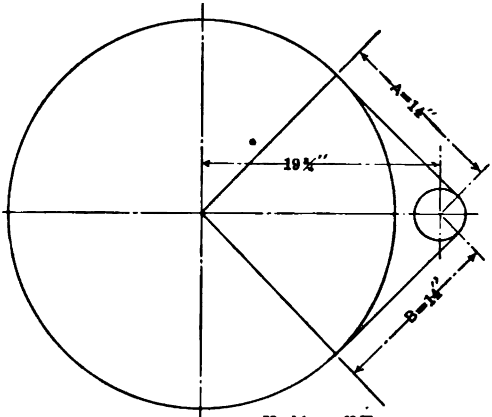


Fig. 1.



Machinery,N.Y.

Fig. 2.

The thing to do, then, is to make the middle grades of these cones (or the two 18-inch pulleys) enough larger than 18 inches diameter to just take up this 10½ inches of belt, and if this were the only case we had to deal with, it would be very easy to settle it by saying that as half the circumference of two 18-inch pulleys is the same as the whole circumference of one 18-inch pulley, we should make the two 18-inch pulleys enough larger in diameter to make an additional circumference of 10½ inches; and as 3⅜ inches is nearly the diameter of a 10½-inch circumference pulley, by making the middle of both cones 18 plus 3⅝ inches diameter (that is, 21⅝ inches diameter) our trouble would be ended in this particular case. It is easy enough to see, by looking at Fig. 2, that the belt being obliged to go three quarters of the way around the large pulley, is what makes it so much too long to go around the two middle pulleys, where, of course, it goes but half way around each. But, of course, what we want is some way of calculating the diameters to turn any pair of cones, running at any distance apart.

If we were to draw these same 32- and 4-inch pulleys twice 19¾ inches apart, and then three times 19¾ inches apart, and so on, until we got them far enough apart so that the belt would practically run half way around each, and should calculate the diameter of the middle grade of the cone to fit each distance, we would probably formulate a rule that would work for any distance apart, with this particular cone; but as it is evident that the further apart the cones are to run, the nearer to the nominal diameter of 18 inches must the middle of the cones be turned, so also must it be evident that the less difference between the largest and smallest diameter of the cone, the less must also be the excess over nominal diameter of the middle of the cones.

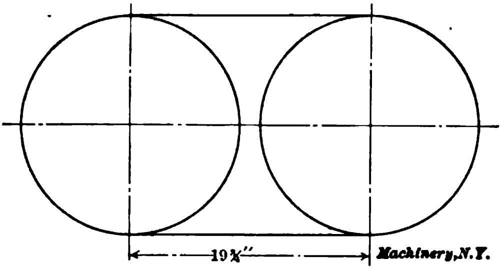


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Fig. 3.

Any method, then, of calculating such problems must take both of these things into consideration. The nominal diameter of the middle of any cone will be equal to half the sum of the diameters of the largest and smallest part respectively. This is almost self-evident, and no proof of it is necessary in this connection. What we want, then, is some way to find out how much larger than the nominal diameter to turn any one cone or cones to fit the conditions under which they are to run. The following formula is the result of a thorough investigation of this subject by Prof. Rankine, and has proved itself to be practically correct in the shop, as well as satisfactory to those mathematicians who are competent to criticise it. This formula is:

$$R = \frac{R_1 + R_2}{2} + \frac{(R_1 - R_2)^2}{2\pi C}.$$

This formula translated into plain English means that the radius of the center of a cone will be equal to the radius of the smallest part, added to the radius of the largest part, and this sum divided by 2, and added to this the difference in radii between the largest and smallest part squared, and then divided by twice the center distance between the cones multiplied by 3.1416. That is, the first half of the formula gives the radius at the center of a cone, when the largest and smallest radii are known, and, of course, if the middle radius is equal to the smallest radius added to the largest radius and the sum divided by 2, it follows that the middle diameter is equal to half the sum of the diameters of the largest and smallest part, respectively, as mentioned above. The second part of the formula allows us to calculate how much larger than this nominal diameter to make the middle of a cone, no matter what the size or center distance.

Applying this formula to the case of the cones shown in Figs. 1, 2 and 3, we find the radius of the middle of the cone to be 10 6/10 inches, or, what is the same thing, the diameter to be 21 2/10 inches, which, in view of the extreme case in consideration, is very near the first result obtained (21⅝), and shows that the formula is perfectly safe in any case likely to occur in practice.

When this formula is reduced so as to express the numerical value of diameters instead of radii, it takes the following form:

$$\text{Diameter at center of cone} = \frac{D + d}{2} + \frac{(D - d)^2}{12\tfrac{1}{2}C},$$

the 12½ being the nearest value in plain and easy figures to which the quantity containing π in the original formula can be reduced.

Applying this simplified formula to the cone which we have been considering, it will be found that the middle diameter is 21 2/10, the same as by the original Rankine formula.

If a cone has five steps instead of three, it will be practically correct to add half as much to the nominal diameters of the second and fourth grades as was added to the middle grade, or, if it has four grades, add two-thirds of what is found by the calculation to the second and third grades (as there is evidently no middle grade). If more than four or five grades, add to each grade according to the same principle.

We have so far been considering two similar cones, but it often happens that one cone is larger than the other. In such case the problem becomes a little longer to work, and the length of belt necessary to go around each pair of steps of the cones must be used to find the diameters; that is, starting with one end of the cone, find the length of belt, and then calculate how much larger or smaller (as the case may be) than the nominal diameter it is necessary to make each grade, in order to make the same length of belt run properly.

Prof. Rankine has worked out a formula for the length of belt also, which, reduced to diameters, is as follows:

$$\text{Length of belt} = 2C + \frac{11D + 11d}{7} + \frac{(D - d)^2}{4C}.$$

That is, the length of a belt to pass around any two pulleys (and, of course, a cone is simply a set of pulleys) is the sum of the following quantities: First, twice the center distance of the shafts; second, 11 times the diameter of the larger pulley, plus 11 times the diameter of the smaller pulley, and this sum divided by 7. This gives the nominal length of belt, or what would be practically correct if the center distance was fairly great; for the excess, the last part of the formula must be used, which is the difference between the diameters of the larger and smaller pulley squared, and this result divided by 4 times the center distance.

Having found the length of belt to run on one end of the cones, and keeping this for a starter, we can easily find how much to add to, or take from, the nominal diameter of any other part of the cone to make the same belt run, as explained above. If, for instance, we find that the nominal diameters of the next grades that we try bring the length of belt one-half inch shorter than the first calculation, we add enough to one or both diameters to make up one-half inch of circumference, which would be about 5/32 of diameter, and this could all be added to one pulley, or half of it could be added to each pulley, as convenient, and this would be practically correct.

CHAPTER II.

CONE PULLEY RADII.

In the present chapter a method presented by Dr. L. Burmester in his "Lehrbuch der Kinematik," for the solution of the cone pulley problem, has been extensively treated. Dr. Burmester's method is entirely graphical, and is exceedingly simple in application. While it is not theoretically exact, it is, as will be shown later, much more accurate than practice requires.

In order to bring out more clearly the points which will come up in the case of open belts, let us first consider the simple case of crossed belts. It is a well-known fact that in this case the only calculation necessary in order to find the radii of the various steps is to make the sum of the radii of any two corresponding steps a constant. This may be shown in the following manner:

a = radius of step, driving cone.

A = length of belt from contact on driving cone to contact on driven cone.

b = radius of step, driven cone.

E = distance between centers of cones.

K = the constant sum of the radii of two corresponding steps.

θ = angle shown, Figs. 4 and 5.

Then

$$\sin \theta_1 = \frac{a_1 + b_1}{E} = \frac{K}{E}$$

$$\sin \theta_2 = \frac{a_2 + b_2}{E} = \frac{K}{E}$$

Therefore $\theta_1 = \theta_2 = \theta =$ a constant.

Therefore the arc of contact on each pulley $= 180° + 2\theta =$ a constant.

$$\text{Also } \cot\theta = \frac{A_1}{a_1 + b_1} = \frac{A_1}{K} = \frac{A_2}{a_2 + b_2} = \frac{A_2}{K}$$

$$A_1 = A_2 = K \cot\theta = \text{a constant.}$$

$$\text{But length of belt} = 2A + \frac{180° + 2\theta}{360°} \times (2\pi a_1 + 2\pi b_1)$$

$$= 2A + \frac{180° + 2\theta}{360°} \times 2\pi \times (a_1 + b_1)$$

in which, as has been shown above, all the terms are constants, therefore length of belt is constant.

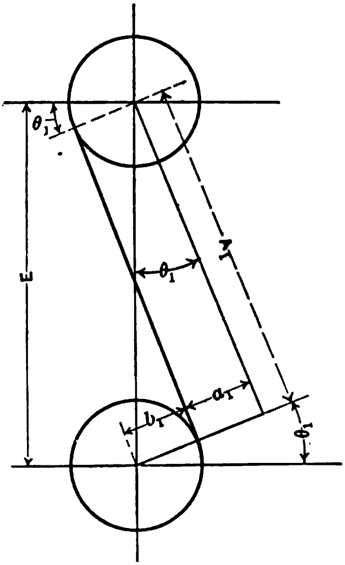

Fig. 4.

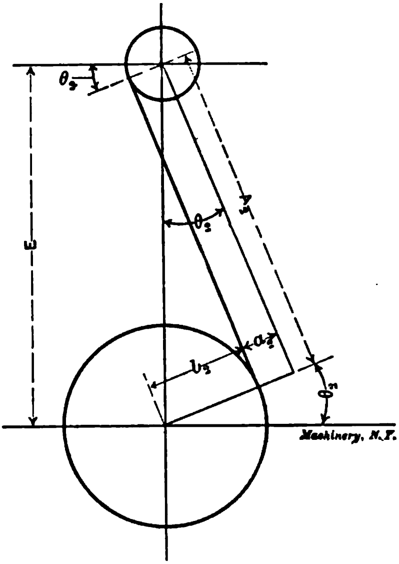

Fig. 5.

Note: The subscript applied to the letters denotes that the letters are used for the corresponding quantities in a special case; thus a_1 refers to a in Fig. 4.

The radii of the various steps may be determined graphically by the following diagram (Fig. 6):

Draw a horizontal line from *A*, and also draw *AC* making an angle of 45 degrees with it. On this line lay off *AS* equal to the distance between the cone centers, using any scale most convenient, bearing in mind, however, that the scale adopted now must be used consistently throughout the diagram. At *S* erect the perpendicular *TST'* to the line *ASC*. From some convenient point on *AC*, as *D*, drop a vertical equal to some known radius of the cone *a*, as *DE*, and then from *E* measure back on this vertical the radius of the corresponding step on cone *b*, as *EF*, and from these points *E* and *F* draw lines parallel to *ASC*. From the point *G*, where the line *FG* intersects the line *TST'*, drop a vertical. This will intersect the line *EH* in *H*. Through *H* draw the horizontal *MN*, *O* being the point where this line intersects the line *TST'*. Then, distances on the line *MO* may be taken to represent radii on cone *a*; and to find the corresponding radii on cone *b* erect perpendiculars at the extremities of these radii, producing them until they intersect the line *TST'*. These perpendiculars then represent the desired radii. It may be shown as follows that the sum of the two corresponding radii, as obtained from this diagram, is always a constant, and the diagram therefore satisfies the conditions for crossed belts.

Let *MJ* represent any radius on cone *a*, then *JI* represents the corresponding radius on cone *b*.

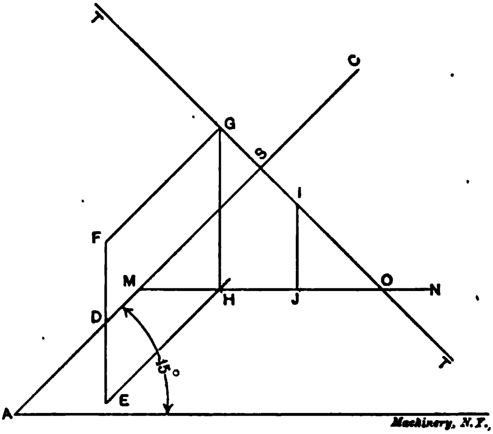

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Fig. 6.

The $\angle JIO = \angle JOI = 45$ degrees.

Therefore $JI = JO$.

Therefore $MJ + JI = MJ + JO = MO =$ a constant.

Dr. Burmester's diagram for open belts is a modification of the diagram just shown, the only difference being that the line *TST'* is replaced by a curve. This curve was determined by plotting a series of points, and after several pages of exceedingly intricate mathematics he arrives at the astonishing result that this curve can be replaced by a simple circular arc without any appreciable error.

The diagram is shown in Fig. 7, and may be drawn as follows: Proceed as in Fig. 6 until the line *TST'* is drawn, then lay off distance *SK* equal to $\frac{1}{2}$ *AS*. Next, with the center at *A*, and a radius equal to *AK*, describe the arc *XY*, and the diagram is ready for use.

In order to give an idea of the extreme accuracy of the diagram, let us observe the values obtained by Dr. Burmester in his calculations.

Let $R = AK$ (Fig. 7).

When $\theta = 0$, $R = E \times 1.11815172$.

$\theta = 15°$, $R = E \times 1.11806842$.

$\theta = 30°$, $R = E \times 1.11798671$.

$\theta = 45°$, $R = E \times 1.11803397$.

The value used for R in the diagram is $E \times 1.11803397$; so the maximum error of R occurs when $\theta = 0$, and is equal to $E \times 0.00011775$, which is much more accurate than the work of the most careful draftsman. Dr. Burmeister gives values of R up to $\theta = 90°$, but as it is evident from Fig. 8 that it would be practically impossible to have a value of θ greater than 45°, the writer has omitted the other values.

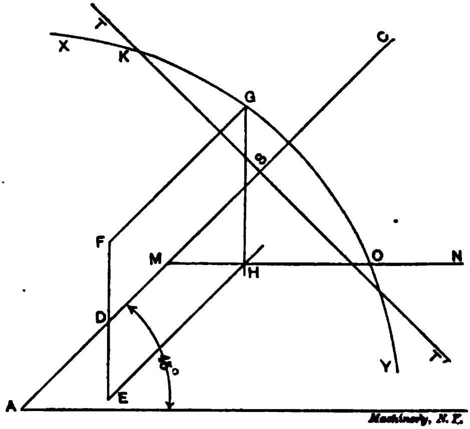

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Fig. 7.

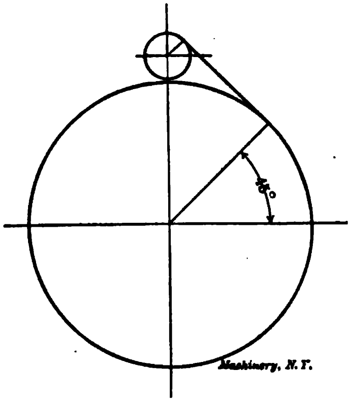

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Fig. 8.

In order to make the use of the diagram perfectly clear, let us solve the following problems:

Problem 1. Fig. 9.

Given:

Distance between centers of cones = 3′ 4″.

Diameters of driving cone, 4", 8", 14", 20".
Diameters of driven cone, X, X, 14", X.

Required:

All diameters of driven cone.

Lay out the diagram and determine the point *M* as previously directed. Now the radii of driving cone may be laid off as abscissas or ordinates, whichever happens to be the more convenient, as the results obtained will be exactly the same in either case. In this particular problem it is evidently more convenient to lay them off as abscissas. Then the ordinates erected at the ends of these abscissas will represent the corresponding radii of the driven cone. The problem is solved in Fig. 9 and the following results obtained:

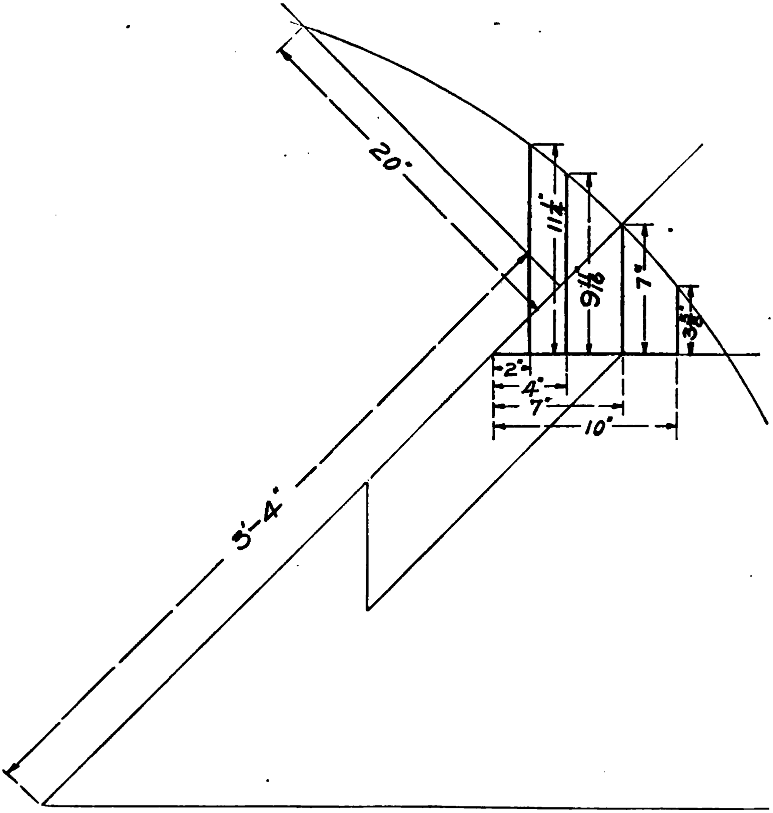

Fig. 9. Solution of Cone Pulley Problem when Diameters of one Pulley and Center Distance are Known.

Results:

Diameters of driven cone, 22½", 19⅜", 14", and 7¼".

This problem does not bring out all of the fine points of the diagram, so let us solve a more complicated one, in which the different steps of the cone are to transmit given velocities.

Problem 2. Fig. 10.

Given:

Distance between centers of cones = 3′ 4″.

Maximum velocity of belt (assumed) 30 feet per second.

R. P. M. of driving cone = 240.

Required:

Driven cone to make 100, 240, 400 and 580 R. P. M.

The maximum belt speed will be attained when the belt is on the largest step of the driving cone.

Therefore

$$\frac{2\pi a_1 \times 240}{12 \times 60} = 30; \quad 2a_1 = 28\tfrac{5}{8}''$$

$$\text{But } \frac{2b_1}{28\tfrac{5}{8}} = \frac{240}{580}; \quad 2b_1 = 11\tfrac{7}{8}''.$$

Now having obtained a value for a_1 and b_1, the point M on the diagram may be found. Next draw a line from M as MO, inclined so that any horizontal projection, as MN, will be to the corresponding vertical projection, NO, as the R. P. M. of the driver are to the R. P. M. of the driven; thus,

$$\frac{MN}{NO} = \frac{\text{R. P. M. of driving cone}}{\text{R. P. M. of driven cone}}.$$

Also from similar triangles

$$\frac{MN}{NO} = \frac{MN'}{N'O'}.$$

But we know that

$$\frac{\text{R. P. M. of driving cone}}{\text{R. P. M. of driven cone}} = \frac{\text{rad. of driven cone}}{\text{rad. of driving cone}}$$

Therefore MN' equals radius of driven cone, while $N'O'$ equals radius of driving cone, thus making, for this case, radii of driving cone vertical and of driven cone horizontal. The problem is solved in Fig. 10 and the following results obtained:

Results:

Dia. of driving cone, $28\tfrac{5}{8}''$, $25\tfrac{5}{8}''$, $20\tfrac{3}{4}''$, $11\tfrac{7}{8}''$.

Dia. of driven cone, $11\tfrac{7}{8}''$, $15\tfrac{3}{8}''$, $20\tfrac{3}{4}''$, $28\tfrac{5}{8}''$.

We have seen that the Burmester diagram is under all conditions much more exact than is required in practice; and a more compact, simpler, or quicker method of finding cone pulley radii could not be desired. An experienced draftsman should be able to solve a problem like No. 2 above in less than 10 minutes, while to obtain the same results by an analytical method would require as many hours. Results of sufficient accuracy can usually be obtained by making the diagram to half scale, although there is no reason for reducing the scale, unless the distance between centers is inconveniently large, and in that case the results do not need to be so accurate, as the belt will stand more stretching.

Another graphical method for laying out a pair of cone pulleys is as follows: First draw straight line *A A*, Fig. 11, supposed to connect the centers of the cones to be laid out; then set off the centers of the cones *B* and *C* on line *A A* (full size is best); then bisect the distance between the centers of the cones and draw perpendicular line *D E*. Now assume the size of the two cones—say the largest is 25 inches and the smallest 3 inches diameter. Then draw a line tangent to the circles, or the line representing the inside of the belt *G*, which will intersect the line *D E* at *E*, and taking the point *E* for a center scribe the circle *F*. Then divide the circle *F*, commencing at the line of the belt *G*, into as many parts as needed, of a length to suit the required speeds.

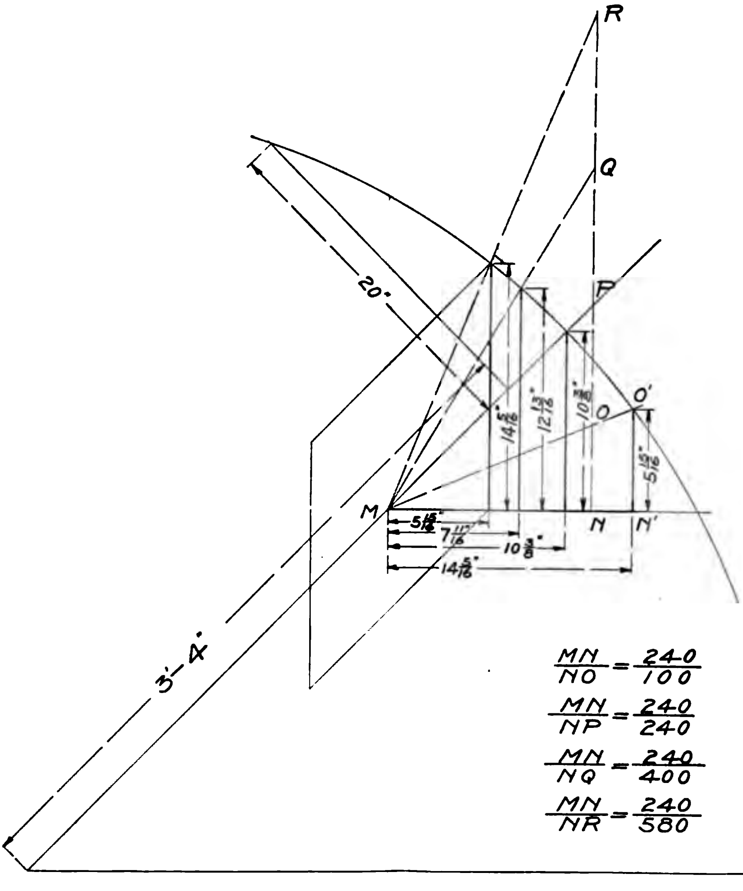

Fig. 10. Solution of Cone Pulley Problem when Velocity Ratios, Maximum Belt Speed, Center Distance, and R. P. M. of Driver are Known.

Draw the other radiating belt lines through the point *E* and the divisions on the circle *F*, extending them toward the cone *B*, and they will be the inside of the other belt lines. Draw circles tangent to these lines. We now have all the diameters of the rest of the steps of the cone to match the first, and the belts will correctly fit all the steps. This is, of course, only an approximation rule. This method was contributed to the June, 1905, issue of MACHINERY by John Swanberg.

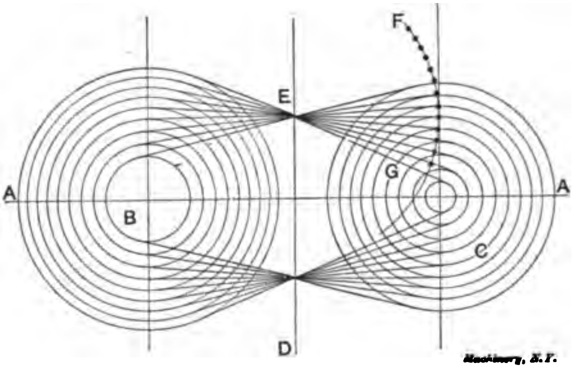

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Fig. 11. Simple Graphical Solution of Cone Pulley Problem.

CHAPTER III.

STRENGTH OF COUNTERSHAFTS.

There is scarcely a shop in existence which has not had a more or less serious accident from a countershaft some time in its history. It may have been caused by a heavy pulley running very much out of balance, or the shaft may have been bent in the beginning. Possibly the shaft was too light, or too long between hangers. The latter is responsible for most of the trouble, and is the one with which this discussion is principally concerned.

There are two methods in vogue for turning cones and pulleys; one is to set the rough casting to run true on the inside, and the other on the outside. This latter method makes a cheaper and an easier job, but when turned, it requires an enormous amount of metal to balance it. And here is the source of considerable trouble. We may balance a large cone perfectly on straight edges, but that is a standing balance only; and when the cone is put in place and speeded up to several hundred revolutions per minute, it shakes, and shows that it is decidedly out of balance. The trouble is that we have not placed the balance weights directly opposite, or in the plane of the heavy portion of the cone. The result is that neither weight, when rotating, has its counterbalance pulling in the same line, and, of course, the pulley is sure to be out of balance. All cones and all other pulleys which have a wide face should be set to run true on the inside before turning.

A certain countershaft failed because it had been welded near the center. The weld twisted and bent open, and some one was badly injured by the fall. A weld in machine steel is so very uncertain that it should never be trusted for such a purpose. The extra expense of a new shaft would not warrant the hazard of such a risk.

In the calculations which follow, the spring of the shaft is limited to 0.06 of an inch. There are plenty of countershafts which have been running for years with about this much spring. Now, from the general formula for the deflection of a simple beam, we have:

$$\text{The deflection, or spring} = \frac{W L^3}{48 E I}$$

in which $W =$ the load at the center in pounds.

$L =$ the length between center of hangers in inches.

$E =$ the coefficient of elasticity $= 29{,}000{,}000$.

$I =$ the moment of inertia of the cross-section of the shaft.

For a round shaft,

$$I = \frac{\pi d^4}{64} \qquad (1)$$

in which d = the diameter of the shaft in inches. We then have:

$$\frac{W L^3}{48 E I} = 0.06 \qquad (2)$$

From (1) and (2), we have $\frac{64 W L^3}{48 E \pi d^4} = 0.06$, and

$$L = \sqrt[3]{\frac{0.06 \times 48 E \pi d^4}{64 W}} = \sqrt[3]{\frac{0.06 \times 48 \times 29{,}000{,}000 \times \pi}{64} \times \frac{d^4}{W}}$$

$$= \sqrt[3]{4{,}100{,}000 \frac{d^4}{W}} \qquad (3)$$

Fig. 12 shows a countershaft which is in actual service, and which is known to be all right. *A* and *B* are keyed to the shaft. *C* and *D* are loose pulleys arranged for open and cross belts.

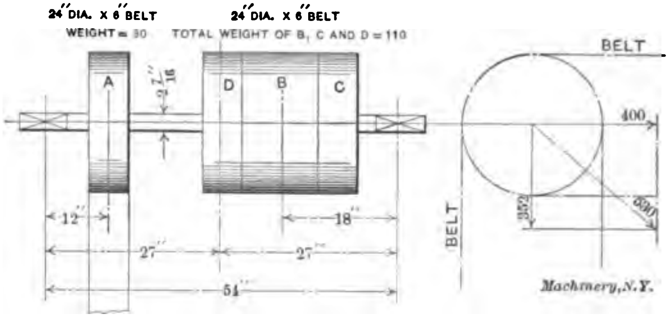

Fig. 12.

A weighs 30 pounds, and *B*, *C* and *D* weigh 110 pounds. The belts run as shown in the figure. If *A* weighs 30 pounds, and the centers of the hangers are 54 inches apart, then by taking the left-hand hanger as the center of moment, we have $30 \times 12 = x \times 27$, when x is the weight at the center. Solving we find

$$x = \frac{30 \times 12}{27} = 13$$

In the same way, by taking the right-hand hanger for the center moment, we find that

$$x_2 = \frac{110 \times 18}{27} = 73$$

As to the belt pull, it is possible for a single belt to run up to 70 pounds per inch of width of belt, and a double belt can be taken at 100 pounds. As a double belt is used in this case, and as the slack side of the belt is very loose when the tight side is pulling its maximum, we will take the pull at the pulley $A = 6 \times 100 = 600$ pounds, and getting this in terms of a load at the center, we have

$$x_3 = \frac{600 \times 12}{27} = 266$$

and the downward pull is $13 + 73 + 266 = 352$.

The pull at the pulley *B* will be $6 \times 100 = 600$, and by transferring this to the center we have

$$\frac{600 \times 18}{27} = 400$$

The resultant of these two forces will be the diagonal of the force diagram, and is equal to 530 pounds, which is equal to *W* in the formula. Introducing these terms in equation (3) we have

$$L = \sqrt[3]{4,100,000 \frac{(2.44)^4}{530}}$$

and by solving we find $L = 65$, which means that for this condition of loading the countershaft would be safe even with the hangers 65 inches apart.

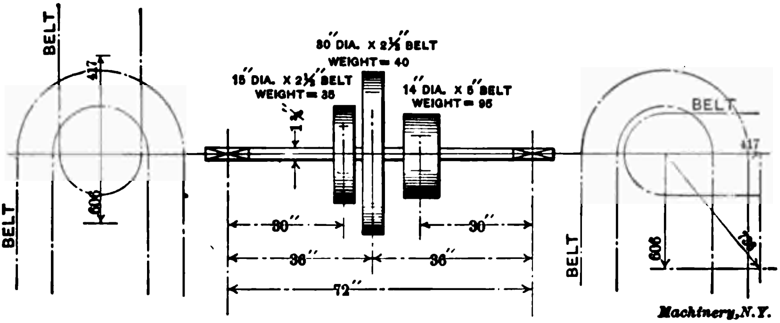

Fig. 13.

Fig. 13 represents another countershaft taken from actual service. It is belted as shown on the left-hand view, and is running all right, although it looks rather flimsy, and one would consider it unsafe. Taking the moments of the weights of the pulleys and belt pull about the right- and left-hand supports, and finding the equivalent pull at the center, we obtain:

Weight at center due to pulleys =		148
Pull on 30-inch pulley = $2\frac{1}{2} \times 100$ =		250
Pull on 15-inch pulley = $2\frac{1}{2} \times 100$;	$\frac{250 \times 30}{36} =$	208
Total downward pull =		606
Pull on 14-inch pulley = 5×100;	$\frac{500 \times 30}{36} =$	417
Resultant downward pull =		189

Introducing this value of *W* in equation (3) we have,

$$L = \sqrt[3]{4,100,000 \frac{(1.75)^4}{189}} = 59$$

This is considerably less than the distance between the hangers, and it shows that it is not safe to place the hangers in this way. If the belts ran as shown at the right-hand side of Fig. 13, we would then have:

Weight due to pulleys (as before) =	148
Pull on 30-inch pulley =	250
Pull on 15-inch pulley =	208
Total downward pull =	606
Horizontal pull on 14-inch pulley =	417

From these two forces we find a resultant of $W = 736$. Substituting this in (3) and solving as before, we find $L = 38$, which is the greatest safe distance between hangers for this condition of loading.

There are cases where one must have an extra long shaft in order to work in the pulleys, cones, etc., as shown in Fig. 14. Here the downward loads amount to 820 pounds, and the pull at right angles amounts to 360 pounds. The resultant 895 pounds $= W$.

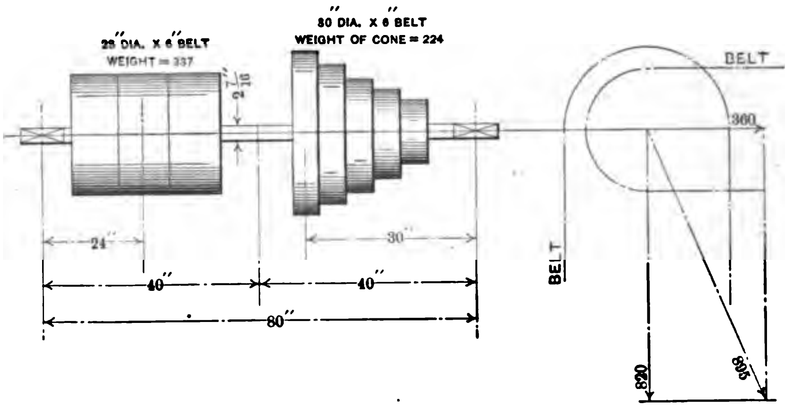

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Fig. 14.

Introducing in the formula we have

$$L = \sqrt[3]{4{,}100{,}000 \frac{(2.44)^4}{895}} = 55$$

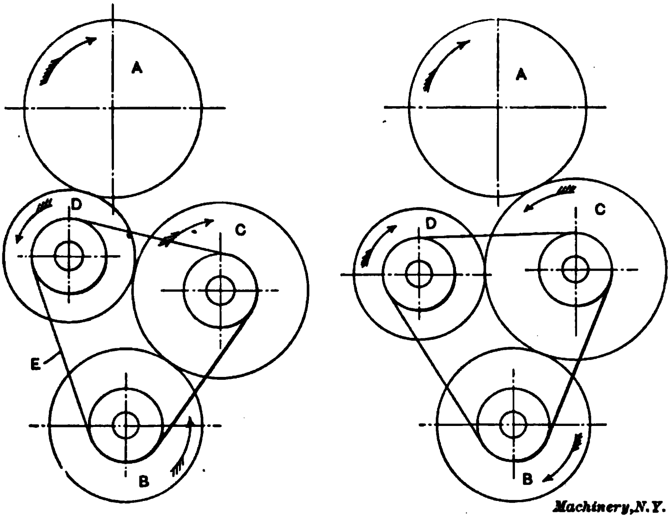
This means that for this condition of loading, the center distance should not exceed 55 inches, and since in this case it could not be made as small as this, the pulleys should be arranged for a third hanger.

In every case, therefore, where the centers are so far apart as formula (3) would indicate to be unsafe, a third hanger should be used. If all the flimsy countershafts had a third hanger added to them there is no doubt but that the number of accidents would be greatly diminished. In the above calculation the weight of the countershaft has not been considered, as it is usually very small. If the belts run at any other angle than that shown, the construction is made in exactly the same way, using the required angle instead of a right angle, the resultant of the two forces being used as W in the formula.

CHAPTER IV.

TUMBLER GEAR DESIGN.

Of the different mechanisms that have been used in the machine tools of the past, one—the tumbler gear—could be found in some form or other in almost every machine. Its office, in most cases, was to reverse the direction of the feed. Fig. 15 shows the usual form in which it is found when used for this purpose. The gears *A* and *B* are to be connected so that motion may be transmitted from one, which runs constantly in one direction, to the other, which it is desired to run in either direction. Suppose that *A* is the driver and runs as shown by the arrow. As connected, *A* drives *B* through the intermediate gears *D* and *C*, *B* rotating in an opposite direction to *A*, as shown by the arrows.



Figs. 15 and 16. Examples of Tumbler Gears.

This mechanism is termed the tumbler gear, because the gears *D* and *C* are supported in a frame which swings about the axis of either the driving or the driven gear. In the case in hand, the intermediate gears are carried in the frame *E*, which rotates about the axis of the gear *B*. Some means, not shown, must be provided by which the rocker frame may be changed from one position to the other, and locked. Fig. 16 shows the mechanism shifted so that the motions of *A* and *B* are in the same direction.

The tumbler gear has been used as a reversing gear ever since present forms of machine tools were first invented. While it has always given considerable trouble, it has shown up to disadvantage mostly when applied to the modern machine with positive gear feed, where great power has to be transmitted by it. It is the purpose here to show where this gear may be used to advantage, and also to explain the theory on which the principles of its design are based.

All of us have met with this mechanism in some form or other, and may have formed an unfavorable opinion. The prejudice thus created keeps us from fully appreciating the tumbler gear, even when properly designed, and when used in the right place. It has been placed by many along with the worm drive and the spiral gear as undesirable, and to be avoided unless it is absolutely impossible to get along without it. This opinion has been responsible for the adoption of many combinations used for purposes that rightly belong in the field of the tumbler gear, and many times, in order to avoid using this mechanism, much unnecessary complication has resulted.

What are the faults of the tumbler reversing gear? That one on So-and-So's lathe used to kick furiously when one tried to throw it over. Then, the one used on the milling machine used to go into mesh easily enough, but when any amount of strain was put onto it, the teeth used to crack and growl, showing that the tendency was to drag the gear farther into mesh, causing the teeth to bind on one another and sometimes break. Let us look into the case represented in Fig. 15. Fig. 17 shows the gear *D* just entering into mesh with *A*. An examination of this figure shows that the tendency is for the teeth of gear *A*, when they strike those of gear *D*, to cause the latter to rotate about the axis of the rocker frame, should the gear *B* be locked against turning. This tendency opposes the motion in the opposite direction necessary to bring the gears wholly into mesh. In practice, *B* is not locked, but it is necessary to overcome a certain amount of resistance in order that it may be set in motion, and the presence of this resistance has the same effect as if the gear were locked. The greater this resistance is, the greater is the effort necessary to bring the gears into working position.

Examining the conditions in the case of Fig. 16, we see that the effect would be just the opposite, that is, the gears would come into mesh of their own accord as soon as a contact is produced between the teeth of *A* and *C*. Practically no effort is necessary to bring the gears into mesh, but, in order to withdraw the gear *C* from *A*, considerable effort would be required. When the gears *C* and *A* are in mesh and transmit power, the tendency for gear *C* is to crowd farther into mesh with *A*, which has the effect of binding the teeth. Should the pressure of contact be sufficient, the binding tendency would cause the motion to cease, or would break the teeth. This is one of the points on which many have based their verdict against the tumbler gear, and when designed so that such results are obtained, it is not to be wondered at.

Direction of Tooth Pressure in Ordinary Cut Gears.

The first consideration in the design of tumbler gears in any form is that of tooth pressure and its line of application. As all cut gears used in machine tools are made to the 14½-degree involute system, we will confine ourselves to that system. In this, the force tending to revolve the driven gear is not a tangential force, applied as a tangent to the pitch circle, but is a force applied at an angle of 14½ degrees to the tangent of the pitch circle, this 14½-degree line being termed the line of pressure. In case that there may be some confusion as to the above statement regarding the tangential force and the line of pressure on the teeth, the case is graphically shown in Fig. 18. The tangential force is equal to the twisting moment divided by the radius of the pitch circle. This force is equivalent to that which transmits motion between two disks by friction alone, the diameters of the disks being equal to the pitch circles of the gears. This force is, in the case of a gear, resolved into two component forces. One component acts perpendicular to the tangential force and tends to force the gears apart; the other acts in the direction of the line of tooth pressure shown in Fig. 18. The tooth pressure thus is somewhat less than the total twisting force, and equals the twisting or tangential force multiplied by the cosine of 14½ degrees.

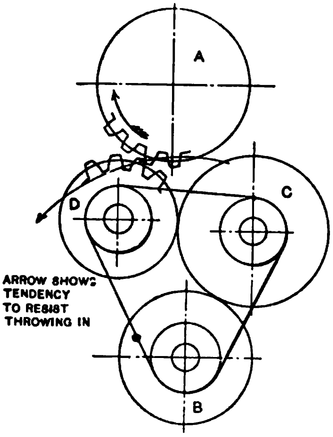


Fig. 17. Illustrating the Tendency to resist Throwing-in of Gear.

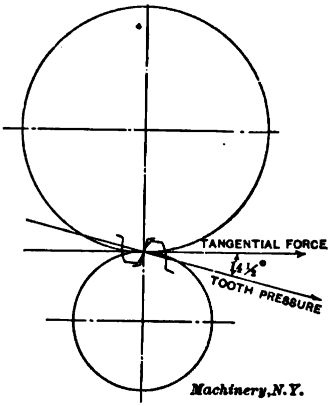


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Fig. 18. Illustrating the Direction of Line of Pressure.

Influence of Direction of Tooth Pressure on Tumbler Gear Design.

To show what effect the line of pressure has upon the layout of the tumbler gear, we will use the simple case shown in Fig. 19. In this figure, *A* is the driving gear and *B* is the driven gear. These gears are connected by means of the intermediate gear *C*, which is carried in the swing frame *E*, which, in turn, swings about the axis of *A*. This mechanism is a simple case of tumbler gear, and while it is little used, it is useful as a means for disconnecting a train of gears when it is desired to stop the motion of the driven section. If we consider the gear *B* locked in the position shown, and exert a turning effort on the gear *A* in the direction indicated by the arrow, this effort is transmitted by the teeth of *A* and *C*, and a pressure is produced between the teeth of *B* and *C*, two of which are shown in the cut. The direction in which this force is applied is shown by the line of pressure *H K*, and is exerted in the direction of *H*. Since every force is opposed by an equal and opposite force when in a state of equilibrium, we have in this instance a force or reaction opposing the force along the line of pressure referred to. It is this reaction that causes our troubles. In the mechanism shown in Fig. 19, the gear *C* and the link *E* are free to rotate about the axis of *A*, and since the line of pressure does not go through the center of gear *A*, the force acting along this line tends to rotate the arm *E* about the axis of *A*, the direction of rotation being dependent on which side of the center of *A* the line falls. Thus in Fig. 19, the line falls in a position that produces a tendency for the arm to force the gear *C* further into mesh with *B*. The twisting moment thus set up is equal to the tooth pressure multiplied by the normal distance from the axis of *A*, or *G L*.

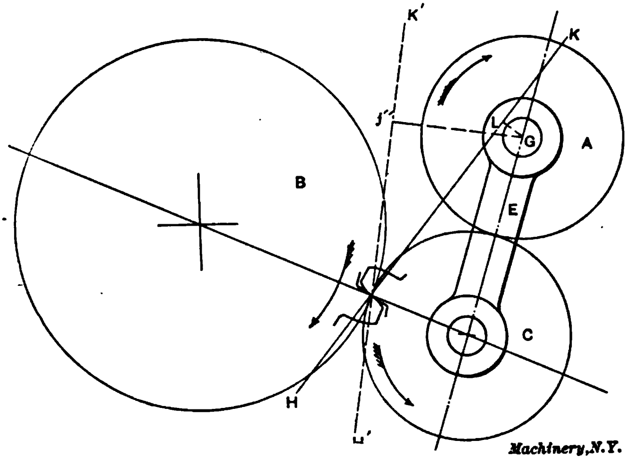


Fig. 19. Objectionable Tumbler Gear Design.

If now, instead of trying to turn the gear *A* in the direction of the arrow, we exert a torque in the other direction, the opposite sides of the teeth would come into contact, and the line of pressure would be located as shown by the dotted line *H′ K′*. The normal distance of this line from the axis of *A* is much greater than in the former case; consequently the twisting moment tending to rotate the arm *E* about the axis is also increased, but the direction in which the torque is applied has changed the direction in which the reacting force along the line of pressure acts, and, since this line falls on the same side of the axis, the tendency of the arm is to rotate in the opposite direction, and to separate the gears *C* and *B*. Had the line of pressure gone directly through the axis of the gear *A*, where *E* is pivoted, the effect of any force acting along it would have had no rotating influence upon the tumbler gear arm. That this would be the ideal case needs not to be mentioned, and it should be the aim of the designer to approach that condition as nearly as possible.

The tendency for the tumbler to crowd the gears into mesh might be of some advantage were it desirable to throw them into mesh while the gears are in motion; but in cases where any considerable amount of power is being transmitted, a very stiff and rigid design will be necessary for the tumbler frame and the locking device. It is also well in such cases, when setting the locking device, to have the gears mesh with plenty of play or backlash, so that, if there be any spring in the frame, the gears will not be likely to bind and cramp. Should *B* be the driver and run in the direction of the arrow, the line of pressure would be *H′ K′*. and the pressure would be in the direction of *H′*. The arm would then tend to carry the gear *C* out of mesh with *B*.

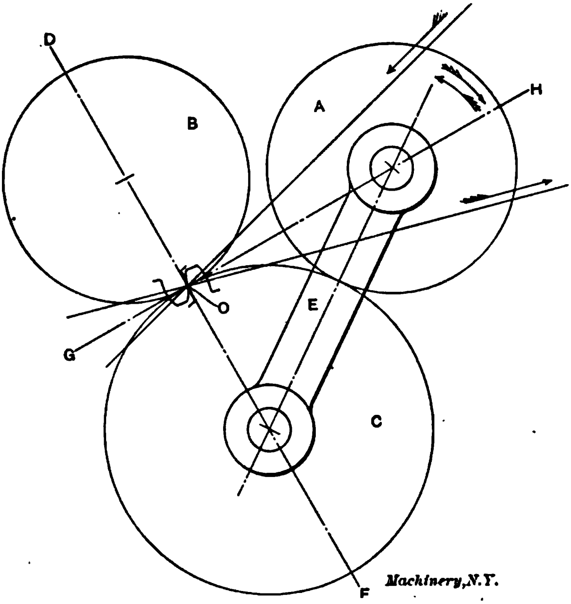


Fig. 20. Correct Design of Tumbler Gear to run in Both Directions.

Should the direction be reversed, *H K* would be the line of pressure, and the tendency would be to crowd the gear in.

The layout in Fig. 19 has two bad features. In the first place, the gears have a tendency to crowd farther into mesh, which limits the amount of power that can be transmitted, and increases the liability of breakage of the gear teeth and of the tumbler frame, should an overload be imposed upon the mechanism. Inaccuracy in the shape and spacing of the teeth aggravates the above conditions. In the second place, the mechanism should be used to transmit motion in but one direction.

In most cases the throwing in or out of the tumbler is a secondary matter, as it is done either when the gears are not in motion, or while not under load, if running. In such cases it should be the aim of the designer to overcome the objection of the crowding of the teeth into mesh by having the line of pressure properly located, so that the tendency is in the opposite direction. When it is desired to provide a tumbler gear that can be run in either direction, the layout in Fig. 20 is recommended. The object in this case is to have the twisting moment equal in either direction, and such that the gears have no crowding tendency. The arrangement in Fig. 20 is laid out as follows: Draw the pitch circles of the gears *B* and *C* and connect their centers by the line *D F*. Through the pitch point *O* draw a line *G H* normal to *D F*. Then locate the gear *A* at some point on *G H* so that its pitch circle will be tangent with that of *C*.

The Single Tumbler Gear.

The single tumbler gear is the basis of many of our modern rapid change speed and feed mechanisms, and the principles treated above apply to this as well as to the regular tumbler gear. Take the simple case shown in Fig. 21, which shows the pitch circles of a four-gear cone and the driver *A* and tumbler gear *C*. It is evident that only one position of the gear *C* can be such that the ideal condition prevails, that is, only when in mesh with one gear of the cone can the line of pressure pass through the axis of the tumbler frame. Fig. 21 shows this to be the case when *C* is in mesh with the gear *B′*. Each subsequent shifting of the tumbler along the cone brings the line of pressure eccentric to the axis, until the position of extreme eccentricity is reached when *C* is in mesh with *B.‴* In mechanisms of this kind, it should always be the aim of the designer to have the line of pressure pass as close to the center of rotation of the tumbler frame as is possible, because the locking devices used with this type of tumbler gear are necessarily of such a design as to be quick in action, and in consequence are not very stiff or rigid. The line of pressure should always be made to fall on that side of the axis where the tendency is to separate the gears rather than to bring them closer together. When the gear *C* is supported in a swinging frame which does not

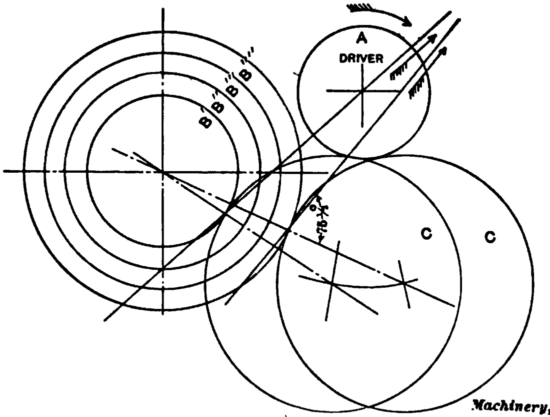


Fig. 21. Single Tumbler Gear in a Feed Box.

slide in a lateral direction, but the changes are made by shifting *C* along an intermediate shaft, the supporting member should be located at the end where the line of pressure has the greatest eccentricity, as the greatest strain comes at that end. Thus, in Fig. 21 the support should be at the same end as *B''''*. The diameter of the intermediate gear *C* has an important effect on the location of the line of pressure. It will be found that it should in most cases be as large as *B''''*, in order that the line of pressure may come right. However, no exact rule can be given by which the diameter of *C* can be calculated, as it depends greatly on the difference in the diameters of *B'* and *B''''*, and also on the diameter of *A*.

Rules for the Design of Tumbler Gears.

What direct rule can be given that may be used as a guide in laying out the tumbler gear? Referring to Fig. 19, we see that the gear *C* is revolving in a direction away from the axis of the tumbler at the point of tangency of the pitch circles of *C* and *B*, and that the reacting force tends to crowd the gears farther into mesh. Had this line of pressure fallen on the other side of the axis of the tumbler frame, the tendency would have been opposite in effect. When the gear *C* is revolving so that a point on the pitch circle travels away from the pivot of the tumbler, and the line of pressure falls somewhere between the pivot point and the axis of the driven gear *B*, the tendency will be to crowd. From this we therefore may formulate the following rules:

Rule I. When the gear about which the tumbler gear swings is the driver, and the line of pressure falls between the axis of that gear and that of the driven gear, the motion of a point on the pitch circle of the tumbler or intermediate gear, when near the contact point, must be toward the axis of the tumbler frame. Should the direction of a point on the pitch circle be opposite, the line of pressure must fall outside of that area included between the axis of the pivot gear and the driven gear.

Referring again to Fig. 19, it is seen that should the driving gear be *B*, the above rule does not apply, but may be altered to read thus:

Rule II. When the gear about which the tumbler gear swings is the driven gear, and the line of pressure falls in the space between the axis of this gear and that of the driving gear, the motion of a point on the pitch circle of the intermediate gear at the contact point must be away from the axis of the pivot gear; when the line of pressure falls outside of this space, this motion must be reversed.

By following these two rules, more as a precaution than as a compulsory condition, much better success may be expected in the results obtained.

CHAPTER V.

FAULTS OF IRON CASTINGS.

POINTS FOR THE MACHINE TOOL DESIGNER.

The most useful and indispensable of all the materials with which the designer has to do, is cast iron. Of all the metals used in the construction of machinery, it is the cheapest. It is the one to which we can the most readily give the form and proportions which we desire. It is, of all the common materials, the most easy to machine. While it is lacking in strength and ductility, its cheapness makes it possible to use it in such ample quantity as to overcome these disadvantages, and in many constructions it may be so shaped and proportioned, or so reinforced by other materials, as to make this lack but a slight detriment. It is therefore a matter of interest to the designer to learn of the various faults to which this valuable material is subject, and the best ways in which they can be avoided or minimized.

Causes of Blow-Holes.

Probably the one fault which spoils more castings than any other, is the result of an outrush of gas from the materials of the cores or the mold, into the molten iron, at the instant of solidification. If the solidification of the iron has proceeded so far that the outrushing gas or steam cannot bubble through it, and escape through the vents which should be provided for the purpose, it will be imprisoned in the substance of the casting, forming one or more holes, according to the special shape of the casting, and the quantity of the escaping gas. These holes, which are known as blow-holes, may not be apparent on the outside, and quite often occur in such a location that they do no particular harm, but it is more often the case that they occur at some point where they become apparent when the metal is being cleaned, or where their presence weakens the casting greatly.

Steam from Moisture in Sand.

The gases which cause blow-holes may come from three sources. They may be, and generally are, caused by the generation of quantities of steam from the moisture contained in the molding sand, by the heat of the iron. In the case of dry sand and loam castings, the quantity of steam so generated is so insignificant, if the molds have been properly heated, that it gives no trouble whatever. In the case of green sand castings, however, the moisture present, and therefore the steam generated, is quite large in amount, and special precautions have to be taken to prevent blow-holes.

When the molten iron pours into a green sand mold, all the moisture in the layer of sand immediately in contact with the iron will at once be transformed into steam. The depth of the sand layer so affected depends on the thickness and extent of the fiery mass to which it is adjacent. The steam so formed must either force its way through the molten iron in the form of a mass of bubbles, or else it must escape through the sand. To facilitate its escape, the mold is vented. That is, after the damp sand has been packed around the wooden pattern by ramming it closely into place, a wire is thrust repeatedly into the mold, making numerous passages for the escape of the steam and gas.

It is obviously impossible that one of these vent-holes should extend to every point in the layer of sand adjacent to the casting, so it is necessary that the most of the steam and gas should force its way for some small distance through the sand, before it can reach a vent-hole. This it can only do when the sand is somewhat porous. If the sand is too tightly rammed, it will lack the necessary porosity, and even though it be unusually dry, and the venting carefully done, the casting will be full of blow-holes. Cases have occurred where molds have been rammed so hard that the castings were nothing better than shells, the whole interior being a mass of blow-holes.

Decomposition of Binder in Cores and Entrapping of Air.

The second cause of blow-holes in iron castings is the decomposition of the material, generally flour or molasses, used as a binder in preparing the cores, and its escape in the form of gas, into the iron, at the instant of pouring. It is impossible to prepare and bake a core in such a way that it will not give off large quantities of gas when the iron is poured, and so means must be provided for the escape of this gas. In order to do this, the cores are prepared with wax strips running through them. When the core is baked, the wax melts, leaving passages for the escape of these gases, known as core vents. If the core is of such form, and so set in the mold, that the gases can escape from these vents in an upward or sidewise direction, and leave the mold without forcing their way through the molten iron, no blow-holes will result.

A third source of blow-holes is the entrapping of air in certain parts of the mold, and its mixing, on expansion, with the iron. This trouble is due to insufficient venting of the mold, and is not a fault to which the designer need pay any particular attention.

Dry Sand or Loam Advisable for Large Complicated Castings.

In the case of large and complicated castings, it is generally advisable to make dry sand or loam castings, in order to avoid, as far as possible the chance of blow-holes. When the mold is very large, it is difficult to vent it thoroughly, and when the work on it extends over a period of three or four weeks, it is impossible to keep the vents from filling up; hence the general use of dry sand work for large castings. Often, however, for the sake of economy, fairly large and complicated pieces must be undertaken in green sand, and it becomes a matter of importance that they be so designed that the molder will not be compelled to invite disaster by keeping his sand too wet, or ramming it too hard, and that there is no part of the mold which may not be thoroughly vented.

Elements of Green Sand Molding.

In order that we may understand thoroughly the effect of the design of a casting on the probability of blow-holes, it is necessary that we review, in a brief way, the elements of green sand molding. The sand is sprinkled with water, and thoroughly mixed and sifted, preparatory to packing or "ramming" it around the pattern. The object of wetting the sand is of course to cause it to stick when it is packed. Up to a certain point, the wetter it is, the better it will stick, but the molder should not wet it any more than is necessary. In the same way, the more tightly the sand is rammed, the better its particles will cohere, and the more easily will the mold be handled, and the pattern drawn. However, tight ramming and wet sand, while they make a solid and easily handled mold, invariably produce blow-holes, and are therefore to be avoided.

It will be apparent then, that if a pattern be of complicated form, or hard to draw, or if when it is drawn it leaves the sand in such a form that the mold will easily fall together at a little jarring, the molder will be compelled to wet the sand more and to ram it harder than usual. Small, deep openings, sharp fillets, and thin walls and partitions of sand, are especially troublesome. Not only do they make it difficult to draw the pattern, and handle the mold, and so make excessive wetting and hard ramming imperative, but they cause spots in the mold which the venting wire is unlikely to reach. For these reasons, they are to be avoided when possible, in any class of molding, whether it be green sand, dry sand, or loam work, and on no account should such work be permitted in the case of large green sand castings.

When designing a casting to be made in green sand, the designer ought to know the position which it will occupy in the mold, when it is poured. In general, the parts of a casting which lie in the bottom of the mold will be the soundest, and those parts which must be machined, or which require the greatest strength, should therefore occupy the bottom of the mold, if possible, when the casting is poured. Having decided which side will be down, the designer needs generally to pay no particular attention to the configuration of the lower part of the mold, provided only that all of the pattern can be drawn, and that there are no sand partitions which overhang, or whose extent is large in proportion to their thickness. To insure a sound casting, the sand in the lower parts of the mold must be comparatively dry, and loosely rammed. This condition of affairs is not generally hard to attain, since all the work on the sand is done with the pattern in place, and that part of the mold is not generally moved or handled after the support of the pattern has been withdrawn. In the lower part of the mold, the sand is generally supported at all points in a very thorough manner by the sand lying under it, and so hard ramming or wet sand is unnecessary. If, however, the pattern must be made with loose pieces, or with sharp fillets, or must leave thin walls or tongues of sand when it is withdrawn, the case is changed. Then hard ramming and wet sand are almost compulsory, and the molder is not to be blamed if he does not produce sound green sand castings. The fault is with the designer.

The upper part of the mold must of necessity be rammed harder than the lower part, since the sand is not supported from beneath, but hangs from above. This is not as great a disadvantage as it might seem to be at first sight, since the escaping gases do not have to make their way through the iron, as they would if they were given off by the sand in the lower part of the mold. The venting, however, must be just as thorough, and it is desirable that the sand should be as dry as possible. The whole arrangement of the upper part of the casting should be such that the sand may be well supported from above. Generously rounded fillets and corners, simple surfaces, plenty of "draft," and an absence of depending walls and masses of sand, make the mold easy to handle, and therefore promote freedom from blow-holes.

When Green Sand and Dry Sand Both May be Used.

It often occurs that the larger part of a casting is of simple form, and easy to mold. A certain part of it, however, may be of a form exceedingly difficult to mold, and therefore likely to give a great deal of trouble. It is not necessary that the whole casting should be made in a dry sand mold, but a core-box may be made to take care of the difficult part of the work, even though the work would ordinarily be done without a core. It is just as easy, and often just as desirable to cast the external face of a casting against a core, as the internal face. While it may not pay to do this if only one casting is wanted, if a great many are wanted it is often the cheapest possible way of making them, and reduces to a minimum both the work of the molder and the chance of a spoiled casting. Often forms may be cast in this way which could not be attempted in any other. If it is desirable to use this method of working, the designer has it in his power to make the construction of the core-box much simpler and cheaper than it might otherwise be, by giving the matter a little thought.

In arranging the coring of a mold, it is always better, if possible, to support the cores at the top. The gases formed in the core, being light, tend to rise, and if the core is supported at the bottom only, they tend to escape into the iron, and to bubble through it. If they can escape at the top, they will pass off without coming in contact with the iron. When it is impossible to support the cores at the top, they should be so arranged that the gases can pass off at the sides, and escape from the mold without coming in contact with the iron.

Sponginess.

A second fault to which iron castings are subject is that of sponginess. Sponginess is due to the formation of gas bubbles in iron, at the instant of solidification. In all ordinary cases this is due to an improper mixture of iron. However, if the casting is very thick at one place, and thin at most others, it will be impossible to obtain a mixture which will have satisfactory properties for general work, and not be spongy at points of extraordinary thickness. It is an excellent rule to allow no part of a casting to be at a greater distance from a sand surface than 2½ inches. In case this rule is strictly adhered to, and the castings are of fairly uniform thickness, no trouble will be experienced from sponginess, except from the use of poor iron. When, however, we are obliged to concentrate a considerable quantity of metal at one place, and give it a greater thickness than 5 or 6 inches, either we must take care that it will be at some point where the sponginess will do no harm, or else we must make provision to do away with it.

The only practical method for doing this is to place a riser immediately over the heavy spot. When the metal is poured, and the riser is full, a rod of wrought iron is inserted and worked up and down until the metal has almost solidified. By so doing, the bubbles have a better chance to escape, and the iron is left perfectly solid. Of course, it is not possible to use a riser effectively in this manner, unless it can be placed directly over the heavy spot. A riser at a point a few inches distant is useless. The use of a riser in this way, and for this purpose, is unnecessary when the part of the casting in which the heavy spot occurs is subject to no particular stress, or is not required to be tight under steam, air, or hydraulic pressure, but nevertheless, a spongy spot is a defect in a casting, which should, if possible, be avoided.

Shrink-holes.

A third fault to which iron castings are subject is that of shrink-holes. A shrink-hole is a cavity in a casting caused by the shrinking away of the metal in cooling. Like sponginess, this defect is especially likely to occur in those parts of a casting which are excessively thick. To avoid this fault, it is best to avoid sudden changes in the thickness of a section. If the part of a casting which is unusually thick does not have to be machined, the difficulty may be overcome by placing in the mold at that point a piece of iron, so that the casting will be caused to solidify at that point first, on account of the chilling effect of the cold iron. If, however, the heavy spot in the casting has to be machined, or if it is subject to heavy stress, this method of preventing shrink-holes is to be avoided, since the chilling of the iron makes it so hard as to be impossible to cut with a tool, and at the same time creates stresses within the metal which weaken it. In such a case, shrink-holes are best prevented in the same manner as has already been described for the prevention of sponginess, namely, the use of a heavy riser, and the working of the iron with a rod when it is cooling.

The designer must therefore avoid heavy spots in castings, whenever possible, for the reason that they are likely to produce two serious faults, sponginess and shrink-holes. He must avoid them especially in those parts of castings which are to be machined or which are subject to heavy stresses. If they cannot be avoided entirely, in such a case, they should be so arranged that risers may be placed immediately over them, so that a rod may be inserted into the riser, and into the heart of the spot where the metal is thickest.

Scabbiness.

A fourth fault often encountered in iron castings is that of scabbiness. Although iron in the molten condition does not permeate the sand of the mold, as water would if it were poured in, nevertheless, on account of the great weight of the fluid, it has a considerable erosive action on the materials of the mold. If, as it flows into the mold, the iron eats away fillets or partitions, or scours away patches of sand, it is obvious that the casting will not be of the proper form, but will have its angles partly filled up, and unsightly protruberances upon its surfaces. Such imperfections as these are known as scabs. The sand so washed from its proper place may float on the iron, and rise to the top of the mold, where it forms a dirty mixture, which, when cleaned away, leaves a rough depression in the surface of the casting, also known as a scab.

The remedy for this trouble is to avoid as far as possible sharp fillets, and thin tongues of sand, projecting into the mold, and to so gate the casting that the current of iron, as it enters the mold, will spread itself out, and not concentrate itself in any particular direction, for if it does, it will eat away the part against which it flows, just as quickly and surely as would a current of water. In general, proper gating is a matter which must be attended to by the molder, but if the designer has arranged things so that proper gating is inconvenient or impossible, the castings will almost surely be scabby.

Sand-holes.

The fifth fault to which iron castings are subject, namely sand-holes, is one which is almost invariably associated with that of scabbiness. If the sand which has been eroded by the entering current of iron does not rise immediately to the surface, the iron may partially solidify before it will float to the top. As a result, it will rise till it strikes a part of the iron which has so solidified, and will remain there, imprisoned within the body of the casting. Sand-holes generally occur in that part of a casting which lies near the top of the mold, and just a little ways under the skin. They may occasionally form large cavities which seriously impair the strength of the casting, but more often they form very small holes, which, being full of sand, destroy the edge of any tool which may be used for the purpose of machining the casting, and leave the finished surface pitted and unsightly.

From this description of their origin, it must be apparent that the cure for sand-holes, as far as the designer's work is concerned, is the same as that for scabbiness. The fault may also be avoided by the use of a riser, so arranged that the current of iron will sweep the loose sand out of the mold and into the riser, where it will do no harm; but while this remedy eliminates sand-holes, it does nothing to remedy scabbiness. which is generally the cause of sand-holes.

Floating Cores.

A sixth difficulty often encountered in the production of sound iron castings, is caused by floating cores. The buoyant effect of the molten iron on a core is equal to about three times the weight of the core, if it is solid, and very much more than that in case the core is hollow. Large cores are generally built up about cast-iron skeletons known as core frames. These core frames are roughly of the same shape as the core, and serve to support it, and to bind it together. Were it not for these frames, heavy cores would fall to pieces by their own weight, and would be broken up in the process of casting, owing to the buoyant effect of the iron. A projecting piece of core having a volume of four cubic feet, for instance, will weigh approximately 500 pounds, and have a buoyant force of about 1,500 pounds thrusting it upward when the mold is poured. If the core frame is not amply strong and stiff, this force will bend the projection upward, or even break it off entirely. Hence the necessity of making large cored cavities of such form that the cores may be rigidly anchored and thoroughly secured. Nor is it sufficient that provision be made for securing the cores, but they must be of such shape, and so reinforced by the frames, that they will be stiff and strong, and not bend appreciably under the tremendous forces which will act on them when they are surrounded by the molten iron.

One of the most difficult things to cast properly is a long iron pipe having a small diameter and thin walls. If such a pipe be cast in the usual position, that is, lying horizontally, there will be an upward thrust along the whole length of the core, tending to bend it. On account of its slenderness, the central portions will be deflected upward, making the walls of the pipe thinner on one side, and thicker on the other. Often the deflection proves sufficient to thrust the core against the side of the mold, if it is long, or, in case the thickness of the wall is great in proportion to the length of the core, to break it off entirely. On this account, pipes and hollow columns of cast iron are often cast on end, thus avoiding any deflection of the core. The same principle may be applied to many other pieces, by taking care to so design them that long and slender cores shall have a vertical position when the mold is poured. If they have such a vertical position, and are supported at both ends, they will have no tendency to deflect one way or another, and this source of trouble may be completely avoided.

Cold Shuts.

The seventh fault is known as a cold shut. A cold shut is caused by the imperfect uniting of two or more streams of molten iron, flowing together, which are too cold to coalesce. Such a fault often occurs on the upper side of a thin cylinder cast horizontally, when the iron is not sufficiently hot at the instant of pouring. It there appears as a seam in the side of the cylinder, where it is very apparent that the metal has united imperfectly. It is not only a weak spot in the casting, along which it will readily split if called upon to sustain any great stress, but it is a spot which will surely leak under pressure, and which it is impossible to calk. The cause of the imperfection is generally improper gating, or else too great thinness of metal. If the iron is obliged to flow in thin streams for long distances, it will be cooled very much, and probably the advancing face will be partially solidified. Consequently, when it meets a similar advancing face of metal, which has been similarly cooled, there is small likelihood of their uniting properly.

The remedy is obviously to so design the casting that the metal will not have to flow in thin streams for long distances. The arrangement of gates and risers is often of great importance in minimizing cold shuts, and if the casting is large, and at the same time has thin walls, the designer must see that the gates may be so arranged that the iron may quickly fill up the mold. While the arrangement of the gating generally depends on the molder's fancy, he may often be limited by the shape of the casting, and obliged to place the gate at some point where the iron, in flowing in, must spread itself into a thin sheet, or pass for a considerable distance through a narrow passage. Under such circumstances, a cold shut is hardly to be avoided.

Shrinkage Strains.

The eighth and last fault is that of shrinkage strain. If we have two pieces of iron fastened end to end, as shown in Fig. 22, one piece being notably thinner than the other, the thinner piece will solidify first in the mold, and cool some hundreds of degrees below its freezing point, before the thicker part solidifies. As a result, the thicker part, when cooled to air temperature, will have, or rather tend to have, a less length than the thinner part, the reason being that at the instant of solidification of the thicker part, both pieces had the same length, although the thinner part was much the cooler. The thin part will then be in compression, while the thick part is in tension, and severe stresses will exist within the piece, which make it weaker than it would otherwise be in most cases.

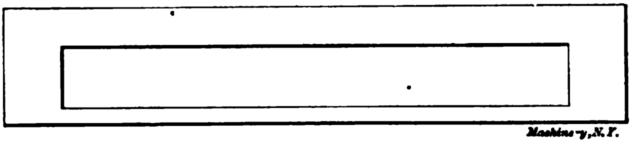


Fig. 22. Casting of such Shape as to be Subjected to Severe Internal Stresses.

Sometimes, however, we are enabled to utilize the shrinkage stresses to advantage. For instance, when cast iron was the standard material for the manufacture of ordnance, guns were cast with cores through which water was circulated, so as to cool the surface of the bore before the outer parts solidified. When a gun is fired, it is known that the inner layers of metal are stretched more than the outer ones. By cooling the inner layers of metal first, shrinkage strains are produced in the walls of the gun, causing the outer layers of metal to compress the inner ones. The combined effect of the shrinkage stresses and the stresses produced by the explosion is to produce a uniform stress throughout the walls of the guns, and so reduce the chance of rupture.

It is not often, however, that we are able to take advantage of shrinkage strains in this way. More often they are troublesome, causing work to warp in the process of machining, or causing mysterious cracks to develop without apparent cause. Since these strains are due to unequal rates of cooling in the different parts of the casting, the best way to eliminate them is to so arrange the thickness of the various parts, that the entire casting shall solidify at the same time. The second best way is to so arrange the parts of the casting that the unequal contraction shall not produce dangerous stresses at any point. In order that the entire casting shall cool at a uniform rate, it is necessary that all parts of it shall be of approximately uniform thickness, and that there shall be no sudden changes of section. In order that unequal contraction shall not produce dangerous stresses in the metal, it is necessary that there shall be no sharp corners, and that the various parts shall be free to expand when necessary. For instance, a wheel or pulley with a solid rim is likely to have severe stresses set up within the arms by unequal cooling, but if the hub be divided as shown in Fig. 23, by means of a thin core, and then bolted subsequently, no shrinkage strains will occur, since the arms are free to expand or contract, independently of the rim.

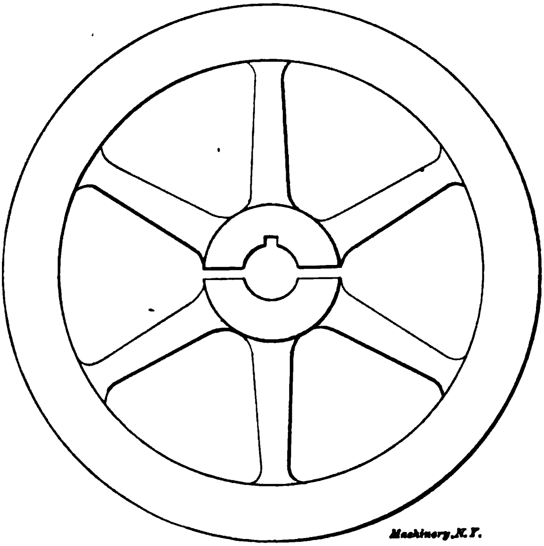


Fig. 23. Method of Obviating Shrinkage Strains in Large Wheels.

Shrinkage strains often become so serious that it becomes necessary to make pieces in two or more parts, which it would be perfectly possible to make, at much less expense, in one piece. Large jacketed cylinders, for steam and gas engines, are good examples of this. When cast in one piece, the shrinkage stresses, together with the stresses set up by the varying temperatures incident to services, are often sufficient to crack them. Were the piece shown in Fig. 22 made in two parts, as shown in Fig. 24, there would be no shrinkage strains in either part, although the cost of machining the surfaces which are fitted together, and of putting in the bolts, would not always warrant the construction.

Relative Economy of Simple and Complicated Castings.

In conclusion, it may be well to state that the most of the faults enumerated will be more likely to occur in a part of a complicated casting, than in a similar part of a simpler casting. For instance, the cylinder of a gas engine will be more likely to have some imperfection if it is cast integral with the frame, than if it is cast separately.

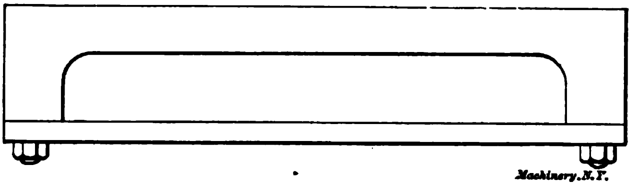

Machinery, N.Y.

Fig. 24. Piece shown in Fig. 22, made in Two Parts.

In the same way, the frame will be more likely to have an imperfection of some kind, than if it were cast separately. Assuming that ten per cent of the cylinders or frames would be lost if they were cast separately, it is more than likely that fifteen per cent of the castings, having cylinder and frame cast together, would be rejected for faults in the frame, and fifteen per cent of the remainder would be rejected for faults in the cylinder. In other words, twenty-eight per cent of these castings would be rejected, against ten per cent of the simpler forms. If more than eighteen per cent of the cost of the castings is saved in machining, or in other ways, by casting cylinder and frame together, it is well to do so, but if the saving is not more than sufficient to balance the loss, it is well to make several simple forms, instead of one complicated one.

CHAPTER VI.

PROPORTIONS OF MACHINES BUILT IN A SERIES OF SIZES.

The problem of cost reduction forces itself, with increasing vividness, upon the mind of every person who has to do with the manufacture of machinery. To the "small shop" people, and to those whose product is unsystematized and whose ideas of methods to pursue are, as yet, vague, this chapter may prove of some assistance.

There are three important means by which the shop product may be systematized: By the use of formulas; by the use of tables; and by the use of charts. As the two latter may be considered as the tabulated, or graphic results of the former, we will deal only with the formulas. In determining sizes, weights, and costs, these formulas are generally most efficient time-savers. For convenience, formulas in this chapter will be divided into two classes: The class used to produce the first of a type of machine we will call fundamental; and the class used to produce several sizes of this type of machine, empirical. Upon seeking fundamental formulas in text books and in mechanical engineers' pocket books we are confronted by a diversity of opinions and tabulated results that are, at least to a novice, a bit confusing. These formulas, it is always to be remembered, have their application in the special case under consideration, and are to be used only as guide posts in our journey of design. It is evident to most designers that some kind of a tentative method must, sooner or later, be resorted to in the type design, for in nearly all machines the governing conditions soon become so numerous or indefinite as to render a subdivision of the problem a necessity. A certain amount of judgment is absolutely essential in the use of most fundamental formulas, and discrimination is always necessary.

Graphically, fundamental formulas can be represented by curves, and will be correct for all sizes under identical conditions, while empirical formulas rest on no such basis and hold true for but a certain series within certain limits. This constitutes the vital difference between fundamental and empirical formulas. A fundamental formula is one found through mathematical reasoning, while an empirical formula is made up by means of trial methods.

Suppose that we have built two or three sizes of a certain type of engine and that they are successful; we desire to put on the market an entire line. Our sizes of this type of engine will run from 10-inch cylinder diameter in the smallest to 30-inch cylinder diameter in the largest. We have built a 12-inch and a 24-inch engine and perhaps an 18-inch. These engines were, as was imperative, tentatively designed. In seeking the derivation of the empirical formula for the length of the cross-head shoe, we find that on our 12-inch engine we have given it an area of 55⅛ square inches, and on our 24-inch engine its area is 190⅛ square inches. In each case the length of the shoe was nearly twice its width, so we decide to make it so in our line of engines; solving for the width, we have in the 24-inch engine

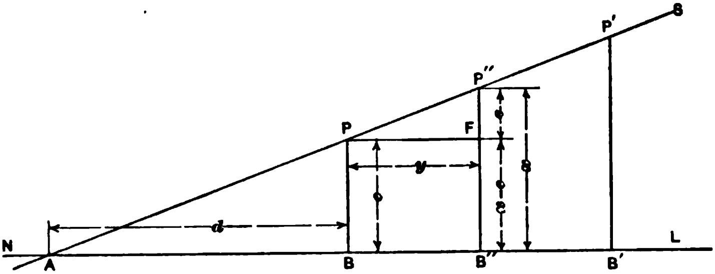
$$2x^2 = 190.125; \quad x = \sqrt{95.0625} = 9.75.$$

making our shoe length for the 24-inch engine 19½ inches, and for the 12-inch engine 10½ inches.

To any scale in Fig. 25, perpendicular to the line *NL*, lay off these shoe lengths *PB* and *P′B′*—10½ inches, and 19½ inches, respectively—making the distance *BB′* equal to 12 inches, the difference between our sizes 12 inches and 24 inches. Through points *PP′* draw line *SA* intersecting *NL* at *A*. At *B″*—for our 18-inch size—erect a perpendicular *B″P″*. Draw *PF* intersecting *B″P″* at *F*. Using the notation given in the figure, we get the simple equations

$$e:y = c:d; \quad ed = cy; \quad e = \frac{c}{d}y;$$

$$x - c = e; \quad x - c = \frac{c}{d}y; \quad x = \frac{c}{d}y + c.$$



Machinery,N.Y.

Fig. 25.

In this last formula many will recognize an old acquaintance—the equation for a straight line. Let us now analyze this equation. From the figures it is seen that $x =$ the desired dimension and that $\frac{c}{d} =$ the rate of increase in the slope of the line. If now we measure the distances and substitute their values for c and d we may determine the ratio $\frac{c}{d}$, which we will call f.

Then $f = \frac{c}{d} = \frac{10.5}{14} = \frac{3}{4}$, and

$x = fy + c$, or $x = \frac{3}{4}y + c$.

In interpreting our empirical formula $x = fy + c$, we have

$y =$ a common unit to which all other sizes are to be referred,

$x =$ desired dimension,

f = a factor of y,
c = a constant increment to be added in each case.

The unit of value y, as generally selected, is a bolt or cylinder diameter, or the capacity of the machine. Obviously, in our line of engines, we select the cylinder diameter D as our value of y, and our unit formula then becomes $\frac{3}{4}D + c$. The value of c is now determined by direct substitution in the following manner: x being the shoe length, we substitute for it 19½ inches (its length on the 24-inch crosshead); then

$$\tfrac{3}{4} \times 24 + c = 19\tfrac{1}{2}; \quad c = 19\tfrac{1}{2} - 18 = 1\tfrac{1}{2}.$$

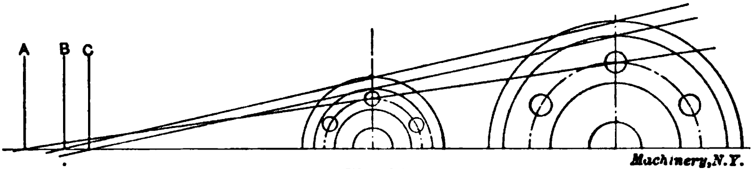


Fig. 26.

Note, that while we have assigned to y and c other values, we have not altered the relations; our formula for this particular crosshead dimension now becomes $\frac{3}{4}D + 1\frac{1}{2}$ inches.

For convenience in charting these sizes, some point is determined upon as a pole about which these lines (represented by our formulas) are drawn as vectors, the ordinate length for a particular size giving the desired dimension. If now in the determination of other formulas it be found, as is likely to be the case, that these lines do not all pass through a common point, it becomes necessary to select one. In well-designed machines the intersection of these lines with the base line will come close together, and an average of these intersections is selected as a pole. Figs. 26 and 27 will serve to illustrate the purport of this paragraph.

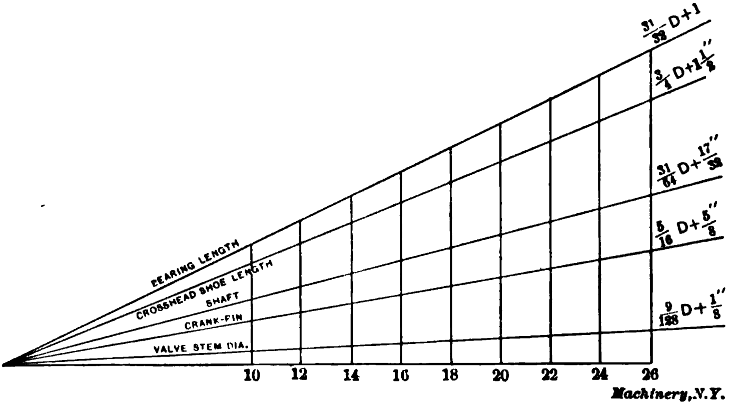


Fig. 27.

Experienced designers are well aware that the final test of any dimension in a design is that of satisfying all fundamental calculable conditions; nevertheless, the instances where our empirical formulas prove incorrect are very few indeed. With the design for our line of engines thus systematized, let us consider what are to be the advantages that will naturally result from it. In the first place, the weights of any particular parts, or details of any size in our line of engines may be determined prior to its design or manufacture. In the determination of weights, cubic contents, and similar processes, the use of "differences" as applied to higher mathematics, will not only prove an efficient time-saver, but relieve much of the drudgery attendant upon such operations.

A brief explanation of the use of "differences" is as follows: When we have a series of numbers connected by a regular, though not obvious law, the nature of that law may be discovered by forming a new series of differences between each two terms of the original series, and then treating the new series (which we may call the series of first differences) in the same way, until we reach a series of differences, the law of which is obvious. In the table above will be found both the arithmetic and algebraic solutions of problems by "differences."

TABLE SHOWING PRINCIPLE OF THE METHOD OF DIFFERENCES.

Arithmetic Solution.				Algebraic Solution.				
Col. 4.	Col. 3.	Col. 2.	Col. 1.	Term	Col. 5. Original Ser.	Col. 6. 1st Ser. Differences.	Col. 7. 2d Ser. Dif.	Col. 8. 3rd Dif.
			7	1	W			
		5				y		
	3		12	2	$W+y$		x	
1		8				$y+x$		c
	4		20	3	$W+2y+x$		$x+c$	
1		12				$y+2x+c$		c
	5		32	4	$W+3y+3x+c$		$x+2c$	
1		17				$y+3x+3c$		c
	6		49	5	$W+4y+6x+4c$		$x+3c$	
1		23				$y+4x+6c$		c
	7		72	6	$W+5y+10x+10c$		$x+4c$	
1		30				$y+5x+10c$		c
	8		102	7	$W+6y+15x+20c$		$x+5c$	
1		38				$y+6x+15c$		c
	9		140	8	$W+7y+21x+35c$		$x+6c$	
1		47				$y+7x+21c$		c
	10		187	9	$W+8y+28x+56c$		$x+7c$	
		57				$y+8x+28c$		
	11		244	10	$W+9y+36x+84c$			
		68						
			312					

In column 1 of the table is given a series of numbers, which we suspect follows some definite, though not obvious law, and which we desire to discover. We here take the differences between each two terms in column 1 and put them down in column 2. Having proceeded with the two orders of differences, the law becomes apparent early in the process of determining the values in column 3. Referring again to the table, it is evident that the next term of column 3 must be 11, which gives 68 (57 + 11) as the next term of column 2 (the series of first differences) and 312 (244 + 68) for the original series. Note that this series can thus be obtained indefinitely, and that ultimately, in any regular series, some one series of differences will become a constant. It is on the principle of differences that calculating machines are constructed to compute logarithmic tables, etc.

In the algebraic solution of such problems as involve the determination of weights and volumes, it will be necessary to calculate these weights or volumes for the 1st, 2d, 3d, and 4th terms of our given series. By substituting in the formulas in column 5 the numerical value of W, which is the first term in our given series, we may equate these expressions and our calculated values for the 2d, 3d, and 4th terms, and determine, by simple algebraic processes, the values of c, x, y, and ultimately, those values which we are requiring in the original series, column 5.

The computations concerning the cost of materials logically follow the determination of volumes and weights and are made with comparative ease. However, our next problem concerning the determination of the cost of labor is a more difficult one to solve. Formulas should express this cost in so many cents per pound of product, including all shop charges, and be established partially by experience and partially by methods suggested in this chapter.

In many instances it will be found both desirable and convenient to have this cost formula embody the unit dimension. When this is the case, the formula is, as are most cost formulas, established by the tentative methods to which we have just alluded. As the methods employed in the deduction of these formulas render them purely empirical, one or another form of expression may have to be adopted. However, formulas of this class usually assume the form of, or at least may be solved into, the familiar type form

$$ax^2 - bx + c,$$

where a and b are factors of the unit dimension, and c is a constant.

For the purposes of illustration we will assume that the formula for the cost of labor which we have established is

$$\frac{3}{2} D^2 - 15D + 314.$$

In this form the formula gives the total cost of labor in dollars for the size desired. The cost of labor C for our 18-inch engine would then be

$$C = \frac{3}{2} (18)^2 - 15 \times 18 + 314 = 530.$$

The computation of a final cost formula, embodying the unit dimension, is the last process in our development of shop formulas; this formula is derived directly from those relative to the costs of material and labor.

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No. 15

SPUR GEARING

CONTENTS

First Principles of Gearing - - - - - - 3
Formulas for Dimensions of Spur Gears, by ARTHUR B. BABBITT - - - - - - - - - 9
Design and Calculation of Gear Wheels, by C. F. BLAKE 21
Strength of Gear Teeth, by J. J. FLATHER - - 33
Variation of the Strength of Gear Teeth with the Velocity, by RALPH E. FLANDERS - - - - - - 46

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CHAPTER I.

FIRST PRINCIPLES OF GEARING.

Gear wheels are such common objects about the machine shop, and are manufactured with such rapidity and ease by the aid of the modern automatic gear cutter, that many seldom stop to think what they really are, why the teeth must be constructed with certain curves, and what it is desired that they shall accomplish. In a following chapter we shall take up some of the practical questions, touching upon the calculations that come up in the shop, but will here deal chiefly with a few of the theoretical points of the subject that are seldom explained in a simple manner for the benefit of those who have had neither the time nor the opportunity to look into matters of this kind.

Suppose there are two wheels arranged as in Fig. 1 with their faces in close, frictional contact, and that both are exactly the same size, so that when the crank is turned around once, wheel *B* will turn exactly once also, provided, of course, there is no slipping between the two wheels. It must be noticed, moreover, that if the crank be turned uniformly, wheel *B* will not only make the correct number of revolutions relative to *A*, but it will revolve uniformly, as well; that is, both its total motion and the motion from point to point will be correct.

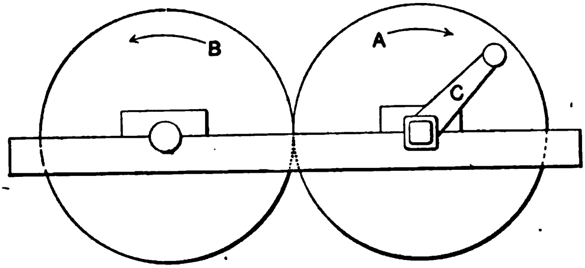


Fig. 1.

Now there are many places in machine construction where the slipping inseparable from friction wheels cannot be tolerated, and this difficulty might be overcome by fastening small projections to one of the wheels, as on *A* in Fig. 2, and cutting grooves in the other wheel, *B*. Then, if the crank were turned, wheel *B* would always make just the right number of turns, even if considerable power were transmitted. It is probable, however, that these projections and grooves would not fulfill the purpose of gear teeth. What is wanted of gear teeth is that they shall give exactly the same kind of motion as corresponding friction wheels, running without slipping. They must not only keep the number of revolutions right, but they must give a perfectly even and smooth motion from point to point or from tooth to tooth.

Fig. 3 will show clearly how such a result is obtained. It represents the friction wheels with teeth fastened to them, the teeth, of course, extending all the way around instead of part way as shown. These teeth are set so as to be partly without and partly within the edges of the two wheels, as obviously they will give better results thus arranged than with all the projections on one wheel and all the grooves or depressions on the other, as in Fig. 2.

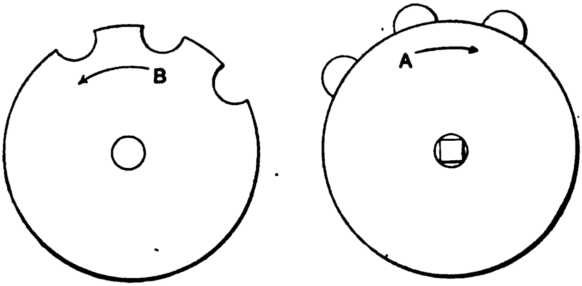


Fig. 2.

With the wheels fitted in this way it can be proved that the only conditions which must be fulfilled in order that the teeth shall give wheel *B* the same motion that it would have if it were driven by frictional contact with wheel *A* is that a line drawn from the point *O*, where the two wheels meet, to the point where the tooth curves touch shall be at right angles to both tooth curves at this point, whatever the position of the gears. For example, in Fig. 3, two of the teeth touch at *h*. If the curves are of the right shape, a line *m n*, drawn through *h* and *O*, will be at right angles to both curves at point *h*. This is the law of tooth curves, and it makes no difference what the shape of the teeth is, so far as their correct action is concerned, if this law holds true for every successive point where the teeth come in contact.

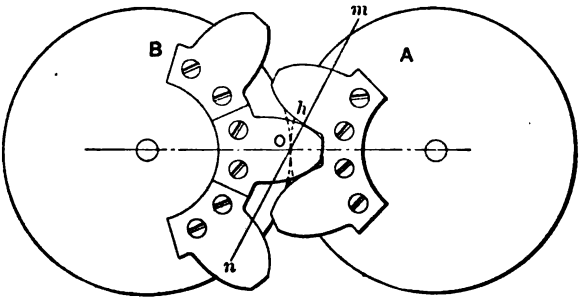


Fig. 3.

In technical language the "friction wheels" mentioned are known as "pitch cylinders," and they are always represented on a gear drawing by a line—usually a dash and dot line—called the "pitch line." As teeth are generally proportioned, this line falls nearly, but not quite, midway between the tops and bottoms of the teeth, the inequality being due to the space left at the bottom of the teeth for clearance. The diameter of the pitch cylinder is called the "pitch diameter."

Involute System.

We are now ready to consider the particular forms of teeth most often used. The one that is at present most in favor is the involute tooth, the term "involute" being the name of a curve described by the end of a cord as it is unwound from another curve. For example, to draw an involute, wind a cord around a circular disk of any convenient material, and make a loop in the outer end of the cord. Lay the disk flat on a piece of paper, and with a pencil in the loop, unwind the string, keeping it drawn tight, and let the point of the pencil trace a curve, which will then be an involute.

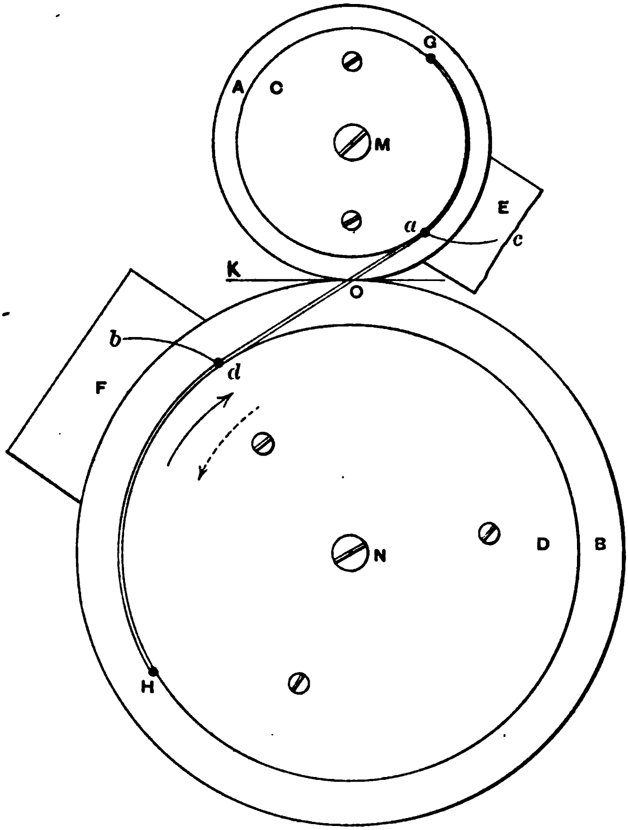


Fig. 4.

In Fig. 4 is shown how the same principle is applied to forming tooth curves. *A* and *B*, with centers at *M* and *N*, are two disks which serve the purpose of pitch cylinders. *C* and *D* are two smaller disks fastened to the larger ones and around which a cord is stretched and fastened at points *G* and *H*. When either disk is turned, the cord is supposed to pull the other one around at the same speed that it would go if moved solely by frictional contact between disks *A* and *B*. To do this, it is simply necessary to have the disks *C* and *D* in the same ratio as *A* and *B*. If *A*, for example, is half as large as *B*, then *C* must be half as large as *D*.

To make room for drawing the curves, let pieces *F* and *E* be fastened to the large and small wheels, respectively. With a pencil fixed at point *d* on the cord, turn the wheels in the direction of the solid arrow, meanwhile moving the pencil outward, and the curve *db* will be described, which will be a suitable tooth curve for the larger wheel, and which it can be proved will answer the requirements of the general law. Starting again with the pencil at *a*, and turning the wheels in the direction of the dotted arrow, and moving the pencil outward, a similar curve, *ac*, for the smaller wheel will be traced.

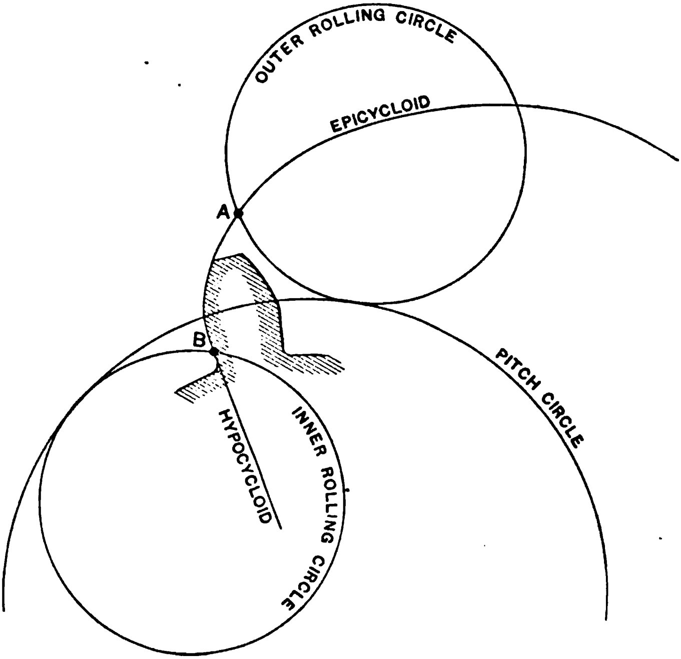


Fig. 5.

The circles representing the disks *C* and *D* are called "base circles," and in practice are drawn at a distance from the pitch circle of about one-sixtieth of the pitch diameter. This brings the angle, *KOd*, called the angle of obliquity, in Fig. 4, about 14½ degrees; and although it is not by any means certain that this is the best angle, it is the one commonly used.

Cycloidal System.

Take a silver dollar and roll it along the edge of a ruler, holding the point of a pencil at the rim of the dollar, so that as the latter rolls, the pencil will trace a curve. This curve is a cycloid. Should the dollar be rolled on the edge of a circular disk, however, the curve traced would be an epi-cycloid, and should it be rolled on the inside of a hoop, it would be called a hypo-cycloid. These curves are employed for the teeth of the cycloidal system of gears.

In Fig. 5 it is shown how the face or the outer portion of the tooth is rolled up by the point *A* on the outer rolling circle, and how the flank or inner portion is generated by point *B* on the inner rolling circle. In this case the hypo-cycloid and flank are straight lines, the reason for this being that, as drawn, the diameter of the rolling circle is one-half the diameter of the pitch circle of the gear, and the hypocycloid generated under these conditions becomes a straight line.

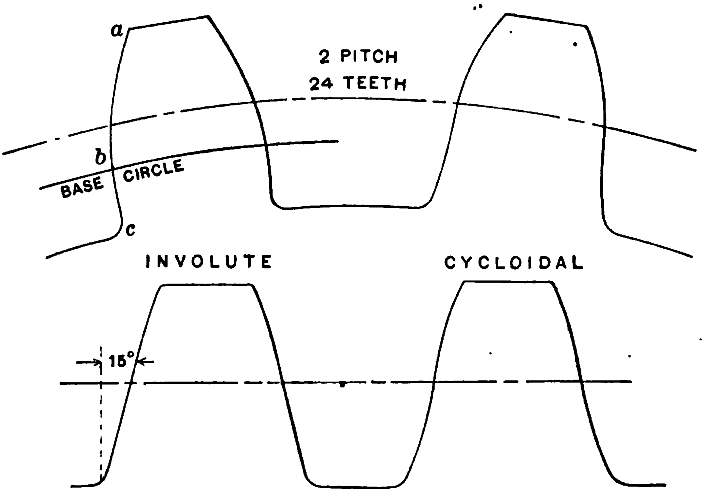


Fig 6.

The involute and cycloidal systems are the only two that are used to any extent, and in Fig. 6 a gear tooth and rack tooth of both are shown for comparison. The involute gear tooth has the involute curve from point *a* to point *b* on the base circle, and from *b* to *c* at the bottom of the tooth the flank is a straight, radial line. One difficulty with the involute system is that with the standard length of tooth the point *a* will interfere when running with gears or pinions having a small number of teeth. To avoid this, the point is rounded off a little below the involute curve. In general appearance the tooth seems to have a broad, strong base, and a continuous curve from *a* to *c*. A strong feature of the involute gearing is that it will run correctly even if the distance between the centers of the wheels is not exactly right. This will be evident by referring to Fig. 4, where it will appear that the relative velocities of the two wheels will be the same however far apart they may be, and if involute teeth are used in place of the string connection there shown, the action will be just the same. The involute rack tooth has straight sides at an angle of 14½ degrees, with the points rounded off.

Of the cycloidal teeth but little need be said except that they have two distinct curves above and below the pitch line, as previously explained, and that in the rack tooth the two curves are just alike, but reversed.

Whatever system is used, it is essential that all the wheels of a given pitch should be capable of running together. To make this possible with the involute, all the wheels must have the same angle of obliquity; and with the cycloidal system the same size rolling or describing circle must be employed for all sizes. The circle generally chosen is one having half the diameter of a 12-tooth pinion, which makes the flanks of this pinion radial. In Fig. 5, if the diameter of the rolling circle had been either greater or less than half the diameter of the pitch circle, the flank of the tooth would have been curved, and in the case of the greater circle, the curve would have fallen inside of the radial flank drawn in the figure, causing a weak, under-cut tooth. With the smaller circle, the curve would fall outside, making a strong tooth.

CHAPTER II.

FORMULAS FOR DIMENSIONS OF SPUR GEARS.

When we consider the number of gears used in machinery, and the number of men employed in the manufacture of machines using gears, it is rather surprising to find men who are unable to find the outside diameter, having given the pitch diameter and pitch, or to find the distance between centers of two gears, having given the number of teeth and pitch, and similar problems. The object of this chapter is to explain in as clear and practical a manner as possible the underlying principles of gearing, and to give concise rules or formulas for the solution of problems which arise in our everyday work upon gears.

Pitch Diameters.

Two shafts *A* and *A′* (Fig. 7) carry rollers *B* and *B′*. By having pressure on the shafts as indicated by the arrows, and revolving *A*, the friction of the rolls at the point of contact, *X*, will cause *A′* to revolve, but we can readily see that if any great amount of power is to be transmitted, the rolls are liable to slip at the point of contact *X*, which will not give a positive motion; that is, it will require more than one revolution of the shaft *A* to produce one revolution of the shaft *A′*.

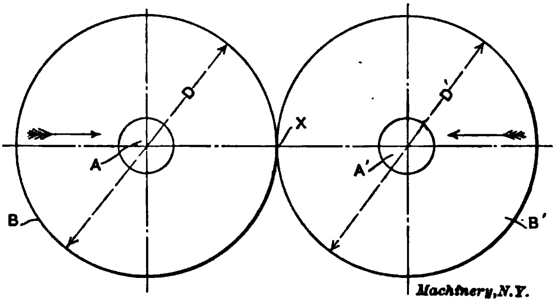


Fig. 7.

Suppose, as shown in Fig. 8, that we put projections on the surface of the roller *B* and cut recesses in the roller *B′*, making them of such shape that the sides of the projections on roller *B* will slide with as little friction as possible upon the sides of the projections caused by cutting the recesses in roller *B′*. Then, when shaft *A* is revolved, shaft *A′* must also revolve. The identity of the rollers *B* and *B′* is not lost, for we have simply added a number of projections to one, and cut the same number of recesses in the other, and the point of contact of the two rollers is still at *X*, but in this case there is no special pressure required to keep the rollers together as in the preceding case, nor is there any slip, and consequently shaft *A′* will make one revolution in the same time that shaft *A* does.

In Fig. 9 we have changed Fig. 8 by adding projections between recesses in roller *B′*, and by cutting recesses on roller *B* between projections, and we have the regular gear tooth. We have now no visible part of the original rollers *B* and *B′* left, but we have in their places imaginary rollers, the diameters of which are the pitch diameters of the gears. Thus we might have called our original rollers pitch rollers, and then proceeded to put on our projections and cut our recesses, which would have given us the gear wheel. This has already been explained in a general way in Chapter I.

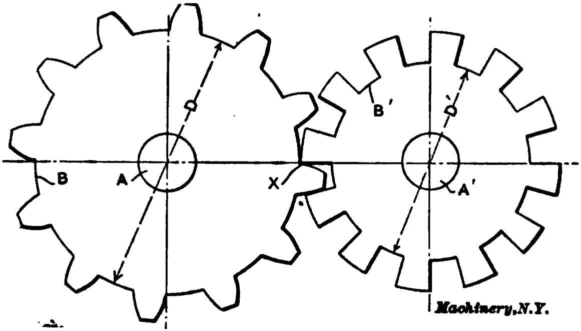


Fig. 8.

Of course, in practice gears are never made in this way; the gear blank is first turned up to the correct diameter, and then the space between the teeth is cut. The method of finding the outside diameter will be given later, this illustration being used simply to show the evolution of the gear wheel from the friction disks or pitch rollers.

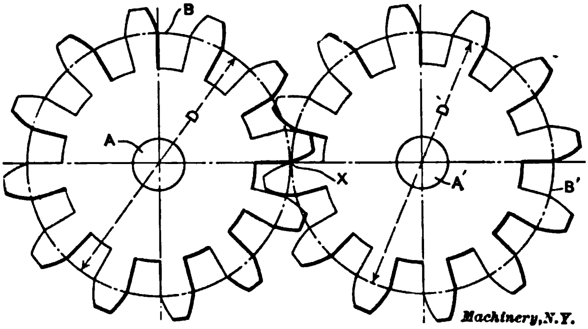


Fig. 9.

Pitch.

When we speak of the pitch of a gear, the diametral pitch is generally referred to. The gear really has two pitches, diametral and circular. The *diametral pitch* of a gear is the number of teeth for each inch of pitch diameter. If a gear has 20 teeth and the pitch diameter is 2 inches, the diametral pitch would equal 20 ÷ 2, or 10; or there are 10 teeth in the gear for each inch of pitch diameter which it contains, and we would call it a 10-pitch gear. The *circular pitch* of a gear is the distance from the center of one tooth to the center of the next adjacent tooth, measured on the pitch lines. It is very seldom that circular pitch is used in describing cut gears.

It can readily be seen that the circular pitch being equal to the distance from the center of one tooth to the center of the next, must be the result of dividing the circumference of the pitch circle by the number of teeth in the gear. Should an occasion arise where it would be necessary to obtain the circular pitch, having the diametral pitch given, divide 3.1416 by the diametral pitch, and the quotient will be the circular pitch, or, expressed in its simplest form,

$$\frac{3.1416}{P} = P_1 \qquad (1)$$

in which P = diametral pitch; P_1 = circular pitch.

Example.—If the diametral pitch of a gear is 4, and it is required to find the circular pitch, divide 3.1416 by 4, and the quotient, 0.7854, is the circular pitch of the gear.

If the circular pitch be given, to find the diametral pitch, we can readily see that formula (1) would have to be transposed and would read thus:

$$\frac{3.1416}{P_1} = P \qquad (2)$$

P and P_1 representing the same as before.

Now, having given the rules, we will proceed to explain how they were obtained. We know that the distance around the circumference of a circle is equal to 3.1416, multiplied by the diameter of the circle; consequently, for every inch of diameter we have 3.1416 inches of circumference. If the diametral pitch of a gear is equal to the number of teeth for each inch of pitch diameter, and each inch of diameter is represented by 3.1416 inches of circumference, then the diametral pitch equals number of teeth for each 3.1416 inches of circumference. As the circular pitch is the distance from the center of one tooth to the center of the next, then the circular pitch must be equal to 3.1416 divided by the number of teeth in that 3.1416 inches of circumference, and, as we have shown that the diametral pitch is equal to the number of teeth in each 3.1416 inches of circumference, then the circular pitch must equal 3.1416 divided by the diametral pitch, which proves formula (1).

It may not be actually necessary to show how we obtain the diametral pitch from the circular pitch, but we will endeavor to explain everything as we go along. As in the preceding case, we begin with the ratio of the circumference of the circle to its diameter, which is 3.1416. In each 3.1416 inches of circumference we have a certain number of teeth, which is the diametral pitch of the gear. Now, having given the circular pitch, if we divide 3.1416 by that, we obtain the number of teeth for 3.1416 inch of the circumference, which is the diametral pitch of the gear, which proves formula (2).

The accompanying tables will facilitate the finding of corresponding diametral and circular pitches. Table I gives the even diametral pitches with the corresponding circular pitches, while Table II gives the even circular pitches with the corresponding diametral pitches.

TABLE I. DIAMETRAL PITCH CONVERTED INTO CIRCULAR PITCH.

Diametral Pitch.	Circular Pitch.	Diametral Pitch.	Circular Pitch.
2	1.571 inch.	12	0.262 inch.
2¼	1.396 “	14	0.224 “
2½	1.257 “	16	0.196 “
2¾	1.142 “	18	0.175 “
3	1.047 “	20	0.157 “
3½	0.898 “	22	0.143 “
4	0.785 “	24	0.131 “
5	0.628 “	26	0.121 “
6	0.524 “	28	0.112 “
7	0.449 “	30	0.105 “
8	0.393 “	32	0.098 “
9	0.349 “	36	0.087 “
10	0.314 “	40	0.079 “
11	0.286 “	48	0.065 “

TABLE II. CIRCULAR PITCH CONVERTED INTO DIAMETRAL PITCH.

Circular Pitch.	Diametral Pitch.	Circular Pitch.	Diametral Pitch.
2 inches.	1.571	7/8 inch.	3.590
1 7/8 “	1.676	13/16 “	3.867
1 3/4 “	1.795	3/4 “	4.189
1 5/8 “	1.933	11/16 “	4.570
1 1/2 “	2.094	5/8 “	5.027
1 7/16 “	2.185	9/16 “	5.585
1 3/8 “	2.285	1/2 “	6.283
1 5/16 “	2.394	7/16 “	7.181
1 1/4 “	2.513	3/8 “	8.378
1 3/16 “	2.646	5/16 “	10.053
1 1/8 “	2.793	1/4 “	12.566
1 1/16 “	2.957	3/16 “	16.755
1 “	3.142	1/8 “	25.133
15/16 “	3.351	1/16 “	50.266

Pitch Diameter.

Having given the diametral pitch and number of teeth in a gear, to find the pitch diameter, divide the number of teeth by the pitch, and the quotient will be the pitch diameter, which, expressed in its simplest form, is:

$$\frac{N}{P} = D \qquad (3)$$

in which $N =$ number of teeth; $P =$ pitch (diametral); $D =$ pitch diameter.

Example.—A 10-pitch gear has 35 teeth, what is the pitch diameter? Divide 35 (the number of teeth) by 10 (the pitch), and the quotient 3½ is the pitch diameter of the gear.

The definition of diametral pitch proves this formula. If the diametral pitch equals the number of teeth to each inch of pitch diameter, then dividing the number of teeth in the gear by the diametral pitch will give the number of inches of the pitch diameter. If the circular pitch and number of teeth are given, first find the diametral pitch, and proceed as given above.

Addendum.

The addendum of a gear tooth is the distance from the pitch circle to the outside circumference of the gear. This distance is always equal to the reciprocal of the diametral pitch, or 1 divided by the diametral pitch, and, expressed as a formula, is:

$$A = \frac{1}{P} \qquad (4)$$

in which $A =$ addendum; $P =$ diametral pitch.

Outside Diameter.

When we start to make a gear, we first wish to know the outside diameter. If we have the pitch and number of teeth given, this may easily be found by the following rule: Add 2 to the number of teeth, and divide by the pitch. This, expressed as a formula, is:

$$\frac{N+2}{P} = D_1 \qquad (5)$$

in which $N =$ number of teeth; $P =$ diametral pitch; $D_1 =$ outside diameter.

Example.—Given a gear of 20 teeth and 4 pitch, to find the outside diameter. The number of teeth, 20, plus 2 equals 22, and 22 divided by 4 (the pitch of the gear) equals 5½, the outside diameter of the gear.

This formula is simply a combination of formulas 3 and 4, for we first find the pitch diameter, and then add the addendum twice, for it must be added on each side of the pitch diameter. The mathematical solution is as follows:

$$\frac{N}{P} = D; \quad D + \frac{1}{P} + \frac{1}{P} = D_1$$

$$D_1 = D + \frac{2}{P}; \; D_1 = \frac{N+2}{P} \qquad (5)$$

Dedendum and Clearance.

The dedendum is the working depth of the tooth below the pitch line, and must be equal to the addendum or $\frac{1}{P}$, for the pitch circles of two gears are tangent (touching), so the addendum of one will give the working depth of the other below the pitch line. The clearance is the distance from the end of the dedendum to the bottom of the space between the teeth. There is no common standard for this distance, different gear makers using different distances, yet the difference between them is very slight.

The Brown & Sharpe formula for this distance is:

$$F = \frac{0.157}{P} \qquad (6)$$

in which F = clearance; P = diametral pitch.

The Geo. B. Grant formula is:

$$F = \frac{A}{8} \qquad (7)$$

in which F = clearance; A = addendum.

Thickness of Tooth.

The thickness of tooth and width of the space of a gear are always equal at the pitch line, and if the circular pitch is the distance from the center of one tooth to the center of the next tooth measured on the pitch line, tooth and space being equal, then the thickness of tooth must be equal to one-half the circular pitch, or

$$T = \frac{P_1}{2} \qquad (8)$$

in which T = thickness of tooth at pitch line; P_1 = circular pitch.

We know by formula (1) that

$$P_1 = \frac{3.1416}{P} \qquad (1)$$

and substituting this value for P_1 in formula (8) we have:

$$T = \frac{\frac{3.1416}{P}}{2}$$

and this formula resolved to its simplest form is:

$$T = \frac{1.5708}{P} \qquad (9)$$

in which T = thickness of tooth at pitch line; P = diametral pitch.

Example.—Given a gear 1 3/16 circular pitch, what is the thickness of tooth at the pitch line? 1 3/16 (the circular pitch) divided by 2 gives 19/32, the thickness of tooth at the pitch line.

Example.—Given a 6-pitch gear to find the thickness of tooth at the pitch line. 1.5708 divided by 6 (the diametral pitch of the gear) gives 0.262, the thickness of tooth at the pitch line.

Table III gives the thickness of tooth at the pitch line for the different diametral pitches.

Depth of Tooth.

After we get the gear blank turned up, we next want to know how deep to run the gear cutter in order to get a perfect tooth. The working depth of the tooth we have shown to be equal to the sum of the addendum and dedendum, or $\frac{1}{P} + \frac{1}{P} = \frac{2}{P}$, and the whole depth of the tooth must equal $\frac{2}{P}$ plus the clearance.

Using the Brown & Sharpe standard, we have $\frac{2}{P} + \frac{0.157}{P} =$

$$E = \frac{2.157}{P} \qquad (10)$$

in which $E =$ full depth of tooth; $P =$ diametral pitch.

Example.—Given a gear of 6 diametral pitch, to find the depth of cut to be taken to get a perfect gear tooth.

Divide 2.157 by 6 (diametral pitch) and the quotient 0.359 is the depth to be cut in the gear.

If we had the circular pitch given, to find the depth of tooth, we could substitute in formula (10) the value of P as given in the formula (2), and we would have

TABLE III. THICKNESS OF TOOTH AT PITCH LINE.

Diametral Pitch.	Thickness of Tooth at Pitch Line.	Diametral Pitch.	Thickness of Tooth at Pitch Line.
2	0.785 inch.	12	0.131 inch.
2¼	0.697 "	14	0.112 "
2½	0.628 "	16	0.098 "
2¾	0.570 "	18	0.087 "
3	0.523 "	20	0.079 "
3½	0.448 "	22	0.071 "
4	0.393 "	24	0.065 "
5	0.314 "	26	0.060 "
6	0.262 "	28	0.056 "
7	0.224 "	30	0.052 "
8	0.196 "	32	0.049 "
9	0.175 "	36	0.044 "
10	0.157 "	40	0.039 "
11	0.143 "	48	0.033 "

$$E = \frac{2.157}{3.1416 \div P_1}$$

which, reduced to its simplest form, is:

$$E = 0.6866\, P_1 \qquad (11)$$

in which E = depth to be cut in gear;
P_1 = circular pitch.

Example.—Given a gear 1½ inch circular pitch, to find the depth to be cut.

Multiply 0.6866 by 1½ (circular pitch), and the product 1.030 is the depth to be cut in gear.

Table IV gives the depth to be cut in a gear for different diametral pitches.

Distance Between Centers.

Having given the number of teeth and diametral pitch of two gears, to find the distance between centers, add the number of teeth together, and divide by twice the diametral pitch, or

$$\frac{N+n}{2P}=C \qquad (12)$$

in which N = number of teeth in one gear.
n = number of teeth in other gear.
P = diametral pitch.
C = distance between centers.

This formula is obtained from formula (3):

$$\frac{N}{P}=D.$$

This formula gives us the pitch diameter of one gear, and, if we get the pitch diameters of two gears and add them together, we have twice the distance between centers, for the sum of the pitch diameters is twice the sum of the pitch radii, which is the distance between centers.

TABLE IV. DEPTH OF TOOTH.

Diametral Pitch.	Depth to be cut in gear.	Diametral Pitch.	Depth to be cut in gear.
2	1.078 inch.	12	0.180 inch.
2¼	0.958 "	14	0.154 "
2½	0.863 "	16	0.135 "
2¾	0.784 "	18	0.120 "
3	0.719 "	20	0.108 "
3½	0.616 "	22	0.098 "
4	0.539 "	24	0.090 "
5	0.431 "	26	0.083 "
6	0.359 "	28	0.077 "
7	0.308 "	30	0.072 "
8	0.270 "	32	0.067 "
9	0.240 "	36	0.060 "
10	0.216 "	40	0.054 "
11	0.196 "	48	0.045 "

We have now traced, by the aid of a few "rules," the proportions of a gear tooth, having given the pitch and number of teeth, through pitch diameter, addendum, dedendum, clearance, width of tooth and depth to be cut, up to the distance between centers. We now give some formulas for the solution of problems in which some of the quantities which were known in preceding problems are unknown.

Pitch.

1 To find the pitch, having given the pitch diameter and number of teeth. Divide the number of teeth by the pitch diameter, and the quotient will be the pitch. The proof of this assertion is derived from the formula:

$$D=\frac{N}{P} \qquad (3)$$

If the pitch diameter equals the number of teeth divided by the pitch, then the pitch diameter multiplied by the pitch must equal the number of teeth; therefore the pitch must equal the number of teeth divided by the pitch diameter, and this, expressed in its simplest form, is:

$$P = \frac{N}{D} \qquad (13)$$

in which $P =$ pitch (diametral); $N =$ number of teeth in gear; $D =$ pitch diameter.

Example.—A gear, 3 inches pitch diameter, has 36 teeth. Find the diametral pitch.

Divide 36 (the number of teeth) by 3 (the pitch diameter), and we have 12, the diametral pitch of the gear.

2. Having given the outside diameter and number of teeth, to find the diametral pitch. Add 2 to the number of teeth, and divide by the outside diameter, and the quotient will be the pitch of the gear.

In formula (5) we have:

$$\frac{N + 2}{P} = D_1 \qquad (5)$$

If the number of teeth + 2 divided by the pitch equals the outside diameter, then the outside diameter multiplied by the pitch must equal the number of teeth + 2, and then the pitch must equal the number of teeth + 2 divided by the outside diameter, which, expressed as a formula, is:

$$\frac{N + 2}{D_1} = P \qquad (14)$$

in which $N =$ number of teeth in gear; $D_1 =$ outside diameter; $P =$ diametral pitch.

Example.—Given a gear of 36 teeth and 3 1/6-inch outside diameter; to find the diametral pitch.

36 (the number of teeth) + 2 = 38.

38 ÷ 3 1/6 = 12, the diametral pitch of the gear.

Pitch Diameter.

1. Having given the outside diameter and the pitch, to find the pitch diameter. The distance from the pitch diameter to the outside diameter is $\frac{1}{P}$, as explained in formula

$$A = \frac{1}{P} \qquad (4)$$

and as this is to be added on each side of the center, the outside diameter of the gear must be equal to the pitch diameter plus $\frac{2}{P}$. If this is so, then $\frac{2}{P}$ subtracted from the outside diameter will give the pitch diameter, or

$$D = D_1 - \frac{2}{P} \qquad (15)$$

in which D = pitch diameter; D_1 = outside diameter; P = diametral pitch.

Example.—Given a gear 3 1/6 inches outside diameter and 12 pitch; to find the pitch diameter.

3 1/6 inches (the outside diameter) — 2/12 = 3 inches, the pitch diameter of the gear.

2. Having given the outside diameter and number of teeth, to find the pitch diameter. Multiply the outside diameter by the number of teeth, and divide by the number of teeth plus 2.

We have shown in formula (5) that the outside diameter equals the number of teeth + 2 divided by pitch, or

$$D_1 = \frac{N + 2}{P} \qquad (5)$$

and in formula (13) that pitch equals the number of teeth divided by the pitch diameter, or

$$P = \frac{N}{D} \qquad (13)$$

Now, if the outside diameter equals the number of teeth plus 2 divided by the diametral pitch (and the diametral pitch equals the number of teeth divided by the pitch diameter), then the outside diameter must be equal to the number of teeth plus 2, divided by a fraction with the number of teeth as numerator and the pitch diameter as denominator. This is simply substituting the value of the pitch as shown in formula (13) for the pitch in formula (5), and expressed as a formula, is:

$$D_1 = \frac{N + 2}{N \div D}$$

Multiplying both sides of the equal sign by $\frac{N}{D}$ we have

$$D_1 \times \frac{N}{D} = N + 2, \text{ or } \frac{D_1 \times N}{D} = N + 2,$$

and now, multiplying both sides by D, we have

$$D_1 \times N = (N + 2) \times D$$

and dividing both sides by $N + 2$ we get

$$\frac{D_1 \times N}{N + 2} = D, \text{ or } D = \frac{D_1 \times N}{N + 2} \qquad (16)$$

in which D = pitch diameter; N = number of teeth; D_1 = outside diameter.

Example.—Given a gear 3 1/6 inches outside diameter and 36 teeth. To find the pitch diameter.

3 1/6 (the outside diameter) multiplied by 36 (the number of teeth) equals 114. 36 (the number of teeth) + 2 = 38. 114 ($D_1 \times N$) divided by 38 (N + 2) = 3 inches, the pitch diameter of the gear.

Number of Teeth.

1. Having given the pitch diameter and pitch, to find the number of teeth. Multiply the pitch diameter by the pitch, and the product will be the number of teeth in the gear.

The diametral pitch of a gear equals the number of teeth for each inch of pitch diameter; hence, if we multiply the pitch by the number of inches of pitch diameter we will have the number of teeth in the gear, which, expressed as a formula, is:

$$N = P \times D \tag{17}$$

in which P = diametral pitch; D = pitch diameter.

Example.—Given a gear 3 inches pitch diameter and 12 diametral pitch, to find the number of teeth. 3 (pitch diameter) multiplied by 12 (diametral pitch) = 36, the number of teeth in the gear.

2. To find the number of teeth, having given the outside diameter and pitch. Multiply the outside diameter by the pitch and subtract 2, or

$$N = (D_1 \times P) - 2 \tag{18}$$

in which N = number of teeth; D_1 = outside diameter; P = diametral pitch.

This formula is simply the reverse of formula

$$\frac{N+2}{P} = D_1 \tag{5}$$

If the outside diameter equals the number of teeth + 2 divided by the pitch, which we have already proved, then the number of teeth plus 2 must equal the outside diameter multiplied by the pitch, and subtracting 2 from this result we have the number of teeth in the gear.

Example.—Given a gear 3 1/6 inches outside diameter and 12 pitch, to find the number of teeth. Multiply 3 1/6 (outside diameter) by 12 (the pitch) and we have 38, and subtracting 2 from this result we have 36, the number of teeth in the gear.

Outside Diameter.

To find the outside diameter having given the pitch diameter and pitch. Divide 2 by the pitch and add to the pitch diameter, or

$$D_1 = D + \frac{2}{P} \tag{19}$$

in which D_1 = outside diameter.
D = pitch diameter.
P = pitch.

The addendum of a gear is $\frac{1}{P}$ [formula (4)] and this, added on each side of the pitch diameter, gives the outside diameter.

Example.—Given a gear 3 inches pitch diameter and 12 pitch; to find the outside diameter.

3 (pitch diameter) plus 2/12 $\left(\frac{2}{P}\right)$ = 3 1/6 inches, the outside diameter of the gear.

Summary of Formulas.

Having given the general principles of the proportions of gear teeth, we will now group the formulas (which we have proved to be correct) under one head, so that they may be more easily found when wanted.

In the following formulas,

P = diametral pitch.
P_1 = circular pitch.
D = pitch diameter.
D_1 = outside diameter.
N = number of teeth in one gear.
n = number of teeth in mating gear.
A = addendum.
T = thickness of tooth at the pitch line.
E = full depth of tooth.
C = distance between centers.
F = clearance.

$$P_1 = \frac{3.1416}{P} \qquad (1)$$

$$P = \frac{3.1416}{P_1} \qquad (2)$$

$$D = \frac{N}{P} \qquad (3)$$

$$A = \frac{1}{P} \qquad (4)$$

$$D_1 = \frac{N + 2}{P} \qquad (5)$$

$$F = \frac{0.157}{P} \text{ or } \frac{A}{8} \qquad (6 \text{ and } 7)$$

$$T = \frac{P_1}{2} \qquad (8)$$

$$T = \frac{1.5708}{P} \qquad (9)$$

$$E = \frac{2.157}{P} \qquad (10)$$

$$E = 0.6866\, P_1 \qquad (11)$$

$$C = \frac{N + n}{2\,P} \qquad (12)$$

$$P = \frac{N}{D} \qquad (13)$$

$$P = \frac{N + 2}{D_1} \qquad (14)$$

$$D = D_1 - \frac{2}{P} \qquad (15)$$

$$D = \frac{D_1 \times N}{N + 2} \qquad (16)$$

$$N = P \times D \qquad (17)$$

$$N = (D_1 \times P) - 2 \qquad (18)$$

$$D_1 = D + \frac{2}{P} \qquad (19)$$

CHAPTER III.

DESIGN AND CALCULATION OF GEAR WHEELS.

The complete calculations required for the design of a *pair* of gears according to the usual shop practice are few and simple, and it is proposed to put these calculations into easily-understood form in the present chapter. The calculations should be made in the following manner: First—Find out if the gears are intended to give a certain *velocity ratio* between two shafts, or a certain *power ratio* between the shafts; and, assuming the number of teeth in one gear, make the number of teeth in the other gear such as to have the *required ratio* to the number of teeth in the first gear. Second—Assume the pitch of the gears, and calculate the pitch diameters of the two gears, and the distance between the centers of the shafts. Third—Calculate the width of face required to give the gears proper strength. Fourth—Lay out the gears and the tooth forms. The relations of these several steps one to another are such as to make some assumptions necessary, and these depend upon the judgment and experience of the designer, especially when the distance between the shafts is approximately settled, and certain ratios are to be obtained without materially changing the shaft centers. In the case of the younger designers, however, these assumptions are all made beforehand and given to them with instructions to lay out the gears. The several steps will now be taken up and each explained.

Speed Ratios.

Fig. 10 represents two shafts connected by a pair of spur gears, A being the driven shaft, and B, the driving shaft. If shaft A is required to revolve half as fast as shaft B, it is easily seen that the gear on A must be twice as large, and, being of the same pitch, must have twice as many teeth as the gear on B. If n and n_1 represent the number of teeth in each gear, respectively, we have the proportion,

$$y : x = n : n_1 \text{ or } \frac{x\,n}{n_1} = y.$$

If, now, a third shaft were to be driven by gears from shaft A, we would assume A to be the driver revolving y times a minute, and by the above proportion determine the revolutions of the third shaft, and so continue indefinitely for as many shafts as are geared together in any one train. Thus follows the rule: *The speed of the last shaft equals the speed of the first shaft multiplied by the product of the number of teeth in the driving gears and divided by the product of the number of teeth of the driven gears.*

Power Ratios.

In case a certain ratio of power is wanted, we shall find some sort of a crank or pulley, the radius R of which is known, upon the power shaft B, and upon the driven shaft there will also be some sort of a crank, pulley or drum, the radius r of which is known. We can now make the equation (referring to Fig. 10):

$$\frac{P R n_1}{n r} = Q$$

This expression may be made general by following through as before from shaft to shaft, and may be given as the following rule: *The power, multiplied by the power arm, times the product of the number of teeth in the driven gears, and divided by the weight arm times the product of the number of teeth in the driving gears, equals the weight.*

Fig. 11 shows in tabular form several different forms of gear trains with their formulas for speed and power ratios. It is to be noted that in place of using the numbers of teeth in these ratios we may use the pitch diameters of the gears, but as these diameters are very often expressed in fractional parts of an inch, while the number of teeth is always a whole number, it is found more convenient to use the latter. Idlers are often used, as shown in the sketch in section 2, Fig. 11, and they have no effect upon either the speed or power ratios. They are introduced either to connect two shafts where the great distance between centers would involve very large gears if geared directly together, or to effect a change in direction of motion, as may be seen by the arrows in Fig. 12. An inspection of this engraving proves the rule that an even number of idlers does not change the direction of motion between two shafts, while an odd number of idlers reverses the direction of motion.

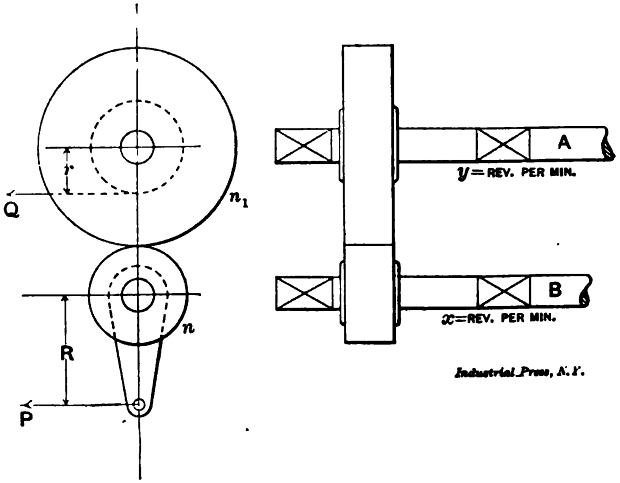
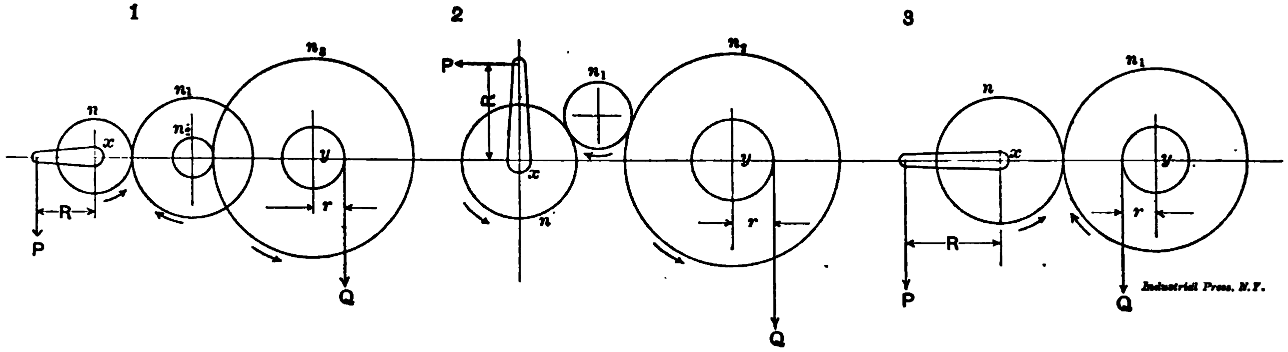


Fig. 10. Speed and Power Ratios.



First Illustration.

Speed ratio:

$$\frac{x\,n\,n_2}{n_1\,n_3} = y$$

Power ratio:

$$\frac{P\,R\,n_1\,n_3}{n\,n_2\,r} = Q$$

Second Illustration.

Speed ratio:

$$\frac{x\,n\,n_1}{n_1\,n_2} = \frac{x\,n}{n_2} = y$$

Power ratio:

$$\frac{P\,R\,n_1\,n_2}{n\,n_1\,r} = \frac{P\,R\,n_2}{n\,r} = Q$$

Note.—As the number of idlers cancels out they do not affect the result.

Third Illustration.

Speed ratio:

$$\frac{x\,n}{n_1} = y$$

Power ratio:

$$\frac{P\,R\,n_1}{n\,r} = Q$$

NOTATION.—$n\ n_1\ n_2\ n_3$ = number of teeth in gears. x = rev. per min. of power shaft. y = rev. per min. of driven shaft. R – rad. of power arm. r = rad. of load arm.

Fig. 11. Diagrams Illustrating Speed and Power Ratios of Gearing.

Pitch Diameters.

Having determined upon the velocity or power ratio required of our gears, the next step is to determine the two pitch diameters of the gears. To do this it is necessary to assume the pitch of the gears, and this assumption depends upon the judgment and experience of the designer, although very often it may be confirmed by comparison with gears of about the same size and doing about the same work as those to be designed. After assuming the pitch and finding the pitch diameters, a calculation for the strength of the gears will show whether the assumed pitch is right, and if it then proves to be too small or too large, the calculation may be repeated with another assumed pitch.

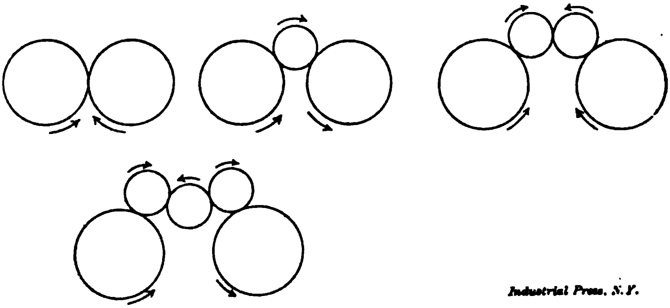


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Fig. 12. Illustration of the Effect of Idlers.

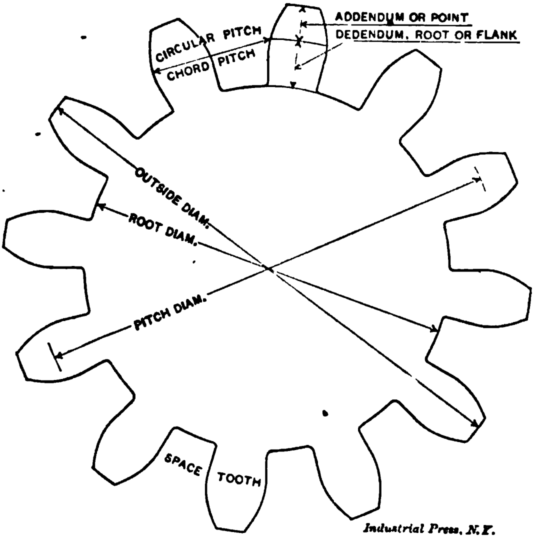

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Fig. 13. Terms used to denote Gear Quantities.

Pitch of Gears.

It is first necessary to understand what is meant by the pitch of a gear, and its relation to the diameter. This has been explained in the previous chapter, but in order to make the present discussion a complete whole by itself, the most important definitions and formulas have here been repeated. Fig. 13 shows a gear of twelve teeth (such a small gear is often called a pinion), and the names of the different parts are clearly indicated. As will be seen, the circular pitch is the distance on the pitch circle from a point on one tooth to the corresponding point on the next tooth. The circumference of the pitch circle is equal to the pitch multiplied by the number of teeth and dividing this by 3.1416 gives the diameter of the pitch circle, or

$$D = \frac{P_1 N}{3.1416}$$

when, D = the diameter of the pitch circle,

N = the number of teeth,

P_1 = the circular pitch.

TABLE V. CONSTANT FOR DETERMINING CHORD PITCH.

Number of Teeth.	Constant K.	Number of Teeth.	Constant K.	Number of Teeth.	Constant K.
12	.258	31	.102	52	.059
13	.239	32	.097	54	.057
14	.222	33	.094	56	.055
15	.207	34	.093	58	.053
16	.195	35	.089	60	.052
17	.184	36	.087	62	.049
18	.173	37	.084	64	.048
19	.165	38	.082	66	.045
20	.156	39	.080	68	.044
21	.148	40	.078	70	.043
22	.141	41	.076	75	.041
23	.136	42	.075	80	.039
24	.130	43	.073	85	.036
25	.125	44	.071	90	.034
26	.120	45	.069	95	.032
27	.115	46	.067	100	.031
28	.112	47	.066	125	.025
29	.107	48	.065	150	.019
30	.104	49	.063	175	.017
		50	.061	200	.015

After having determined the pitch diameter and drawn the pitch circle, we must divide the pitch circle into as many equal parts as the number of teeth, or, what is the same thing, lay off the circular pitch upon the pitch circle. In the case of a small pinion, such as Fig. 13, this may be most easily done by trial with a pair of dividers. It very often happens, however, that a gear is so large as to make this method impracticable, because only a portion of the gear, showing a few teeth, will be drawn. It thus becomes necessary to have some method of accurately laying off the circular pitch upon the pitch circle when only a portion of the circle is drawn. From Fig. 13 it is evident that if we set our dividers to the circular pitch, and attempt to step off the spaces, what we shall actually be stepping off will be chords instead of circular arcs, and the resulting arcs will be greater than the circular pitch. In very large gears this error is very small, but in ordinary gears it is quite appreciable, and the dividers should be set to the chord pitch. Table V has been computed to enable the chord pitch to be easily determined, as the pitch diameter multiplied by the constant *K*, opposite the number of teeth in the table, equals the chord pitch.

Relation between Number of Teeth, Pitch, and Pitch Diameter of Gears.

Molded or rough-cast gears are usually designed by circular pitch, but cut gears are designed by what is known as diametral pitch. Since the number of teeth bears a fixed relation to the pitch circumference, and the pitch diameter bears a fixed relation to the pitch circumference, it follows that the number of teeth bears a fixed relation to the pitch diameter. This being so, we may divide the pitch diameter expressed in inches by the number of teeth, and the result will be what is termed the diametral pitch. It is also evident that if the number of teeth bears a fixed relation to the pitch circumference and pitch diameter, the circular pitch and diametral pitch must have some fixed relation to each other. These different relations are most conveniently given for use as follows:

Circular pitch = P_1

$$D = \frac{P_1 N}{\pi}$$

$$P_1 = \frac{D \pi}{N}$$

$$N = \frac{D \pi}{P_1}$$

$$D_1 = D + 0.6\, P_1$$

$$C = \frac{D + D_2}{2} = \frac{P_1 N_1}{2 \pi}$$

Diametral pitch = P

$$D = \frac{N}{P}$$

$$P = \frac{N}{D}$$

$$N = P D$$

$$D_1 = \frac{N + 2}{P}$$

$$C = \frac{D + D_2}{2} = \frac{N_1}{2 P}$$

Relation of Circular and Diametral Pitch.

$$P_1 P = \pi \qquad P_1 = \frac{\pi}{P} \qquad P = \frac{\pi}{P_1}$$

π = 3.1416,
P_1 = circular pitch,
P = diametral pitch,
D = pitch diameter,
D_2 = pitch diameter of mating gear,
D_1 = outside diameter,
N = number of teeth,
N_1 = number of teeth in a pair of gears = sum of the teeth in each gear,
C = distance between centers of shafts.

When designing cut gears it is not necessary to lay out the form of the teeth, as these are formed by the gear-cutting process, and it is only necessary for the designer to calculate the pitch diameters that will give the required ratios, and then to find the outside diameter of the blank from which the gear is to be cut. For such gears diametral pitch is a great convenience, as the relations of pitch, diameter and number of teeth are so simple.

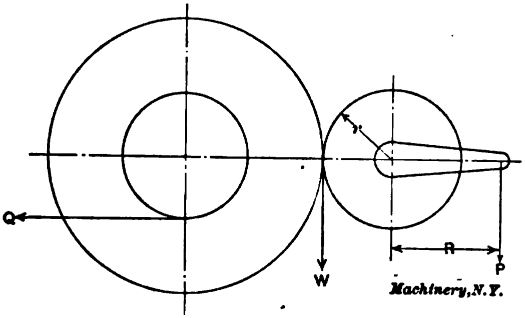


Fig. 14.

Strength of Teeth and Width of Face.

Before proceeding further, it is well to know if our assumed pitch for the gears will give strong enough teeth without requiring a wider face than is practical, and it becomes necessary to know the force or power transmitted by the gears. The most convenient way to do this is to get the force in pounds which is carried by the gear at the pitch line. In Fig. 14, let it be supposed that the power ratio is such as to make it just possible to move the load Q with a force P upon the crank. Then the force W at the pitch line of the gears will be,

TABLE VI. FACTOR FOR CALCULATING WIDTH OF FACE OF TEETH.

Radial Flanks.		Involute 15°.	
Number of Teeth.	Factor Y.	Number of Teeth.	Factor Y.
12	052	38	.107
13	053	43	.110
14	.054	50	.112
15	.055	60	.114
16	.056	75	.116
17	057	100	.118
18	.058	150	.120
19	.059	300	.122
20	.060	Rack	.124
21	.061		
23	.062		
25	.063		
27	.064		
30	.065		
34	.066		

$$W = \frac{P R}{r}.$$

Having found W we may calculate the required width of face for our gears having the assumed pitch, and if the required face proves to be too wide, we shall have to assume a larger pitch, in order to get stronger teeth. The most widely used formula for the strength of gears is that proposed by Mr. Wilfred Lewis and given in Kent's handbook, page 901, as

$$W = SPFY,$$

in which W = force on pitch line in pounds,
P = circular pitch,
S = allowable fiber stress for the material used,
F = width of face of gear,
Y = variable from Table VI.

S may be assumed as 3,500 pounds for cast iron, and 8,000 pounds for cast steel, and as the pitch has been already assumed, the formula may be changed to give the required width of face thus,

$$F = \frac{W}{SPY}.$$

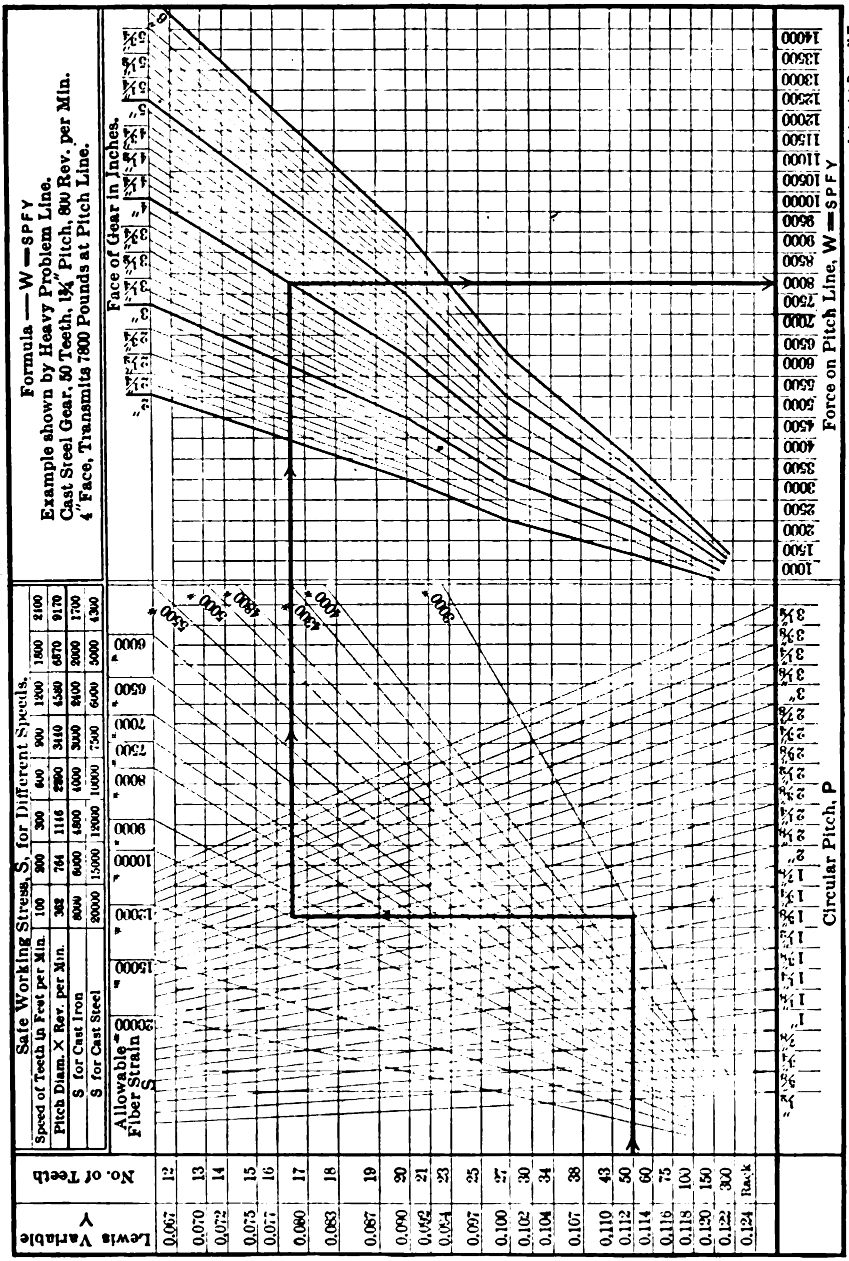
Substituting in this formula the values already obtained for W, S, P, and Y, we find the required face for the gears, and if this is too great, a larger pitch must be used.

Chart for Strength of Spur Gears.

The accompanying chart, Fig. 15, for the strength of spur gears, enables problems to be solved easily and quickly, and the result of any changes in pitch or face to be quickly seen. The heavy line with the arrows shows the method of working out the problem for a 50-tooth gear as stated in the upper right-hand corner of the chart. The chart also shows the safe working loads for different speeds as given by Mr. Lewis. Enter the chart on the left at the number of teeth; then follow over to the diagonal line for the pitch of the gear, then up or down to the diagonal line for the allowable fiber stress for the material of which the gear is to be made, then over to the diagonal line for the face of the gear, then down, and read the load in pounds that the gear will carry at the pitch line. If the load to be carried and the number of teeth required are known, the chart may be entered at each end, that is, at the number of teeth and at the load, and by then following each way, the face required for a certain pitch, or the pitch required for a certain face are easily seen, and thus the best combination of face and pitch for any case is easily determined without any calculations.

Laying out the Tooth Outlines.

Having gone through the steps described previously, we shall know the pitch diameter, pitch, number of teeth, and face of the gear to be designed, and are ready to lay out the gear and tooth outlines. As has been mentioned in Chapter I, two forms of teeth are in general use, the cycloidal and the involute, each having its champions among able designers. The involute form has the advantage of being the more easily ground to an approximately correct tooth form in the case of



Formula — W = SPFY

Example shown by Heavy Problem Line.

Cast Steel Gear, 50 Teeth, 1¼" Pitch, 800 Rev. per Min.

4" Face, Transmits 7800 Pounds at Pitch Line.

Safe Working Stress, S, for Different Speeds.

Speed of Teeth in Feet per Min.	100	200	300	600	900	1200	1800	2400
Pitch Diam. X Rev. per Min.	382	764	1146	2290	3440	4580	6870	9170
S for Cast Iron	8000	6000	4800	4000	3000	2400	2000	1700
S for Cast Steel	20000	15000	12000	10000	7500	6000	5000	4300

Industrial Press, N.Y.

Fig. 15. Chart for Proportioning Spur Gears.

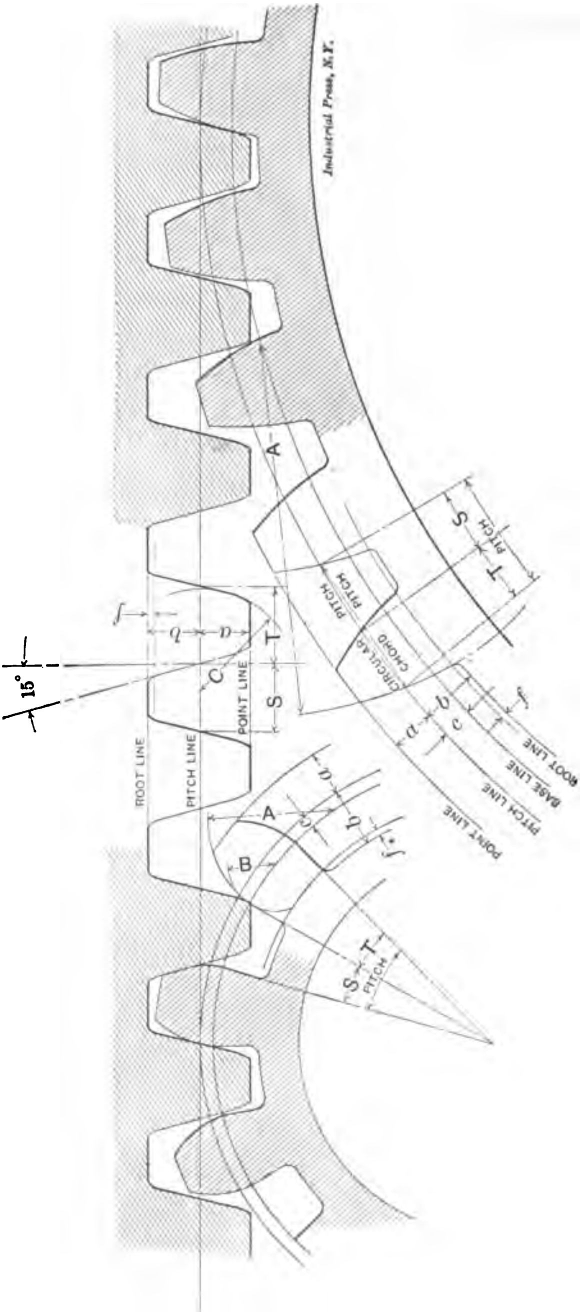


Fig. 16. Two Gears in mesh with a Rack. The Gear to the Left shows the Method of laying out a Gear having twelve to thirty-six Teeth; the View to the Right, a Gear having thirty-six Teeth or more.

molded gears, and of running well with a small deviation from the true distance between centers. The involute form only will be considered, as it is believed that this form is becoming very much the more common of the two. The tooth outlines are most conveniently laid out by the use of an odontograph table (Table VII), in which the dimensions shown in Fig. 16 are given in terms of the pitch. Two gears are shown in mesh with a rack—that on the left showing the method of laying out a gear having from 12 to 36 teeth, and that on the right showing the method for a gear having 36 teeth or more.

TABLE VII. DIMENSIONS AND CONSTANTS FOR LAYING OUT GEAR TEETH.

Circular Pitch.		Diametral Pitch.
0 3 × pitch.	*a*	1 + pitch.
0.4 × pitch, below 1″.	*b*	1.15 + pitch.
0.375 × pitch, above 1″.	*b*	
0.016 × pitch diameter.	*c*	
0.53 × pitch.	*S*	
0.47 × pitch.	*T*	
0 1 × pitch.	*f*	

Multiply by Number of Teeth.			Multiply by Number of Teeth.		
Pitch.	A.	B.	Pitch.	A.	B.
¾″	0.03	0.015	1″	0.125	0.062
⅞″	0.035	0.0175	1¼″	0.100	0.050
1″	0.04	0.02	1½″	0 083	0.041
1⅛″	0.045	0.0225	1¾″	0.071	0.035
1¼″	0.05	0.025	2″	0.062	0.031
1⅜″	0.055	0.0275	2¼″	0.056	0.028
1½″	0.06	0.03	2½″	0.050	0.025
1⅝″	0.065	0.0325	2¾″	0.045	0.022
1¾″	0.07	0.035	3″	0.042	0.021
1⅞″	0.075	0.0375	3½″	0 036	0.018
2″	0.08	0.04	4″	0.031	0 015
2⅛″	0.085	0.0425	5″	0.025	0.012
2¼″	0.09	0 045	6″	0.021	0.010
2⅜″	0.095	0.0475			
2½″	0.100	0.05			

When laying out a molded gear, first draw the pitch circle of a diameter equal to the pitch diameter previously determined. Draw the point circle outside the pitch circle, and a distance from the pitch circle of 0.3 times the pitch, and draw the root circle inside the pitch circle and a distance from the pitch circle of 0.4 times the pitch. These two distances are given as *a* and *b*, respectively, in Fig. 16, and in Table VII it will be seen that as the point is 0.3 of the pitch outside the pitch circle, while the root is 0.4 of the pitch inside the pitch circle, the teeth of the two meshing gears will have a clearance between point and root of 0.1 of the pitch. For gears having greater than 1-inch pitch, this clearance will be greater than necessary even for rough gears, and will not look well; so for gears above 1-inch pitch, 0.375 instead of

0.4 may be the multiplier for the pitch, as will be found noted in the table. The base line is now drawn at a distance 0.016 times the *pitch diameter* from the pitch circle. This base line may sometimes come inside the root circle. The pitch is then laid off upon the pitch circle as before described, and the spaces thus made upon the pitch circle are to be divided into tooth and space parts. The tooth part will be 0.47 times the pitch, and the space part will be 0.53 times the pitch, thus giving the tooth a small clearance in the space. We are now ready to draw in the tooth outlines, which are circular arcs drawn from centers on the base line. In the case of a gear of less than 36 teeth the tooth outline will be composed of two arcs, while for all gears of 36 teeth or more the tooth outline is only one arc. The radii for these arcs are found by multiplying the number of teeth in the gear by the constants found in the odontograph table opposite the pitch of the gear. These radii are designated in Fig. 16 and Table VII as A and B. In the case of gears of less than 36 teeth the tooth outline is completed by radial lines as shown in Fig. 16.

CHAPTER IV.

STRENGTH OF GEAR TEETH.

In considering the strength of gear teeth we shall, in the first place, neglect the actual shape of the tooth and assume it to be rectangular in every section, as shown in Fig. 17. Further, for the sake of simplicity, it will be assumed that the load, *P*, acts on the outer circumference of the gear, and hence at the extremity of the tooth. If we consider the conditions under which the load is assumed to be applied in actual practice, we shall find that: (1) A rough cast gear may have the whole load concentrated upon one corner of a tooth; or the gear itself may be well made, but may be out of alignment, due to springing of shaft, bad workmanship, or other causes; in this case also the load may be concentrated upon the corner of one tooth. (2) The gear may be well made, with accurately cut or cast teeth, well mounted upon heavy shafts having proper alignment, in which case the load may be distributed over the entire width of a single tooth. In determining a formula, then, for the pitch of a gear tooth, we must know the kind of gears we have to deal with, and what considerations affect their strength.

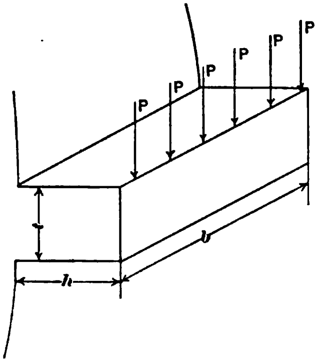
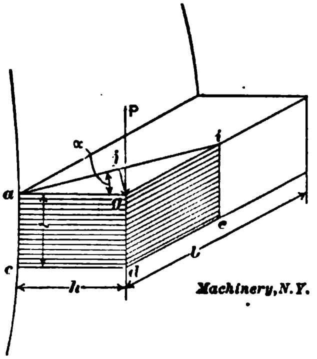


Fig. 17.



Machinery,N.Y.

Fig. 18.

In the first case, if the gear is rough cast or placed so that the load may come upon one corner of a tooth, we must consider not only that the tooth is a cantilever, as in Fig. 17, but that the corner of the tooth is itself a cantilever, as in Fig. 18, in which the whole load, *P*, is concentrated at the point, *g*, so that the moment tending to break the corner off from the tooth is *P* multiplied by the perpendicular distance from *P* to the line, $a\,i$; hence $P \times g\,j = P\,h\,\sin a =$ bending moment. Now, the resistance which the tooth offers depends upon the character of the metal, as well as its actual section, which in the present case is the rectangle *a c e i*. It can be shown that the resistance of this corner to breaking across is equal to

$$1/6\, f\, t^2 \times a\, i = 1/6\, f\, t^2 \frac{h}{\cos a} \qquad (20)$$

in which f = stress in the metal. Equating this to the bending moment, we have

$$Ph \sin a = 1/6\, f\, t^2 \frac{h}{\cos a} \qquad (21)$$

or

$$f = \frac{6\, P \sin a \cos a}{t^2} \qquad (22)$$

Noting that $\sin a \cos a = \frac{1}{2} \sin 2\, a$, and substituting in (22) we have:

$$f = \frac{3\, P}{t^2} \sin 2\, a$$

When $a = 45$ degrees, we have $\sin 2\, a$ = a maximum = 1; therefore

$$f = \frac{3\, P}{t^2};\ \text{or}\ P = f \frac{t^2}{3};\ \text{and}\ t = \sqrt{\frac{3\, P}{f}} \qquad (23)$$

For new teeth, rough cast, in which $t = 0.48\, p_1$ (p_1 being the circular pitch) we have:

$$0.48\, p_1 = \sqrt{\frac{3\, P}{f}};\ \text{or}\ p_1 = 3.6 \sqrt{\frac{P}{f}} \qquad (24)$$

For wooden cogs working with cast iron teeth, in which $t = 0.6\, p_1$, we have:

$$p_1 = 2.9 \sqrt{\frac{P}{f}} \qquad (25)$$

If in any case the width of face, *b*, is less than the height of tooth, the above formulas do not apply. In this case we have used the thickness of tooth without considering loss of strength, due to wear; but we have also assumed the whole load as concentrated upon the corner, whereas, as wear occurs, although the tooth itself is reduced in section, yet the load is better distributed, and the tooth may be actually stronger.

In the case of rough cast iron and mortise gears in which the whole load may come upon the corner of one tooth, the width of gear does not affect its strength, and there is no advantage, as far as strength is concerned, in making such a gear wider than $h \times \frac{1}{\cos 45\ \text{deg.}}$, or $1.41\, h$; as h, the height of the tooth, is frequently made equal to $0.7\, p_1$, we have the safe width of tooth, $b = 1.41 \times 0.7\, p_1 = p_1$, very nearly.

In order to prevent excessive wear, such gearing is frequently made so that its breadth of face $b = 2\, p_1$. So far we have assumed that the tooth section was a rectangle, but in practice the tooth may have the shape shown in Fig. 19, or that in Fig. 20, which represent a wide range in the strength of two gears, when accurately made and aligned, but for a load concentrated upon one corner the difference is not so marked as the shape of the teeth would indicate.

Investigation shows that if the smaller of two gears has at least 18 teeth, and not more than 50, the strength of the tooth will be practically the same as that determined by the above formulas. Since these values may be said to represent fairly well the limits of the number of teeth in the smaller of two gears in those cases where rough gearing is permissible, we shall not at the present time discuss the influence of shape upon the strength of this kind of gearing.

Of more importance is the consideration of that case in which the load is supposed to be equally distributed between two gear teeth.

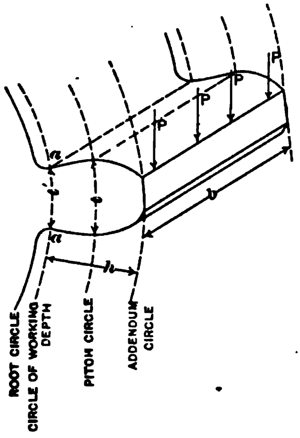


Fig. 19.

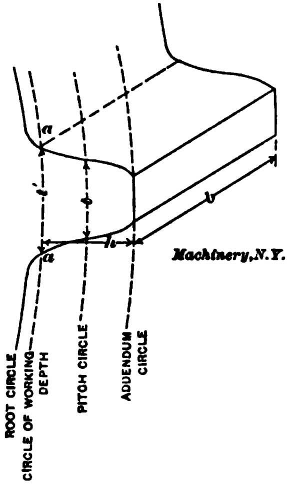


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Fig. 20.

This matter relates to the strength of all gear teeth, and we shall discuss its effects in general.

The point of contact between tooth surfaces in correct gearing can readily be determined when the system upon which the gears have been designed is known. Thus in the involute system, the point of contact is located on the straight line passing through the line of centers at the point of intersection of the pitch circles, and making an angle with the horizontal of 14½ degrees to 22½ degrees, as shown by $P_1 T P'$ in Fig. 21.

On the other hand, the point of contact between two tooth surfaces of the cycloidal system will always be upon a reverse curve, which is a portion of the describing circle for the faces and flanks of the tooth; thus, in the interchangeable system of 12-tooth base, *i. e.*, in that system where the smallest pair of gears in the set is assumed to have 12 teeth and these have radial flanks, the diameter of rolling circle will evidently be equal to the radius of the 12-tooth pinion; and since the tooth profiles are generated by rolling this circle upon the pitch circles of any pair of gears in the set, the point of contact will always fall upon the circumference of this describing circle when the center of the latter lies upon the line joining the centers of the two gears, as shown in Fig. 22. If in Figs. 21 and 22 $M'N'$ and $R'S'$ represent the addendum circles, or the circles drawn through the tops of the teeth of the two gears, we shall have the two loci $P'TP_1$, of the points of contact, as shown in heavy lines; that is, contact will begin in each case at P' and, as the teeth move around, the point of contact between the surfaces will travel along the line or reverse curve, $P'TP_1$, until contact ceases. Now, considering Fig. 22, if the pitch were equal to the arc $P'TP_1$, one tooth would just be beginning action at P', while another would be quitting at P_1. In practice, in order to obtain smoothness of action the effective height of tooth is usually taken at about six-tenths of the circular pitch in cast gears, and about 0.64 of the circular pitch in cut gears. If $p =$ diametral pitch, $p' =$ circular pitch, and $h =$ effective height of tooth, we have: $h = 0.6\,p'$ for cast gears; and

$$h = 0.64\,p' = \frac{2}{p} \text{ for cut gears.}$$

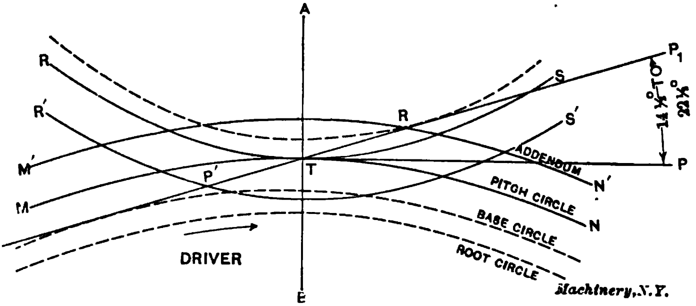

Fig. 21.

We would here note that the effective height of tooth is not the total height, as an additional amount, frequently $0.1\,p'$ for cast gears, is allowed for clearance between the root circle and the tops of the teeth of the mating gear. With an addendum equal to half the above height of tooth, it will be found that the average arc of contact for gears between 12 teeth and a rack, in both the involute and cycloidal systems, has a length equal to about one and two-thirds times the circular pitch. In any case, by stepping off the pitch on $P'TP_1$, Fig. 22, from both P' and P_1, we shall obtain the points a and b, which indicate that with perfect gearing the two pairs of teeth will be simultaneously in contact at P' and at a and will remain in contact until b and P_1 are reached; the distance from b to a will be traversed with only one pair of teeth in contact; however, in this latter case, it will be noticed that the leverage of the tooth is very much reduced. For a 12-toothed pinion (cycloidal system), this leverage, shown in Fig. 22 as h', is $0.41\,p'$; and for a rack it is $0.35\,p'$. Under these conditions, then, the dimensions of the tooth would be determined upon the supposition that the whole working load is carried by one tooth, and that the force is applied with a leverage of $0.35\,p'$ to $0.41\,p'$, the constant depending upon the number of teeth. For the 15-degree involute system the effective height for a 12-toothed pinion is the same as for the cycloidal system, *viz:* $0.41\,p'$, but for a 15-degree involute rack there are always two teeth in contact.

This determination is correct for all gear teeth properly shaped and spaced, but as Mr. Wilfred Lewis has so ably pointed out, it must be admitted that mechanical perfection in forming and spacing has not yet been reached, and that the slightest deviation in either respect is sufficient to concentrate the entire load at the end of one tooth. Even with cut gears the same conditions obtain, for the cutters ordinarily employed are correct only for a single gear, although they are used within certain equidistant limits for various other gears. To what extent the elasticity of bronze, copper, and steel influences the distribution of pressure on cut gears is not known, and it is, therefore, unsafe to consider its effect. As the teeth become worn, the concentration of the load may be reduced, but until this wear takes place, the whole load should be assumed as acting at the extreme end of the tooth.

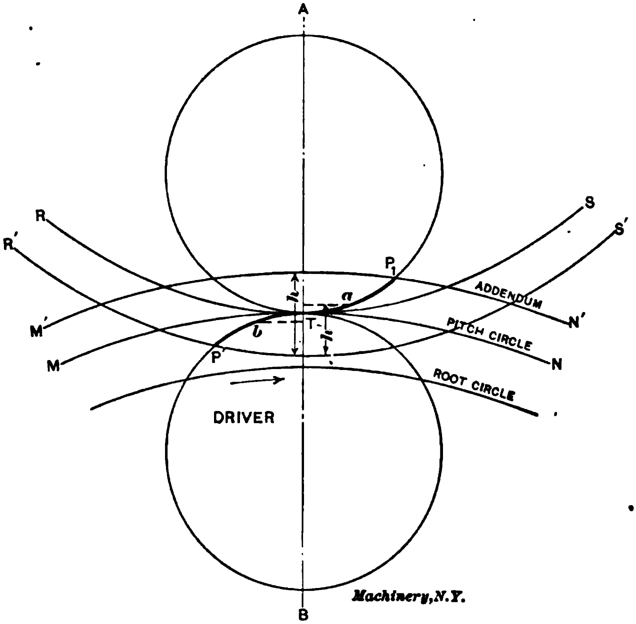

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Fig. 22.

For rough cast iron and mortise gears, as we have shown, it is possible for the whole load to be concentrated on the corner of one tooth, and this may occur even with the best cut gears, owing to careless alignment, or lack of stiffness in the shafting and supports; but when iron patterns are used with accurately cut teeth, especially if the mold is baked, or when the gear is machine molded, or the teeth are cut, the contact should be fairly uniform, and the load evenly distributed across the face of the tooth. It must be understood, however, that in many cases circumstances will arise when it is impossible to secure these favorable conditions; particularly is this true when the gearing is subjected to shock and variable loads. However, with careful work, stiff shafts and not too wide a face, the assumption of fairly uniform distribution of pressure across the tooth may be considered as satisfactory for general practice.

The pressure which comes upon the tooth in the direction of the line of thrust (the common normal to the tooth surface) is greater

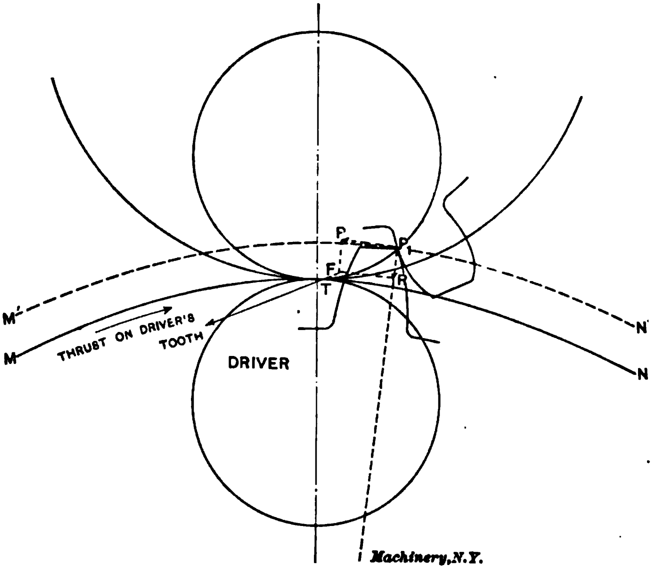

Fig. 23.

than that which transmits motion, for if we resolve the force, F, along and perpendicular to the radius at the point of contact, P_1, Fig. 23, the radial component, R, does not tend to produce rotation; it will, however, exert a pressure on the bearings, and tends to crush the tooth; as the compressive force has little effect upon the strength of the tooth, especially with cast iron gears, its influence may be neglected. The other component, or P, acts at right angles to the radius and produces rotation by its pressure on the tooth, and thus may be considered as the effective working load.

Now considering the tooth as a rectangular cantilever with the load P uniformly distributed across the outer edge, as in Fig. 17, the thickness, t, assumed equal to $0.49\,p' - 0.02$ inch for well-made cast gears, may be obtained by equating the bending moment to the moment of resistance of the tooth; that is

$$P h = f \times \frac{1}{6} b t^2 \qquad (26)$$

Assuming the effective height of tooth $h = 0.6\,p'$ (circular pitch); breadth of face, $b = x\,p'$; thickness of tooth, $t = 0.49\ p' - 0.02$, therefore $t^2 = 0.24\ (p')^2 - 2 \times 0.0098\,p' + 0.0004$, in which we may neglect the last two terms, as they will not appreciably affect the result.

Substituting these values in (26) we have:

$$P \times 0.6\,p' = \frac{f\,x\,p' \times 0.24\ (p')^2}{6}$$

$$P = f\,\frac{x \times 0.24\ (p')^2}{3.6} = 0.066\,f\,x\ (p')^2 \qquad (27)$$

$$p' = \sqrt{\frac{P}{0.066\,f\,x}} = \sqrt{\frac{15\,P}{f\,x}} \qquad (28)$$

For cut gears the addendum is usually made equal to the reciprocal of the diametral pitch, or $1/p$; but since $p\,p' = \pi = 3.14$ we have $1/p = 0.32\ p'$, hence the effective height of tooth now becomes $0.64\ p'$. The thickness of tooth in this case is $0.5\ p'$; therefore the load, from (26) is:

$$P = 0.065\,f\,x\ (p')^2 \text{ and } p' = \sqrt{\frac{15.4\,P}{f\,x}},$$

results practically the same as those just obtained. For cast gears, if the breadth of face equals twice the pitch

$$\text{or } x = 2, \quad \text{then } p' = 2.74\sqrt{\frac{P}{f}};$$

$$\text{if } x = 2\tfrac{1}{2}, \text{ then } p' = 2.45\sqrt{\frac{P}{f}};$$

$$\text{if } x = 3, \quad \text{then } p' = 2.24\sqrt{\frac{P}{f}};$$

$$\text{if } x = 3\tfrac{1}{2}, \text{ then } p' = 2.07\sqrt{\frac{P}{f}};$$

It will be noticed that the formulas thus deduced-apply only to rectangular teeth, as in Fig. 17, whereas in practice we have to deal with such forms as given in Figs. 19 and 20. In Fig. 19, the width of tooth is considerably less measured on the working depth circle than on the pitch circle, and fracture would occur along the line $a\,a$, where the thickness is t'.

With a uniformly distributed pressure, P, acting at the extremity of the tooth as shown, the relative strength of this tooth compared with one of rectangular section having the uniform thickness t varies as $\left(\frac{t'}{t}\right)^2$. Thus if a gear tooth of 1 diametral pitch ($p' = 3.14$) measures 1.52 inch on the pitch circle and 1.34 inch on the working depth circle between the points *a* and *a*, Fig. 19, the strength of the tooth will be only $\left(\frac{1.34}{1.52}\right)^2 = 0.78$ of that which it would have if the tooth had the constant thickness 1.52 inch. In the same way, if the thickness of tooth at *a a*, Fig. 20, measures 1.76 inch, then its relative strength will be $\left(\frac{1.76}{1.52}\right)^2 = 1.34$.

This suggests a convenient method not only of ascertaining the load which a given gear will sustain, but also one which will give a suitable pitch when the load and number of teeth are known.

If in the first case the pitch had been determined from formula (28), the tooth would have only about three-fourths of its assumed strength for a given load *P*, whereas in the second case, the formula would give a tooth about a third stronger than necessary for the same load *P*.

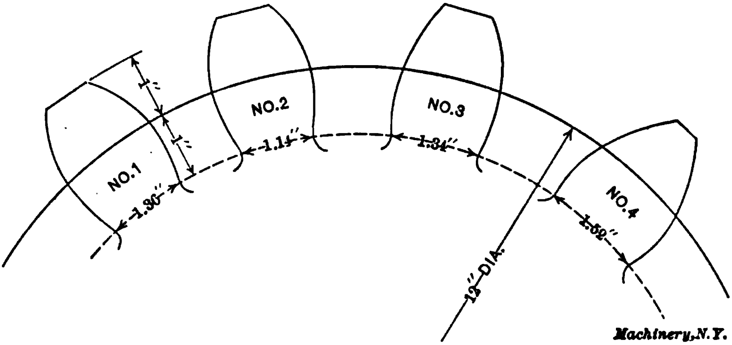

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Fig. 24.

Since the pitch varies as the square root of the pressure, it will be evident that if we multply the pitch in each case by the inverse ratio of $\frac{t'}{t}$ we shall obtain a pitch giving the actual strength required in both cases. Therefore, since

$$\frac{1.52}{1.34} = 1.14, \text{ and } \frac{1.52}{1.76} = 0.86,$$

the respective pitches will be 1.14 p' and 0.86 p', in which

$$p' = \sqrt{\frac{15\,P}{f\,x}}$$

from formula (28).

It is evident that the method of laying out the gear tooth will have some influence upon its strength; this is very clearly shown in Fig. 24, which represents a twelve-tooth pinion of 3.14 inches pitch accurately drawn for the four systems represented, *viz*: (1) cycloidal, 12 base, *i. e.*, the smallest pair of gears is assumed to have twelve teeth.

and these have radial flanks, so that the diameter of rolling circle is one-half of the diameter of pitch circle of a twelve-tooth pinion; (2) cycloidal, 15 base; (3) involute, 15 degrees line of action, and (4) involute, 22½ degrees lines of action. By laying out a series of different gears, according to the different systems represented, the exact thickness of t' and t may be determined graphically, from which the values of the ratios $\frac{t}{t'}$ and $\left(\frac{t'}{t}\right)^2$ may be readily ascertained.

This has been done, and the results are given in Tables VIII and IX, in which the ratio is presented in the form of a coefficient corresponding to the number of teeth in the gear.

TABLE VIII. VALUES OF COEFFICIENT $C_p = \frac{t}{t}$

No. of Teeth in Gear		Value of Coefficient C_p			
		Involute System		Cycloidal System	
Exact	Intervals	15 deg.	22½ deg.	12 Tooth Base	15 Tooth Base
12	12	1.14	1.00	1.12	1.33
13½	13 – 14	1.10	0.96	1.08	1.19
15½	15 – 16	1.07	0.92	1.04	1.10
17½	17 – 18	1.04	0.88	1.00	1.03
20	19 – 21	1.01	0.84	0.96	0.98
23	22 – 24	0.98	0.80	0.92	0.95
27	25 – 29	0.95	0.76	0.88	0.92
33	30 – 36	0.92	0.73	0.85	0.89
42	37 – 48	0.88	0.70	0.82	0.86
58	49 – 72	0.84	0.68	0.79	0.83
97	73 – 144	0.80	0.66	0.76	0.80
290	145 – Rack	0.76	0.65	0.73	0.77

Combining these coefficients with formulas (27) and (28) we obtain

$$P = 0.066\, f\, x\, C_s\, (p')^2 \qquad (29)$$

and

$$p' = C_p \sqrt{\frac{15\,P}{f\,x}} \qquad (30)$$

In the same way formulas may be deduced for wooden cogs, working with cast iron, by assuming the width of tooth equal to 0.6 p'.

In these formulas, as previously stated, the effective or working height of tooth was used instead of the total height, which is always somewhat greater, but as the strength of the tooth varies with the square of its thickness, and as this value increases below the working depth circle more rapidly than the increased height of tooth on account of the fillet, we are justified in neglecting that portion of the tooth between the working depth and root circles, assuming that a suitable fillet has been used at the base of the tooth.

The formula here presented will give a close approximation to the working strength of a gear tooth when the pitch and number of teeth are given, provided we know the working stress in the material of which the tooth is composed. This is, however, such a variable factor that it is possible only to suggest limitations covering general conditions; and the judgment of the designer, based on a knowledge of the conditions under which the gear is to work, must modify the general values, as occasion may require. Indeed, it is much more satisfactory to the designer to use a general formula involving a choice of stress than to employ an entirely empirical rule in which the stress is unknown, and which offers no opportunity for variation under varying conditions.

Professor Unwin assumes that for cast iron f may have the three values 9,600, 6,100, and 4,300 pounds per square inch for "little shock," "moderate shock," and "excessive shock," respectively, for those gears in which, from inaccuracy of form or mounting the pressure may come on a corner of the tooth. For carefully fitted gearing, however, in which the pressure is assumed to be distributed along the whole width of the tooth, he suggests 4,350 and 2,780 pounds per square inch, "the latter to apply to cases where there is some vibration and shock."

TABLE IX. VALUES OF COEFFICIENT $C_2 = \left(\frac{t'}{t}\right)^2$

No. of Teeth in Gear		Value of Coefficient C_2			
		Involute System		Cycloidal System	
Exact	Intervals	15 deg.	22½ deg.	12 Tooth Base	15 Tooth Base
12	12	0.77	1.00	0.79	0.56
13½	13 – 14	0.83	1.08	0.85	0.70
15½	15 – 16	0.87	1.18	0.92	0.83
17½	17 – 18	0.92	1.29	1.00	0.94
20	19 – 21	0.98	1.42	1.08	1.04
23	22 – 24	1.02	1.56	1.18	1.11
27	25 – 29	1.10	1.72	1.29	1.18
33	30 – 36	1.18	1.87	1.38	1.26
42	37 – 48	1.29	2.03	1.48	1.35
58	49 – 72	1.42	2.16	1.60	1.45
97	73 – 144	1.56	2.29	1.73	1.56
290	145 – Rack	1.72	2.39	1.87	1.69

In selecting these values he assumed that the load is divided between two pairs of teeth in all cases, so that the pressure on each tooth is 2-3P. Multiplying these several values of f by 2-3, we obtain the stress in pounds per square inch, assuming the total load to be carried on one tooth.

TABLE X. VALUES OF STRESS f FOR CAST IRON (UNWIN).

	Little Shock	Moderate Shock	Excessive Shock
Inaccurate gearing	6,300	4,000	2,850
Well formed gearing........	2,875	....	1,850

It will be noted that the real stress in the well formed gearing may be considerably less than that given, on account of the influence of increased thickness of tooth at the base. Professor Reuleaux, in his "Constructor," states that the dimensions of gear wheels must, for the same pressure on the teeth, be increased to meet shock in proportion to the increase in initial velocity. For slow-running gears, however, he neglects this action and divides gears into two classes, *viz.*, hoisting gears, and transmission gears, and includes under hoisting gears all those having a linear velocity at the pitch circle of not more than 100 feet per minute, and under transmission gears all those running at a higher velocity. For hoisting gears he recommends a fiber stress of about 4,200 pounds per square inch for cast iron, and states that an increase of one-fourth in the permissible stress would reduce the pitch only 7 per cent, but on the other hand, he notes that too low a value of f causes unnecessary increase in the size and weight, not only of the gears, but also of the bearings, frame work and other parts of the machine. In assuming this value of f, Reuleaux does not consider any increase in strength due to increased section at the base of the tooth, although he states that "the actual stress is properly somewhat less, because the thickness of the tooth at the base is usually more than one-half the pitch as assumed in the formula."

For transmission gears Reuleaux states that the fiber stress should be taken smaller for a given force P, as, when the circumferential velocity increases, the dynamic action of shock and vibration also increases.

For cast iron he recommends

$$f = \frac{9{,}600{,}000}{V + 2{,}164}$$

in which V is the velocity of the pitch circle in feet per minute. For steel he states that f may be taken 3 1/3 times, and for wood, 6/10 times the value thus obtained.

Arranged in tabular form this gives results as shown in Table XI.

TABLE XI. VALUES OF SAFE STRESS f FOR TRANSMISSION GEARS (REULEAUX).

Velocity in Feet per Minute.	100	200	400	600	800	1000	1500	2000	2500
For cast iron f =	4240	4060	3744	3478	3238	3034	2620	2302	2068
For steel f =	14112	13020	12467	11565	10782	10103	8725	7665	6886
For wood f =	2544	2436	2246	2083	1943	1820	1572	1381	1240

Mr. Wilfred Lewis in his paper on Gearing, read before the Engineers' Club of Philadelphia, in 1893, makes the following pertinent remarks:

"What fiber stress is allowable under different circumstances and conditions cannot be definitely settled at present, nor is it probable that any conclusions will be acceptable to engineers unless based upon carefully made experiments. In the article referred to, certain factors are given as applicable to certain speeds, and in the absence of any later or better lights upon the subject. Table XII has been constructed to embody in convenient form the values recommended.

It cannot be doubted that slow speeds admit of higher working stresses than high speeds, but it may be questioned whether teeth running at 100 feet a minute are twice as strong as at 600 feet a minute, or four times as strong as the same teeth at 1,800 feet a minute. For teeth which are perfectly formed and spaced, it is difficult to see how there can be a greater difference in strength than the well-known difference occasioned by a live load or a dead load, or two to one in extreme cases. But, for teeth as they actually exist, a greater difference than two to one may easily be imagined from the noise sometimes produced in running, and it should be said that this table is submitted for criticism rather than for general adoption. It is one which has given good results for a number of years in machine design, and its faults, such as they may be, are believed to be in the right direction."

TABLE XII. SAFE WORKING STRESS *f* FOR DIFFERENT SPEEDS (LEWIS).

Speed of Teeth in Feet per Minute	100 or less	200	300	600	900	1,200	1,800	2,400
Cast iron	8,000	6,000	4,800	4,000	3,000	2,400	2,000	1,700
Steel	20,000	15,000	12,000	10,000	7,500	6,000	5,000	4,300

If the formulas and coefficients presented by the writer are followed in the determination of the proportions of gear teeth, the permissible stress may be considerably greater than the real, as determined from the thickness of tooth near the base; this accounts in a large measure for the high stresses assumed to be carried by some gears. Thus, for instance, a gear having sixty-two teeth, 3.6-inch pitch, and 15-inch face, runs at 280 feet per minute and carries a load of 22,240 pounds. The working stress f is assumed to be 7,250 pounds, but if the increased section on the working depth circle be taken into consideration the working stress is reduced to 5,200 pounds.

While high rim speed does not necessarily imply shock, yet the effect of shock is more liable to be disastrous if high speed obtains, and for this reason the stress should diminish as the speed increases. Another reason for this is found in the fact that the stress is increased, due to the centrifugal force set up in the metal itself, which varies as the square of velocity; while this is not very great at ordinary speeds, it is still of sufficient importance to be considered for high speeds.

The influence of change of load is well understood in engineering construction, and usually a factor of safety is adopted for such loads from two to three times greater than would be the case for a steady or dead load, the value depending largely upon the range of stress involved. There seems good reason to believe that a similar relation modified by the effect of speed should obtain in selecting a suitable value for the working stress in gear teeth. From the foregoing considerations, then, it would appear that the working value of f should be chosen both with reference to the velocity and to the character of the acting force. With this in view the author of the present chapter has deduced the following formula for working stress:

$$f = \frac{50{,}000}{k + \sqrt[3]{V}}$$

in which f = the allowable stress in pounds per square inch;

V = velocity at pitch circle in feet per minute;
k = a constant having the values: 5 for little shock, 10 for moderate shock, and 15 for violent shock.

From this formula Table XIII has been calculated, and is offered as a guide to the designer in selecting suitable values of f under varying conditions.

TABLE XIII. VALUE OF STRESS f FOR CAST IRON (FLATHER).

Velocity in Feet per Minute	Value of Stress in pounds per square inch.		
	Little Shock	Moderate Shock	Violent Shock
100	5,200	3,400	2,550
200	4,600	3,150	2,400
400	4,050	2,900	2,250
600	3,700	2,700	2,150
800	3,500	2,600	2,050
1,000	3,300	2,500	2,000
1,400	3,100	2,350	1,900
1,800	2,900	2,250	1,840
2,400	2,700	2,150	1,770
3,000	2,600	2,050	1,700

While the previous discussion and the formulas given are of interest to the student of the subject of strength of gear teeth, as indicating what has been done and proposed along this line, it should be mentioned here that the Lewis formula given on page 28 is the formula now almost exclusively used. Table XIV gives the factor Y necessary for use with this formula for a complete range of number of teeth, and for both involute and cycloidal tooth-forms.

TABLE XIV. OUTLINE FACTORS FOR USE WITH LEWIS FORMULA.

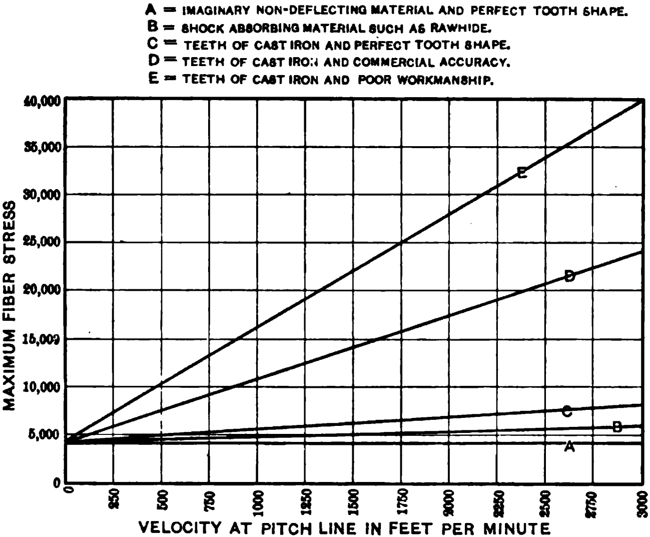
No of Teeth.	Factor for Strength, Y.		No. of Teeth.	Factor for Strength, Y.	
	Involute 20° Obliquity.	Involute 15° and Cycloidal.		Involute 20° Obliquity.	Involute 15° and Cycloidal.
12	.078	.067	27	.111	.100
13	.083	.070	30	.114	.102
14	.088	.072	34	.118	.104
15	.092	.075	38	.122	.107
16	.094	.077	43	.126	.110
17	.096	.080	50	.130	.112
18	.098	.083	60	.134	.114
19	.100	.087	75	.138	.116
20	.102	.090	100	.142	.118
21	.104	.092	150	.146	.120
23	.106	.094	300	.150	.122
25	.108	.097	Rack.	.154	.124

CHAPTER V.

VARIATION OF THE STRENGTH OF GEAR TEETH WITH THE VELOCITY.

The generally accepted formula for calculating the strength of gear teeth is that proposed by Mr. Wilfred Lewis, first published in the Proceedings of the Engineers' Club of Philadelphia, January, 1893, and referred to in the preceding chapter.

The merit of this formula lies in the great number of variables taken into account as compared with other rules in more or less common use, and in the fact that these variables are rationally considered. The effect of each of them can be calculated with some assurance, with the single exception of the influence of the velocity on the safe stress. In the fifteen years since the formula was first proposed, the original values for the stress as affected by the velocity have been largely used. Many designers, however, have felt that these values are rather unsatisfactory, although most of them will agree that they err rather on the side of safety than otherwise. By referring to Mr. Lewis' original paper it will be seen that these values were not given as being definitely determined, but merely as agreeing well with successful cases met with in his own practice. The following is a general analysis of the conditions involved.



Machinery,N.Y.

Fig. 25. Hypothetical Diagram showing the Relation of the Velocity to the Fiber Stress.

A variation in the strength of the teeth of a gear, due to a variation in the velocity, can be due, of course, to but one thing—impact. To illustrate this idea, and to show the cause of the impact, we will study the action of gearing under three different conditions.

1. *Gears of an imaginary undeflectable material.*—In Fig. 25 is a diagram in which the horizontal distances give velocity in feet per minute, and vertical distances give stresses in pounds per square inch, starting in this case at 4,000, which is assumed to be the maximum fiber stress in the gear we are considering, due to the load at the pitch line, which is supposed to be constant at all speeds. If the teeth of this gear are perfectly formed and well fitted together, so that there is no back lash, if the power is delivered to them steadily and smoothly, and the mechanism they drive runs without shock, any disturbance of the even movement will be impossible, and impact will be entirely absent. In the diagram in Fig. 1, then, there will be no rise of maximum fiber stresses with the velocity, so that the horizontal line *A* will show the conditions for this imaginary case.

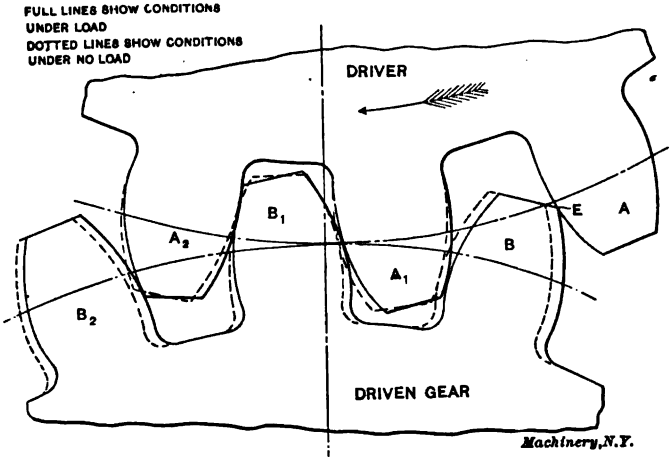

FULL LINES SHOW CONDITIONS UNDER LOAD
DOTTED LINES SHOW CONDITIONS UNDER NO LOAD
DRIVER
B$_1$
A$_2$
A$_1$
B$_2$
E
A
B
DRIVEN GEAR
Machinery,N.Y.

Fig. 26. The Action of Gear Teeth under Load, Greatly Exaggerated.

2. *With commercial material and theoretically accurate workmanship.* The conditions in this case are shown in Fig. 26, with all the phenomena greatly exaggerated. The full lines show the conditions under load, while the dotted outlines show the conditions when the load is removed from the driven gear. The teeth A_1, B_1, and A_2, B_2, carrying the load, are deflected by it, as shown. Tooth *B*, just about to come into contact with tooth *A*, is on that account shifted from its normal position; it should be located as shown by the dotted lines. If it were in this position, it would come in contact with tooth *A* under mathematically perfect conditions, and there would be no shock of engagement. As it is, the two come suddenly into action as shown at *E*, under different conditions than those contemplated by the design, thus the contact takes place in the form of a slight blow, after which the

teeth are deflected more and more, until they have taken up their share of the load, as shown later at A_1 and B_1. If the gears are moving very slowly, the deflection takes place very slowly, and the problem is practically a static one. If the gears are running at a high velocity, the problem becomes essentially a dynamic one, and the stresses are greater than with the slow speed. The increase in stress with the increase in speed for this second case could probably be represented by a line something like *C*, in Fig. 25.

3. *With commercial materials and commercial accuracy.* This is, of course, the practical case to consider. A line to show the relation of the velocity to the maximum fiber stress for a given gear, would very probably look something like *D* in Fig. 25. This is, in fact, approximately the line which embodies the conclusions of the Lewis tables for a static stress of 4,000 pounds. It is considerably higher than line *C*, because impact due to irregular tooth outlines is added to the impact due to the deflection.

Practical Considerations Affecting Design.

The fact that the variation of the strength with the velocity is due to impact, suggests also a number of points relating to design.

1. *Value of accuracy.* It is evident that this theory of impact puts a premium on accuracy in workmanship for gears that are to run at high speed under a heavy load. It is probable that the strength of a given pair of gears may be cut in two if the tooth outlines are not carefully determined, and if the cutter is not set centrally. This suggests the desirability of a greater sub-division of the standard cutter series for work of this kind.

2. *Resilience of design and materials.* In high-speed gearing it is evident that the shock due to the impact should be absorbed as quickly and as fully as possible. This suggests the use, at abnormally high speeds, of rawhide, wood, etc., for one of the members of the pair of gears. The introduction of spring couplings or similar devices may also be desirable, especially where the other parts of the mechanism are liable to transmit shock to the gearing.

3. *Easing off the points of the tooth.* There has always been a sort of superstition that the points of the tooth should be eased off to make the action smoother. This is done, of course, in standard involute gears, though for another reason, that of avoiding interference with the flanks of the pinions. It can now be seen that there is a solid basis for this practice in all cases where gears are to run at such speeds that severe impact is liable to take place. Referring to Fig. 26, teeth *A* and *B* are taking up the load very suddenly, owing to the fact that they are out of step, due to the deflection of the other teeth momentarily carrying the load. Easing away the points of *A* and *B* would mitigate this sudden reception of the load, allowing the inevitable deflection to take place more slowly, with a consequent gain in the strength of the gear at high speeds.

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Cutting Speeds and Feeds - - - - - - - - 16
Trains of Gears - - - - - - - - - - 20
Lathe Change Gears - - - - - - - - - 26
Speed of Pulleys - - - - - - - - - 31
Indexing Movements for the Milling Machine - - 34
Change Gears for Milling Spirals - - - - - - 36
Square and Square Root - - - - - - - - 40
Use of Formulas - - - - - - - - - - 43
Use of Tables of Sines, Cosines, Tangents and Cotangents 47

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Cutting Speeds and Feeds - - - - - - - 16

Trains of Gears - - - - - - - - - 20

Lathe Change Gears - - - - - - - - 26

Speed of Pulleys - - - - - - - - - 31

Indexing Movements for the Milling Machine - - 34

Change Gears for Milling Spirals - - - - - 36

Square and Square Root - - - - - - - 40

Use of Formulas - - - - - - - - - 43

Use of Tables of Sines, Cosines, Tangents and Cotangents 47

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INTRODUCTION.

In the following, some of the most common shop subjects requiring simple calculations have been treated, and special efforts have been made to treat each subject as simply as possible, so that the present treatise may be of service to those, particularly, who have not previously acquired a great amount of knowledge about handling figures, and who are not familiar with mathematical expressions and usages. In order to fix the processes and rules more firmly in the reader's mind, examples have been given in almost all instances, and in many cases a number of similar examples have been given, so as to permit the repetition of the same calculation a number of times. All formulas in the first part of the pamphlet have been written out in words, as this gives a better idea about what the formula actually means, at least to those not familiar with handbooks. Mathematical signs have also been avoided in the text to a certain extent, and the corresponding words have been written out in full. In short, all precautions have been taken to present the methods in as plain and simple language as possible. Many text-books deal with principles rather than with specific examples, and to a person who is not used to solving problems of the kind that are met with in the machine shop, it is often difficult to apply the principles involved to each particular case. The purpose of this book has therefore been to select the most common specific cases, and show directly how the principles are applied.

While the subject in hand has been treated to accommodate the requirements of those who demand a book that is plain and simple, it has been necessary to presuppose fundamental knowledge in regard to the use of numbers in calculations, that is, the reader must be fairly competent to add, subtract, multiply, and divide whole numbers and decimals, and also have some fundamental ideas of the use of common fractions. If such knowledge has been acquired, no difficulty will be experienced in making use of the rules and formulas given.

It is assumed that the reader is familiar with the common mathematical signs, + (plus) which signifies addition, — (minus) which signifies subtraction, × (times) which signifies multiplication, and ÷ (divided by) which signifies division, as well as with the sign = (equals) which is put between quantities which are equal to one another to signify this condition. But it may be appropriate to call attention to the different methods commonly used for indicating division, as these may not be clear to all. Usually, as we already have said, in arithmetic, division is indicated by the sign ÷, so that we have, for instance,

$$12 \div 3 = 4.$$

A more common method in technical works, however, is to simply write the dividend as the numerator of a fraction and the divisor as the denominator, thus:

$$\frac{12}{3}=4.$$

In that case the fraction indicates a division. This system will be followed in many of the following formulas, and it should therefore be remembered that ***the line between the numerator and denominator in a fraction always indicates a division, the numerator to be divided by the denominator.***

The actual division, however, is not necessarily worked out in every case where division is thus implied. When two divisions are multiplied together, cancellation, and the following operations of addition or subtraction, may make the actual numerical work very simple.

Although knowledge of common fractions is presupposed, as mentioned, it may be well at this point to mention the rules for multiplication and division of common fractions, as in the following many operations of this kind must be made. Two fractions are multiplied by multiplying numerator by numerator and denominator by denominator, (*numerator* being the *upper*, and *denominator* the *lower* quantity in a fraction). For instance, let it be required to multiply ¼ by ⅜. We have then,

$$\frac{1}{4}\times\frac{3}{8}=\frac{1\times3}{4\times8}=\frac{3}{32}$$

If the numbers to be multiplied contain whole numbers, these are first converted into fractions. Let it be required to multiply 1¼ by 3¼. We have then

$$1\frac{1}{4}\times3\frac{1}{4}=\frac{5}{4}\times\frac{13}{4}=\frac{65}{16}=4\frac{1}{16}$$

Division is simply the reverse of multiplication. The number which is to be divided is called the *dividend*, and the number by which we divide is called the *divisor*. If one number is to be divided by another, we simply invert the divisor, and *proceed as in multiplication*. To invert the divisor means that we place the denominator as numerator, and the numerator as denominator, as, for instance, 3/8 inverted is 8/3. Suppose that we wish to divide ¾ by 7/16. We have then,

$$\frac{3}{4}\div\frac{7}{16}=\frac{3}{4}\times\frac{16}{7}=\frac{48}{28}=1\frac{20}{28}=1\frac{5}{7}$$

If the number to be divided contains a whole number besides a fraction, we first convert this to a fraction, and then proceed as before. Suppose that we wish to divide 2¼ by 3¾. We have then,

$$2\frac{1}{4}\div3\frac{3}{4}=\frac{9}{4}\div\frac{15}{4}=\frac{9}{4}\times\frac{4}{15}=\frac{36}{60}=\frac{3}{5}$$

After this introduction, we are ready to take up some of the most commonly occurring shop problems, and apply to them the principles of general arithmetic.

CHAPTER I.

FIGURING TAPERS.

In all circular or round pieces of work, the expressions "taper per inch" and "taper per foot" mean the taper on the *diameter*, or the difference between the smaller and the larger diameter of a piece, measured one inch or one foot apart, as the case may be. Suppose in Fig. 1 that the diameter at *A* is one inch, and the diameter at *B*, one and one-half inch, and that the distance or dimension between *A* and *B* is 12 inches or one foot. This piece, then, tapers one-half inch per foot, because the difference between the diameter at *A*, one inch, and at *B*, one and one-half inch, is one-half inch. In Fig. 2, the diameter at *C* is 7/16 inch, and at *D*, ½ inch, and the distance between *C* and *D* is one inch. This piece of work, therefore, tapers 1/16 inch per inch. Tapers may also be expressed for other lengths than one inch and one foot. In Fig. 3, the diameter at *E* is 1⅛ inch, and at *F*, 1 9/32 inch, and the dimension from *E* to *F* is 5 inches. This piece of work, therefore, tapers 5/32 inch in 5 inches, the difference between 1 9/32 and 1 1/8 being 5/32.

If we know the taper in a certain number of inches, as for instance, that the taper in 5 inches is 5/32, it is easy to find the taper per inch. It is clear that the taper per each inch is one-fifth of what the taper is in 5 inches. We only divide the taper in 5 inches by 5, and we get the taper per inch. In this case, dividing 5/32 by 5 would give us 1/32. If we now want to find the taper per foot we only have to multiply the taper per inch by 12. It is clear that the taper per foot, or the taper in 12 inches, is 12 times the taper in one inch. In this case, therefore, the taper per foot would be equal to 12 times 1/32, or ⅜ inch.

The problems met with in regard to figuring tapers may be of three classes. In the first place we may have given us the figures for the large and small end of a piece of work, and the length of the work, as in Fig. 4, and we want to find the taper per foot. In the second place we may know the diameter at one end, the length of the work, and the taper per foot, as in Fig. 5, and we want to find the diameter at the other end of the work. In the third place we may know the required diameters at both ends of the work, and the taper per foot, as in Fig. 6, and we want to find the dimension between the given diameters, or the length of the piece. We will now treat each of these problems in detail.

1. *To find the taper per foot when the diameters at the large and small ends of the work, and the length, are given.*

Referring to Fig. 4, the diameter at the large end of the work is 2⅝ inches, the diameter at the small end, 2 3/16 inches, and the length of the work 7 inches. The taper in 7 inches is then equal to the difference between 2⅝ inches and 2 3/16 inches, or 7/16 inch. The taper in one inch equals 7/16 divided by 7, or 1/16 inch; and the taper per foot is 12 times the taper per inch, or 12 times 1/16, which equals ¾ inch. The taper per foot in our case in Fig. 4, then, equals ¾ inch.

If the dimension between the small and the large diameter is not expressed in even inches, but is 5 3/16 inches, for instance, as in Fig. 7, the procedure is exactly the same. Here the diameter at the large end is 2.216 inches and at the small end 2 inches. The taper in 5 3/16 inches is, therefore, 0.216 inch. This is divided by 5 3/16 to find the taper per inch.

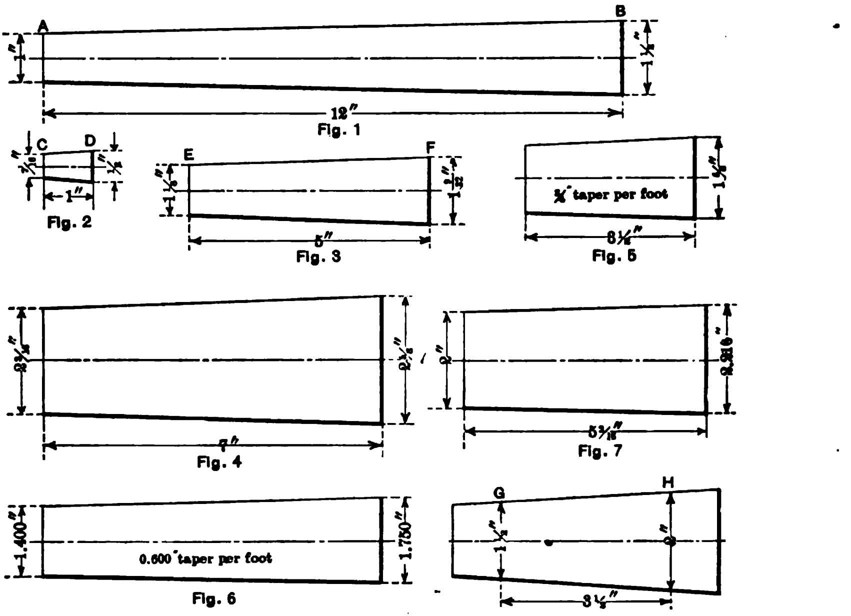

Fig. 1

Fig. 2

Fig. 3

Fig. 5

Fig. 4

Fig. 7

Fig. 6

Fig. 8

Machinery, N.Y.

Figs. 1 to 8.

$$0.216 \div 5\frac{3}{16} = 0.216 \div \frac{83}{16} = 0.216 \times \frac{16}{83} = 0.0416.$$

The taper per inch, consequently equals 0.0416 inch, and the taper per foot is 12 times this amount, or almost exactly ½ inch.

Expressed in a formula, if all dimensions given are in *inches*, the previous calculation would take this form:

$$\text{taper per foot} = \frac{\text{large dia.} - \text{small dia.}}{\text{length of work}} \times 12.$$

It makes, of course, no difference if the large and small diameters are measured at the extreme ends of the work or at some other place on the work, provided the length or distance between the points where the diameters are given, is stated. In Fig. 8, the smaller and larger diameters are given at certain distances from the ends of the work, but the dimension from *G* to *H* is given, and the figuring is carried out exactly as if the work were no longer than between *G* and *H*. The following examples will tend to show how the figuring of the taper per foot enters in actual shop work.

Example 1.—Fig. 9 shows the blank for a taper reamer. The diameters at the large and small ends of the flutes, and the length of the fluted part, are stated on the drawing. It is required to find the taper per foot in order to be able to set the taper turning attachment of the lathe.

Referring to the figures given in Fig. 9, the difference in diameters at the large and small ends of the taper is 15/64 inch. This divided by the length of the flute, 7½ inches, gives us the taper per inch. This we find to be 1/32. The taper per foot is 12 times the taper per inch, or, in this case, then, ⅝ inch. The taper attachment of the lathe is, therefore, set to the ⅝-inch graduation, and the taper turned will be according to the diameters given on the drawing.

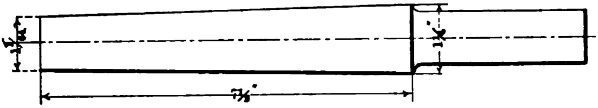

FIG. 9

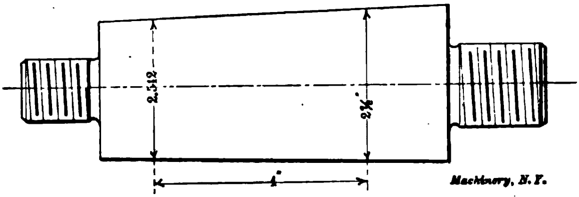

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FIG. 10

Figs. 9 and 10.

Example 2.—Fig. 10 shows a taper clamping bolt, entering into the design of a special machine tool. As seen from the cut, the drawing calls for a diameter of 2⅞ inches a certain distance from the large end of the taper, and for a diameter of 2.542 inches a distance 4 inches further down on the taper. The taper in 4 inches is then 2⅞ inches minus 2.542 inches, or 0.333 inch. The taper in one inch equals this divided by 4, or 0.0833. The taper per foot is 12 times the taper per inch, or 12 times 0.0833, which equals one inch, almost exactly. The taper to which to turn the bolt in Fig. 10 is thus one inch per foot.

2. *If the diameter at one end of the taper is given, and also the length of the work and the taper per foot, to find the diameter at the other end of the work.*

Referring to Fig. 5, the diameter at the large end of the work is 1⅝ inch, the length of the work is 3½ inches, and the taper per foot is ¾ inch. We now want to find the diameter at the small end. In this case we simply reverse the method employed in our previous problems, where we wanted to find the taper per foot. In this case we know that the taper per foot is equal to ¾ inch. The taper in one inch must be one-twelfth of this, or ¾ inch divided by 12, which equals 1/16 inch. Now, the taper in 3½ inches, which we want to find in order to know what the diameter is at the small end of the work, must be 3½ times the taper in *one* inch, or 3½ times 1/16, which equals 7/32. The taper in 3½ inches, then, is 7/32 inch, which means that the diameter at the small end of a piece of work, 3½ inches long, is 7/32 inch smaller than the diameter at the large end. The diameter at the large end, according to our drawing, is 1⅝ inch. The diameter at the small end, being 7/32 inch smaller, is therefore 1 13/32 inch.

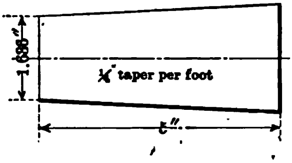

Fig. 11.

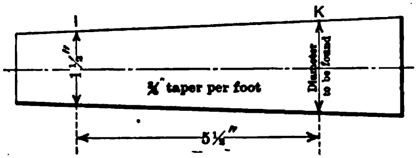

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Fig. 12.

Expressed in a formula, the previous calculation would take this form:

$$\text{dia. at small end} = \text{dia. at large end} - \left(\frac{\text{taper per foot}}{12} \times \text{length of work}\right).$$

If we now take a case where the diameter at the small end is given, as in Fig. 11, and the diameter at the large end is wanted, the figuring is exactly the same, except of course, we *add* the amount of taper in the length of the work to the small diameter to find the large diameter. When the large diameter is given, we *subtract* the amount of taper in the length of the work to find the small diameter. This is so self-evident that no difficulties ought to be experienced on this account.

Referring again to Fig. 11, where the small diameter is given as 1.636 inch, the length of the work as 5 inches, and the taper per foot as ¼ inch, how large is the large diameter of the work? If the taper per foot is ¼ inch, the taper per inch is ¼ divided by 12 which equals 0.0208, and the taper in 5 inches consequently 5 times 0.0208, or 0.104 inch. The diameter at the large end of the work, which we are figuring, is, then, 0.104 inch larger than the diameter at the small end. The diameter at the small end is given on the drawings as 1.636 inch; adding 0.104 inch to this, we get 1.740 inch as the diameter at the large end.

Expressed in a formula, the previous calculation would take this form:

$$\text{dia. at large end} = \text{dia. at small end} + \left(\frac{\text{taper per foot}}{12} \times \text{length of work}\right)$$

It may again be well to call attention to the fact that it makes no difference whether the large and small diameters are figured at the extreme ends of the work or at some other points, as long as the diameter to be found is located at one end of the length dimension, and the diameter stated on the drawing on the other. Thus, in Fig. 12 the diameter stated at *I* is given a certain distance up on the taper, and the diameter at *K*, which is wanted, is not at the end of the taper. But the dimension 5½ is given between the points *I* and *K* where these diameters are to be measured, and in figuring, one may reason as if the work ended at *I* and *K*, the diameter at *I* being the small diameter, the diameter at *K*, the large diameter, and 5½ inches the total length of the work. The following examples of direct practical application to shop work will prove helpful in remembering the principles outlined.

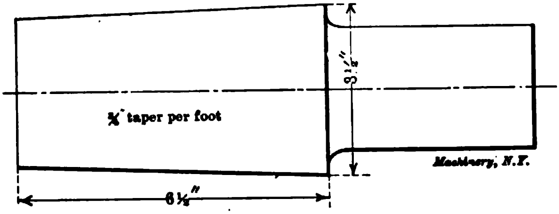

Fig. 13.

Example 1.—Fig. 13 shows a taper tap, the blank for which is to be turned. The diameter at the large end of the threaded part is 3½ inches, as given on the drawing, the length of the thread is 6½ inches, and the taper per foot is ¾ inch. We want to find the diameter at the small end, in order to measure this end and ascertain that the tap blank has been correctly turned.

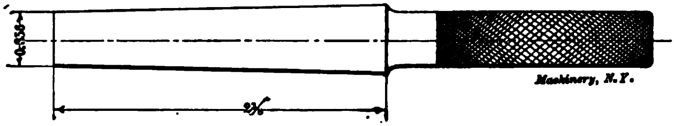

Fig. 14.

The taper per foot being ¾ inch, the taper per inch is ¾ divided by 12, or 1/16 inch. The taper in 6½ inches is 6½ times the taper in one inch, or 6½ times 1/16 inch, which equals 13/32 inch. The taper in 6½ inches being 13/32 inch means that the diameter at the small end of the tap blank is 13/32 inch smaller than the diameter at the large end. The diameter at the small end is, therefore, 3 3/32 inches.

Example 2.—Fig. 14 shows a taper gage for a standard Morse taper No. 1. The diameter at the small end is 0.356 inch, the length of the gage part is 2⅜ inches, and the taper per foot 0.600 inch. We want the diameter at the large end, in the first place in order to know what size stock to use for the gage, and later for measuring this diameter when turned, to see that the taper turned is correct.

A taper of 0.600 per foot, gives us a taper of 0.050 per inch. In 2⅜ inches the taper equals 2⅜ times 0.050, or 0.119 inch. This added to the diameter at the small end gives us the diameter at the large end: $0.356 + 0.119 = 0.475$ inch.

Example 3.—Fig. 15 shows a taper bolt used as a clamp bolt. The diameter 3¼ inches is given 3 inches from the large end of the taper. The total length of the taper is 10 inches. The taper is ⅜ inch per foot. We want to find the diameters at the extreme large and small ends of this piece.

We will first find the diameter at the large end. The taper per foot being ⅜ inch, the taper per inch equals 1/32 inch. The taper in 3 inches is consequently 3/32. This added to 3¼ inches will give us the diameter at the large end, which is 3 11/32 inches.

To find the diameter at the small end, subtract the taper in 10 inches, which is 10 times the taper in one inch, or 10 times 1/32, which equals 5/16, from the diameter 3 11/32 inches at the large end. This gives us the diameter at the small end 3 1/32 inches.

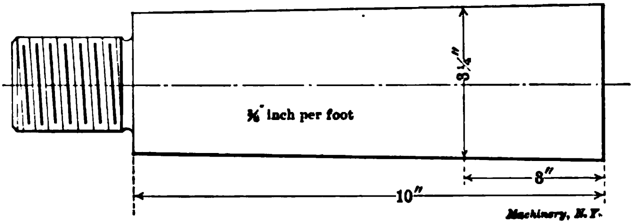

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Fig. 15.

We can also find the diameter at the small end without previously finding the diameter at the extreme large end. The total length of the taper is 10 inches, and the dimension from where the diameter 3¼ inches is given to the large end is 3 inches. Consequently, the dimension from where the diameter 3¼ inches is given to the small end is 7 inches. The taper in one inch was 1/32 inch; in 7 inches, therefore, 7/32 inch. The diameter at the small end of the work is 7/32 inch smaller than 3¼ inches, or 3 1/32 inches, the same as found previously when we figured from the extreme large diameter of the taper.

3. *To find the distance between two given diameters on a tapered piece of work, if the taper per foot is known.*

Referring to Fig. 6, if the diameters at both ends of a tapered piece are known, together with the taper per foot, it is required to find the length of the work. Assume that the diameter at the large end of the piece is 1.750 inch, and at the small end, 1.400 inch. The taper per foot is 0.600 inch. How long is this piece of work required to be, in order to have the given diameters at the ends, with the taper stated? We know that the taper per foot is 0.600 inch. The taper per inch is then 0.600 divided by 12, or 0.050 inch. The difference in diameters between the large and the small ends of the work is $1.750 - 1.400$, or 0.350 inch, which represents the taper in the length of the work.

Now, we know that the taper is 0.050 inch in *one* inch. How many inches does it then require to get a taper of 0.350 inch? This we find by seeing how many times 0.050 is contained in 0.350, or, in other words, by dividing 0.350 by 0.050, which gives us 7 as answer. This means that it takes 7 inches for a piece of work to taper 0.350 inch, if the taper is 0.600 per foot. The length of the work consequently is 7 inches in the case referred to.

Expressed in a formula the previous calculation would take the form:

$$\text{length of work} = \frac{\text{dia. at large end} - \text{dia. at small end}}{\text{taper per foot} \div 12}$$

The taper per foot divided by 12, as given in the formula above, of course simply represents the taper per inch. The formula may therefore be written:

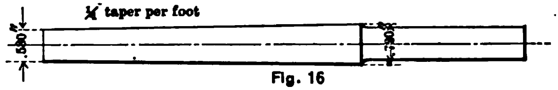

Fig. 16

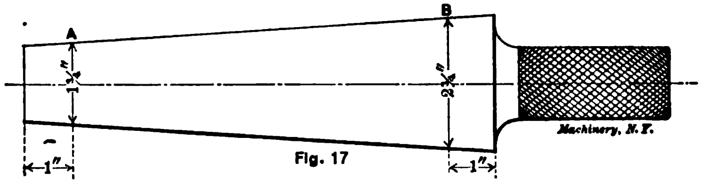

Fig. 17

Figs. 16 and 17.

$$\text{length of work} = \frac{\text{dia. at large end} - \text{dia. at small end}}{\text{taper per inch}}$$

A few examples of the application of these rules will make their use in actual shop work clearer.

Example 1.—A taper reamer, Fig. 16, for standard taper pins, having ¼ inch taper per foot, is to be made. The diameter at the large end of the flutes is wanted to be 0.720 inch. The diameter at the point of the reamer must be 0.580 inch, in order to accommodate the longest taper pins of this size made. How long should the fluted part of the reamer be made?

The taper per foot is 0.250 inch, and the taper per inch, consequently, 0.250 divided by 12, or 0.0208 inch. The taper in the length of reamer required is equal to the difference between the large and the small diameter, or 0.720 — 0.580 equals 0.140 inch. This amount of taper divided by the taper in one inch gives the required length of the flutes. Thus, 0.140 divided by 0.0208 equals 6.731, which represents the length of flutes required. This dimension is nearly 6¾ inches, and, being a length dimension of no particular importance, it would be made to an even fractional part of one inch.

Example 2.—In Fig. 17 is shown a taper master gage intended for inspecting taper ring gages of various dimensions. The smallest diameter of the smallest ring gage is 1¾ inch, and the largest diameter of the largest ring gage is 2¾ inches. The taper per foot is 1½ inch. It is required that the master gage extends one inch outside of the gages at both the small and the large ends, when these are tested. How long should the gage portion of this piece of work be?

The taper per foot is 1½ inch, which is equivalent to ⅛ inch taper per inch. The total taper from *A* to *B* in Fig. 17 is 2¾ minus 1¾, or one inch. Therefore, as the taper per inch, ⅛, is contained in the taper of one inch in the distance from *A* to *B* exactly 8 times, the dimension from *A* to *B* is 8 inches. The gage extends one inch beyond *A* and *B*, respectively, at either end, and the total length of the gage is, therefore, 10 inches.

Rules for Figuring Tapers.

If we formulate the previous discussion for figuring tapers into rules, these may be stated as follows:

1. If the taper per foot is known, the taper per inch is found by *dividing the taper per foot by* 12.
2. If the taper per inch is known, the taper per foot is found by *multiplying the taper per inch by* 12.
3. To find the taper per foot, when the diameters at the large and small ends and the length of the taper are given, *subtract the small diameter from the large, divide the remainder by the length of the taper, and multiply the result by* 12.
4. To find the diameter at the small end when the diameter at the large end, the length of the taper, and the taper per foot are given, *divide the taper per foot by* 12, *multiply the result by the length of the taper, and subtract the resulting dimension from the diameter at the large end.*
5. To find the diameter at the large end when the diameter at the small end, the length of the taper, and the taper per foot are given, *divide the taper per foot by* 12, *multiply the result by the length of the taper, and add the resulting dimension to the diameter at the small end.*
6. To find the dimension between two given diameters of a piece of work, when the taper per foot is given, *subtract the diameter at the small end from the diameter at the large end, and divide the remainder by the taper per foot divided by* 12.
7. To find how much a piece of work tapers in a certain length, when the taper per foot is given, *divide the taper per foot by* 12, *and multiply the result by the dimension of the certain length in which the taper is required.*

CHAPTER II.

SETTING OVER TAIL-STOCK FOR TAPER TURNING.

When the tail-stock of the lathe is set over for turning taper, in cases where no taper turning attachment is available, the amount to set over the tail-stock can be determined, if the taper per foot of the work, and the length, are known. Suppose a piece of work is 7½ inches long, as shown in Fig. 18, and that the taper per foot is ¾ inch. We first require to know how much the work tapers in 7½ inches. According to our previous discussion, Rule 7, page 12, we find that the work tapers ¾ divided by 12, times 7½, or 15/32 inch in 7½ inches. The purpose of setting over the tail-stock is to make the front of the work come parallel with the travel of the lathe carriage, or, which is the same, parallel with the center line of the lathe spindle, as shown in Fig. 19. It is clear, upon examination of Fig. 20, where the dotted lines indicate the original position of the work, that in order to get the front side of the work parallel with the center line of the spindle, its small end must be moved forward a distance equal to one-half the taper in the length of the work. This forward movement is accomplished by moving the tail-stock over an amount equal one-half of this taper, as shown at *a*, Fig. 19. In the case in Fig. 18, where, as we have found, the taper in the length of the work is 15/32 inch, the tail-stock should be moved 15/64 inch sideways in relation to the spindle of the lathe.

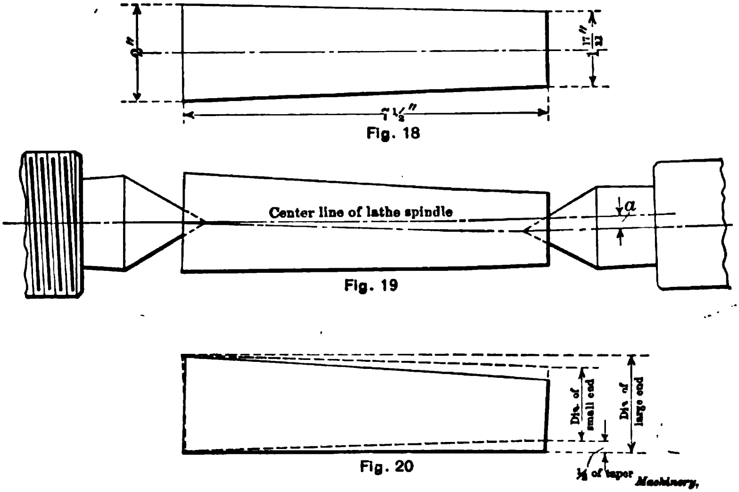

Fig. 18

Fig. 19

Fig. 20

Figs. 18 to 20.

If the diameters at both the large and the small ends of the work are given, we can determine the amount to set over the tail-stock without knowing the taper per foot, because all we need to know is the amount of taper in the length between the centers of the lathe. In Fig. 21, for instance, the diameter at the large end of the work is 1½ inch, and the diameter at the small end is 1¼ inch. The amount to set over the tail-stock will be one-half of the difference between 1½ and 1¼, or ⅛ inch.

If part of the work is turned straight, and part of it turned taper, as shown in Fig. 22, we must determine the amount of taper in the *whole* length of the work, and then set over the tail-stock one-half of this amount. In Fig. 22 the piece of work shown is 1⅜ inch at the small end of the taper. It is then turned taper for 4 inches, and is 1⅝ inch in diameter at the large end of the taper. It is then turned straight for the remaining 6 inches, the total length of the work being 10 inches. In this case we must first find out what the amount of taper would be in 10 inches, if the whole piece had been turned taper, with the same taper as now required for 4 inches. The taper in 4 inches is 1⅝—1⅜, or ¼ inch. The taper in one inch is consequently 1/16 inch, and in 10 inches ⅝ inch. The amount to set over the tail-stock is one-half of this, or 5/16 inch.

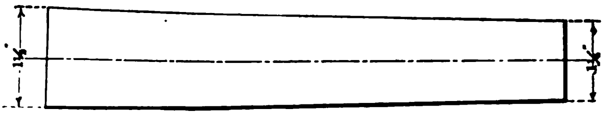

FIG. 21

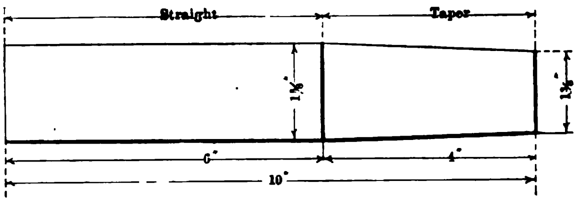

FIG. 22

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Figs. 21 and 22.

If in the case in Fig. 22, the diameter at the small end had not been given, but the taper per foot of the tapered part been stated instead, the diameter at the small end would first have been found according to Rule 4, page 12, and then the taper in the total length of the work, and the amount to set over the tail-stock, would have been found exactly as indicated above.

If we state what has previously been said in formulas, we would get, for the case when the taper per foot and the length of the work are known:

$$\text{amount to set over tail-stock} = \frac{1}{2} \times \left(\frac{\text{taper per foot}}{12} \times \text{length of work} \right)$$

For the case in Fig. 21, we would have:

$$\begin{array}{c}\text{amount to set}\\ \text{over tail-stock}\end{array} = \frac{1}{2} \times \left(\text{dia. at large end} - \text{dia. at small end.}\right)$$

Finally, for the case in Fig. 22, we would have:

$$\begin{array}{c}\text{amount to set}\\ \text{over tail-stock}\end{array} = \frac{1}{2} \times \left(\frac{\text{dia. at large end of taper} - \text{dia. at small end}}{\text{length of taper}} \times \begin{array}{c}\text{total}\\ \text{length}\\ \text{of work.}\end{array}\right)$$

For those who prefer rules in words to formulas, the following rules have been formulated:

1. To find the amount to set over the tail-stock for work tapering for its full length, when the taper per foot and length of the work are known, *divide the taper per foot by* 12, *multiply the result by the length of the work, and divide this result, in turn, by* 2.

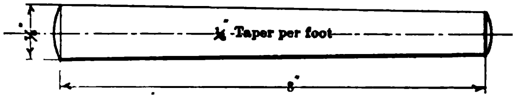

FIG. 23

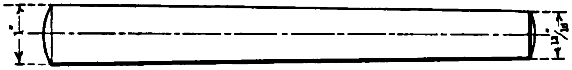

FIG. 24

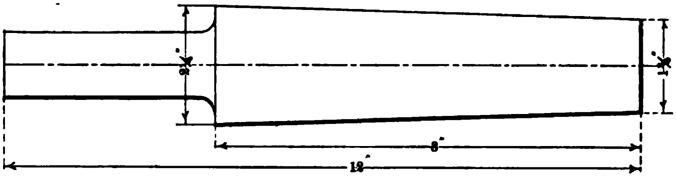

FIG. 25

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Figs. 23 to 25.

2. To find the amount to set over the tail-stock for work tapering for its full length, when the diameters at the large and small ends are known, *subtract the small diameter from the large, and divide the remainder by* 2.

3. To find the amount to set over the tail-stock for work partly tapered and partly straight, when the diameters at the large and small ends of the taper, the length of the taper, and the total length of the work are known, *subtract the small diameter from the large, divide the remainder by the length of the taper, multiply the result thus obtained by the total length of the work, and finally divide by* 2.

The following examples will help to give a clear idea of the application of these rules.

Example 1.—The taper pin shown in Fig. 23 is 8 inches long, and tapers ¼ inch per foot. How much should the tail-stock be set over when turning this pin?

Dividing the taper per foot by 12 gives us 0.0208. Multiplying this figure (which represents the taper per inch) by 8 gives us 0.166 as the taper in 8 inches. Dividing this by 2 gives us the amount required to set over the tail-stock. This amount, then, is 0.083 inch.

Example 2.—Another taper pin, Fig. 24, is 1 inch in diameter at the large end, and 13/16 inch at the small end. How much should the tail-stock be set over for turning this pin?

The total taper of this pin is found by subtracting the diameter at the small end, 13/16 inch, from the diameter at the large end, 1 inch. This gives us a remainder of 3/16. One-half of this amount, or 3/32 inch, represents the amount which the tail-stock should be set over.

Example 3.—A taper gage, as shown in Fig. 25, is to be turned by setting over the tail-stock. The diameter at the large end of the taper is 2¼ inches, the diameter at the small end is 1¾ inch, the length of the taper, 8 inches, and the total length, 12 inches. How much should the tail-stock be set over?

Subtracting the diameter at the small end, 1¾ inch, from the diameter at the large end, 2¼ inches, gives us a taper of ½ inch in 8 inches. Dividing ½ by 8, gives us the taper in one inch, which is 1/16 inch. Multiplying this with the total length of the work, 12 inches, gives us ¾ inch, which, divided by 2, gives us, finally, the required amount to which to set over the tail-stock. This latter is, therefore, set over ⅜ inch.

CHAPTER III.

CUTTING SPEEDS AND FEEDS.

There is a certain mathematical relation between the diameter of the work turned in a lathe or on a boring mill (or the diameter of the drill, or hole drilled, in a drill press), the number of revolutions, and the cutting speed of the work or tool. This relationship is simple, and can be easily explained.

The cutting speed of a tool is the speed with which the tool passes over the surface of the work operated upon, counted in so many feet per minute. Thus, if the point of a lathe tool passes over the surface of a casting turned in the lathe at a rate of 40 feet per minute, this figure expresses the cutting speed. Of course, the tool point is really stationary, and it is the casting surface that passes by the tool point, but it is customary to say that the "tool passes over the work," as it actually does in a shaper, for instance.

The feed of a tool is its sideways motion for each revolution or stroke of the work. Thus in a lathe, if the feed is 1/16 inch, it means that for each revolution of the work the tool moves along the lathe bed 1/16 inch, so as to cut a chip 1/16 inch wide. In a planer, the feed would mean the amount the tool-carrying head is moved sideways for each complete stroke of the table or platen.

Cutting Speeds.

The problems which meet the machinist in regard to cutting speeds may be divided up in two groups: If the diameter of the work is known (which, of course, can always be, at least approximately, measured), and a certain cutting speed is required, how many revolutions per minute ought the work to make; and, if the diameter of the work and the number of revolutions are known, what is the cutting speed? We will now deal with each of these problems in the order they come.

1. *The diameter of the work turned in a lathe or boring mill (or the diameter of the drill, or drilled hole, in a drill press) and the required cutting speed are known. How many revolutions per minute should the work make?*

Assume that the diameter of the work is 5 inches, and the required cutting speed 40 feet per minute. When the diameter of a piece of work is known, its circumference equals the diameter times 3.14. Therefore, the circumference of the work in hand is 15.7 inches. It is evident, that for each revolution of the work the length of its circumference passes by the tool once. Thus, for each revolution 15.7 inches passes by the tool. As the cutting speed is expressed in feet, this length should also be given in feet, and not in inches, when we figure. To transform 15.7 inches into feet we divide by 12, thus obtaining 1.308 feet as the circumference of the work. How many revolutions, each represented by 1.308 feet, does it require to get a cutting speed of 40 feet? This we evidently get by finding how many times 1.308 feet is contained in 40 feet, or, in other words, by dividing 40 by 1.308. Carrying out the division gives us 30.6 revolutions per minute, as required to obtain a cutting speed of 40 feet per minute with a piece of work 5 inches in diameter.

The calculation carried out above is expressed, shortly, by the formula

$$\text{number of revolutions per minute} = \frac{\text{cutting speed in feet per minute}}{\left(\begin{matrix}\text{diameter of}\\ \text{work in inches}\end{matrix} \times 3.14\right) \div 12}$$

A few examples may tend to make this formula clearer.

Example 1.—A tool steel arbor is turned, using an ordinary carbon steel for turning. The diameter of the arbor is 2 inches. The cutting speed, in feet per minute, ought to be about 18 feet. How many revolutions ought the work to make per minute?

The diameter of the work, 2 inches, multiplied by 3.14, gives us a circumference of 6.28 inches. This circumference, expressed in feet, is obtained by dividing 6.28 by 12, getting 0.523 as a result. The cutting speed per minute, 18 feet, divided by the circumference (or the distance traveled for *one* revolution) gives us the number of revolutions. Thus 18 divided by 0.523, which equals 34.4, represents the proper number of revolutions for a piece of work 2 inches in diameter, being cut at a rate of 18 feet surface speed per minute.

Example 2.—A brass rod, 1 inch in diameter, is being turned. The proper cutting speed for this material is 100 feet per minute. How many revolutions should the work make?

The circumference of the work in inches is 3.14, and expressed in feet, 3.14 divided by 12, or 0.262. The cutting speed, 100 feet, divided by 0.262 gives us 382 revolutions, approximately, as the proper number of revolutions.

Example 3.—A ½-inch drill, cutting cast iron, may cut at a speed of 40 feet per minute. How many revolutions ought it to make?

The circumference in feet equals ½ times 3.14 divided by 12, or 0.131. This divided in 40 gives us approximately 305 as the proper number of revolutions.

2. *The number of revolutions which the work makes in a lathe or boring mill, or the number of revolutions of the tool in a drill press, and the diameter are known. What is the cutting speed?*

Assume that the work is a tool steel stud, 4 inches in diameter, and revolving at a speed of 16 revolutions a minute. The circumference of the work is figured as before, and transformed into feet. The circumference, in feet, equals, in this case, 4 times 3.14 divided by 12, or 1.05 foot. This is the distance traversed by the tool for each revolution. For 16 revolutions the distance traversed is evidently 16 times greater, or 16 times 1.05, which equals 16.8. As the work makes 16 revolutions a minute, and during that time the tool traverses 16.8 feet on its surface, that means that the cutting speed is 16.8 feet per minute.

The calculation carried out above is expressed by the formula:

$$\begin{array}{c}\text{cutting speed in feet}\\ \text{per minute}\end{array} = \frac{\text{dia. of work in inches} \times 3.14}{12} \times \begin{array}{c}\text{number of revolutions}\\ \text{per minute}\end{array}$$

The following examples will tend to make clear the use of this formula:

Example 1.—A cast iron pulley, the rim of which is being turned, revolves in the lathe at a rate of 6½ revolutions per minute. The diameter of the pulley is 23 inches. What is the cutting speed?

The circumference of this pulley in feet equals 23 times 3.14 divided by 12, which is 6.02. Multiplying the circumference by the number of revolutions, 6½, gives the cutting speed. We have, then, 6.02 times 6.5, equals 39.13, which is the cutting speed in feet.

Example 2.—A one-inch drill, cutting tool steel, revolves at a rate of 60 revolutions per minute. What is the cutting speed?

The circumference of the drill, in feet, is 3.14 divided by 12, or 0.262. This multiplied by the number of revolutions, 60, gives us 15.72 feet as the cutting speed of the drill.

Rules for Calculating Cutting Speeds.

What has been previously stated in formulas, may be expressed in rules as follows:

1. To find the number of revolutions per minute, when the diameter of work (or drill) in inches and the cutting speed in feet per minute are known, *multiply the diameter by* 3.14, *and divide the result by* 12; *then divide the cutting speed by the figure thus obtained.*

2. To find the cutting speed in feet per minute, when the diameter of the work (or drill) in inches, and the number of revolutions per

minute are given, *multiply the diameter by* 3.14 *and divide the result by* 12; *then multiply the result thus obtained by the number of revolutions per minute.*

Feed of Cutting Tool.

The feed of the cutting tool enters but little in the calculations that a machinist would be required to make. The most common question into which the feed enters is the time required for turning or planing a certain piece of work. In this case both the cutting speed, or the number of revolutions per minute, and the feed would have to be considered.

Suppose for instance that a tool steel arbor is 2 inches in diameter; that its length is 10 inches; that the cutting speed of the work is 18 feet per minute; and that the feed of the cutting tool along the work is 1/16 inch per revolution. How long time would it require to take one cut over the surface of the work?

We first find from our discussion on the cutting speed, and from Rule 1 on previous page, that the revolutions per minute of the work equal 34, approximately. As the tool feeds forward 1/16 inch for each revolution of the work, it is fed forward 34/16 or 2⅛ inches in 34 revolutions, or in one minute. The time required to traverse the whole length of the work, 10 inches, will evidently be found by finding how many times 2⅛ is contained in 10 inches, or, in other words, by dividing 10 by 2⅛. The result of this division is 4.7 minutes. It would therefore take 5 minutes, approximately, to traverse the work once with the cutting speed and feed mentioned.

Expressed in a formula, our calculation would take this form:

$$\text{time required for one cut over the work} = \frac{\text{total length of work (or length of cut)}}{\text{revolutions per minute} \times \text{feed per revolution}}$$

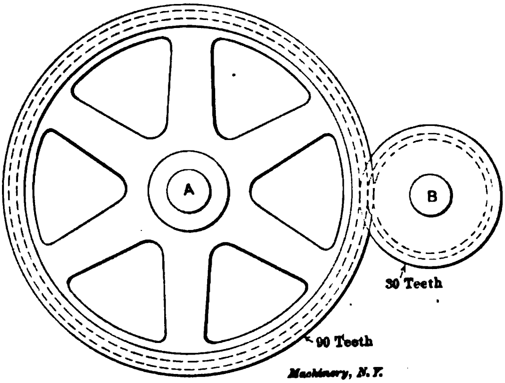
Expressed as a rule, the formula takes this form:

To find the time required for one complete cut over the work, when the feed per revolution, the total length of the cut, and the number of revolutions per minute are given, *divide the total length of the cut by the number of revolutions per minute multiplied by the feed per revolution.* If the cutting speed is given, originally, instead of the number of revolutions, find the latter number first from Rule 1, page 18.

CHAPTER IV.

TRAINS OF GEARS.

Suppose that we have two shafts *A* and *B*, as shown in Fig. 26, and that we want to connect these shafts by gears so that the shaft *A* is making one revolution while shaft *B* makes three. In order to do this we must place a gear on *A* having three times the number of teeth as compared with the number of teeth in the gear on *B*. The stud with the larger gear will always run slower than the stud with a smaller gear. Suppose that we have 90 teeth in the gear on *A*. The gear on *B* must then only have 30 teeth, because the gear on *A* was to have three times the number of teeth in the gear on *B*. Each time the gear on *B* turns around one complete revolution, it engages 30 teeth in the gear on stud *A*. It is then plain that it must turn around three complete revolutions in order to engage all the 90 teeth of the gear on *A*, or, in other words, turn around three times in order to turn the gear on *A* once. The numbers expressing the relationship between the number of times one gear revolves, to the corresponding number of times the other revolves, is called the *ratio* of the gearing. Thus, in the present case, the ratio is 3 to 1, one gear revolving 3 times, while the other revolves once.



Machinery, N.Y.

Fig. 26.

When the ratio of the speed of studs or shafts is given, it is possible to find the gears which will cause the gears to run at the required speed. In Fig. 27, suppose that shafts *C* and *D* are required to run in a ratio of 5 to 1, that is, shaft *C* is to revolve 5 times, while shaft *D* revolves once. What gears should we select to make the shafts run as required? If we place a 20-tooth gear on shaft *C*, the gear on shaft *D* should have 5 times as many teeth, or 100. Then it will revolve but once, when the gear on *C* revolves 5 times. Suppose that we put gears with the number of teeth mentioned on the shafts. Now the distance between the shafts may be such that the gears would not reach together. We can then place an intermediate gear *E*, mounted on stud *F*, in the train, which meshes with the gears on both *C* and *D*. *The intermediate gear simply transmits motion from the gear on C to the gear on D, and does not have any influence on the relative speed ratio.* When the intermediate gear is inserted, the gear on *C* still revolves 5 times while the gear on *D* revolves once.

If we placed a number of intermediate gears, *E*, *F*, and *G*, in the train, as in Fig. 28, the result would still be the same, the gear on *C* would turn 5 times while the gear on *D* turned once, as long as the number of teeth in the gear on *D* is 5 times the number of teeth in the gear on *C*.

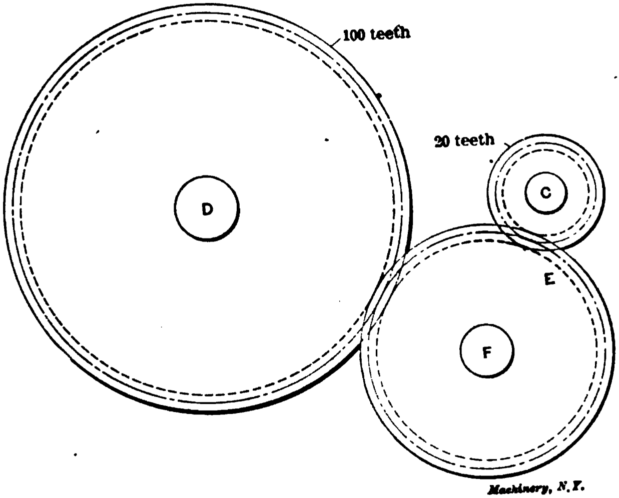


Fig. 27.

In order to prove this, let us assume that in Fig. 28, as before, the gear on stud *C* has 20 teeth, and the gear on stud *D*, 100 teeth, so that consequently the stud *C* makes 5 revolutions, while stud *D* makes one. The intermediate gears, *E*, *F*, and *G*, have 50, 40, and 40 teeth, respectively, as shown in the cut. Now, when the gear on *D* turns around once, the gear *G* must turn 2½ times (100/40 = 2½). The gear *F*, having the same number of teeth as gear *G*, makes one revolution while *G* makes one, and consequently also turns 2½ times while the gear on *D* turns once. The gear on *E*, having 50 teeth, turns 4/5 of a revolution while gear *F* revolves once (40/50 = 4/5), and consequently, while *F* makes 2½ revolutions, gear *E* makes 2 1/2 × 4/5 = 5/2 × 4/5 = 2 revolutions. Thus *E* turns twice while the gear on stud *D* revolves once. Finally, the gear on *C* turns 2½ times to each revolution of gear *E* (50/20 = 2½), or 5 times to 2 revolutions of *E*. But 2 revolutions of *E* correspond, as we have seen, to one revolution of the gear on stud *D*; consequently, the gear on stud *C* makes 5 revolutions to one of the gear on stud *D*, which, as we previously found, is also the case if these two gears had been connected directly without any intermediate gearing.

Principles of Compound Gearing.

Compound gearing consists of a train of gears in which certain gears are placed in pairs on the same stud as shown at *A*, Fig. 29, and turn together, one of the gears, *D*, being driven by another gear *B*, and the other gear, *E*, in turn driving a gear *C*. The simplest, and most common case of compound gearing consists of four gears, as shown in the cut referred to. Suppose that the stud, *F*, carrying gear *C*, Fig. 29, is required to revolve 6 times while stud *G*, carrying gear *B*, revolves 5 times. The ratio in speed of these two shafts would then be 6 to 5. Gears are not available so that studs *F* and *G* can be directly connected, or to make it possible to put in an intermediate gear to transmit the motion directly. In such cases the gears are compounded, that is, the train of gears is made up of two sets of gears, in each of which sets one gear is the driving and one the driven gear. In Fig. 29, *B* and *E* are driving gears, and *C* and *D* are driven gears.

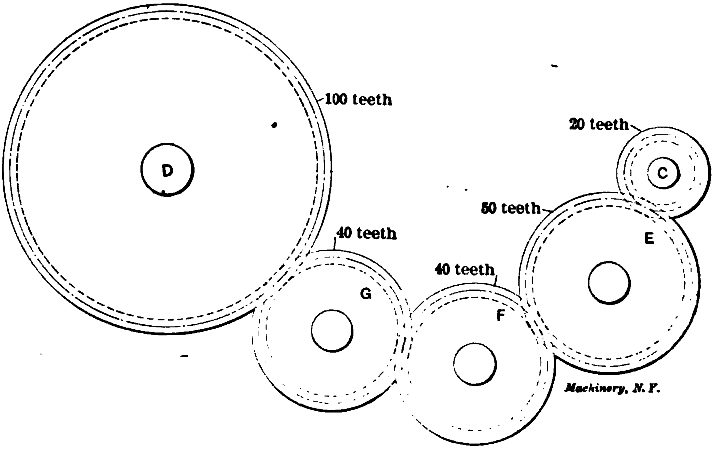


Fig. 28.

Each of these two sets of gears has a speed ratio of its own, which combined with the ratio of the other set gives the total ratio of the whole system. Referring to our specific case in Fig. 29, if gear *B* has 90 teeth, and gear *D*, 45 teeth, then gear *D* revolves two times while gear *B* revolves once, or, as our ratio between *C* and *B* is 6 to 5, let us say that gear *D* turns around 10 times while *B* turns around 5 times. Gear *E* is placed on the same stud as gear *D* and, therefore, also turns 10 times while *B* turns 5 times. Gear *E* drives gear *C*, which is required to turn 6 times to the 5 times of gear *B*. It, therefore, must also turn 6 times to the 10 times of gear *E*. If, then, gear *E* has 60 teeth and gear *C*, 100 teeth, this requirement will be filled. We now have gear *C* turning 6 times while gear *B* turns 5 times.

How could we have found the gears for this compound gearing directly by calculation?

The ratio required was 6 to 5, or if written as a fraction, 6/5. We can multiply numerator and denominator in a fraction by the same number without changing its value. We can also divide up the numerator and denominator in factors, and multiply each pair of factors with the same number. Thus we have:

$$\frac{6}{5}=\frac{2\times 3}{1\times 5}=\frac{(2\times 45)\times(3\times 20)}{(1\times 45)\times(5\times 20)}=\frac{90\times 60}{45\times 100}.$$

Here we have, then, the gears used in our train. The gears in the numerator are the driving gears *B* and *E*. The gears in the denominator are the driven gears *D* and *C*.

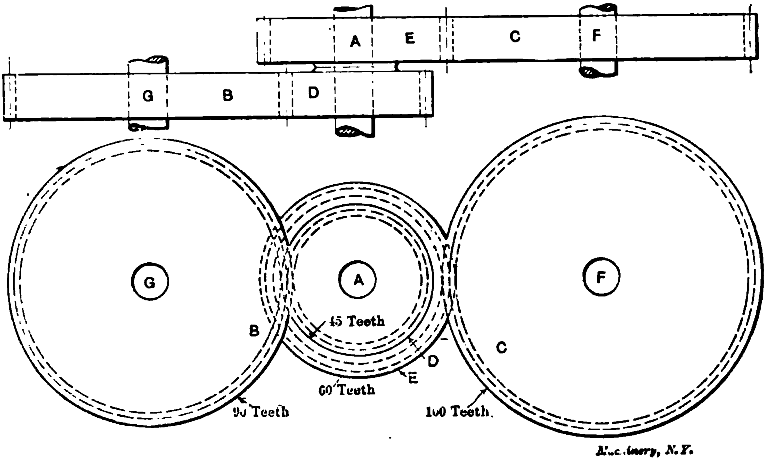


Fig. 29.

We may formulate a rule from the foregoing for finding gears for transmitting motion, in general, when the speed ratios between the driving and driven shafts are known.

1. *Place the number of revolutions of the driven shaft in the numerator, and the corresponding number of revolutions of the driving shaft in the denominator of a fraction* (or, in general, write the ratio in the form of a fraction), *and multiply the numerator and denominator with the same number, until a new fraction is obtained having numerator and denominator expressing suitable numbers of teeth for the gears.* The gear represented by the new numerator is the driving gear, and that represented by the new denominator is the driven gear.

2. If compounding of the gears is necessary or advisable, *divide up both numerator and denominator in the fraction, giving the ratio, in two factors, and multiply each pair of factors* (one factor in the numerator and one in the denominator making "one pair") *by the same numbers, until gears with suitable numbers of teeth are found.*

The most common application of trains of gearing is that occurring in lathes, and used for thread cutting. We shall, therefore, apply the principles of trains of gears to this case. But it should be clearly understood that the same principles hold good, no matter what machine the gears are applied to, and that the fundamental condition which determines the number of teeth in the gears connecting two shafts is the ratio of speed, that is, the number of turns made by one gear to a certain number of turns made by another.

Pitch and Lead of Screw Threads.

The terms pitch and lead of screw threads are often confused, and particularly in the case of multiple threaded screws does this confusion cause difficulties. Before we therefore enter upon the subject of figuring change gears for the lathe for cutting screw threads, it may be well to make clear the real meaning of the words "pitch" and "lead," and their relation to the number of threads per inch.

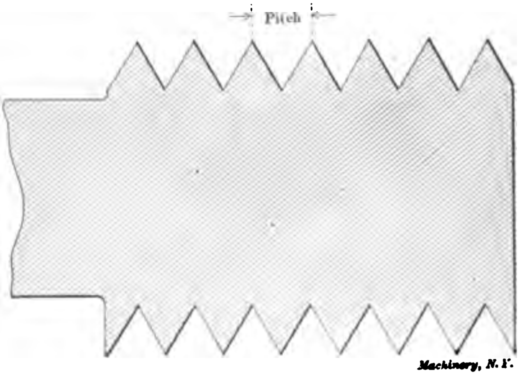


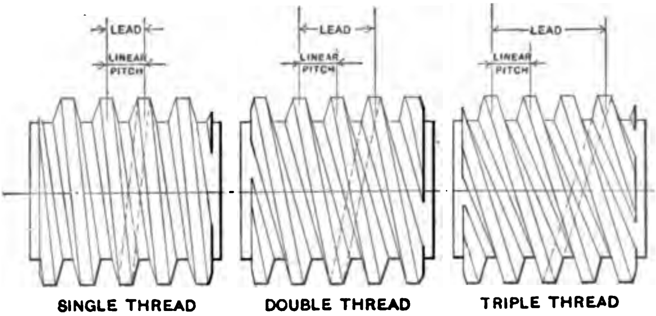
Fig. 30.

The *pitch* of a screw thread is the distance from the top of one thread to the top of the next, as shown in Fig. 30. No matter whether the screw has single, double, triple, or quadruple thread, the pitch is always the distance from the top of one thread to the top of the next. Often, though improperly, the word "pitch" is used in the shop to denote "number of threads per inch." We hear of screws having 12 pitch thread, 16 pitch thread, etc. This is not correct usage of the word pitch, and only tends to cause unnecessary confusion.

The *lead* of a screw thread is the distance the screw will move forward in a nut if turned around one full revolution. It is clear that for a single-threaded screw the pitch and the lead are equal, as the screw would then move forward the distance from one thread to the next if turned around once. In a double-threaded screw, however, the screw will move forward two threads, or twice the pitch, so that in a double-threaded screw the lead equals twice the pitch. In a triple-threaded screw the lead equals three times the pitch, and so forth.

The lead may also be expressed as being the distance from center to center of the *same* thread, after this thread has made one turn around the screw. In the single-threaded screw the *same* thread is the next thread to the one first considered. In a double-threaded screw there are two threads running side by side around the screw, so that the *same* thread here is the second one from the one first considered. In a triple-threaded screw it is the third one, in a quadruple-threaded, the fourth, and so forth. However we consider this, we still see that the lead and pitch are alike for a single-threaded screw, that the lead is twice the pitch for a double-threaded, and three times for a triple-threaded, as already stated. The actual relationship is very plainly shown in Fig. 31, where are shown parts of three screws with Acme threads, the first single-threaded, the second double-threaded, and the last triple-threaded.

The main point to remember, however, is that in *any kind of a screw, the lead is the distance which the screw will move forward in a nut if turned around one revolution.*



Machinery, N.Y.

Fig. 31.

In this connection it may be appropriate to give the rules and formulas for the relation between the lead and the number of threads per inch. If there are 8 threads, single, in one inch, the lead is evidently ⅛ inch. This we found, mathematically, by dividing *one* by 8, which is the number of threads per inch. The formula, therefore, is

$$\text{lead} = \frac{1}{\text{number of threads per inch}}$$

This formula, expressed in words, says: *The lead of a screw equals one divided by the number of threads per inch.*

Confusion is often caused by indefinite designation of multiple thread screws. The most common way to state the lead and the class of thread is perhaps to say ¼ *inch lead, double*, which means a screw with a double thread, which, when cut, has the lathe geared for four threads per inch, but each thread is cut only to a depth corresponding to eight threads per inch. The same condition is also expressed by: 4 *threads per inch, double.* These two ways of expressing the number of multiple threads are both correct, but the expression which ought to be used in order to avoid misunderstanding under any circumstances would be: ¼ *lead,* ⅛ *pitch, double thread.*

CHAPTER V.

LATHE CHANGE GEARS.

While the principles and rules governing the calculation of change gears are very simple, they, of course, presuppose some fundamental knowledge of the use of common fractions. If such knowledge is at hand, the subject of figuring change gears, if once thoroughly understood, can hardly ever be forgotten. It should be impressed upon the minds of all who have found difficulties with this subject that the matter is not approached in a logical manner, and is usually grasped by the memory rather than by the intellect. Before answering the question in regard to any rules for figuring change gears, let us therefore analyze the subject. The lead-screw *B* of the lathe (see Fig. 32) must be recognised as our first factor, and the spindle as the second. If the lead-screw has six threads per inch, then, if the lead-screw makes six revolutions, the carriage travels one inch, and the thread-cutting tool travels one inch along the piece to be threaded. If the spindle makes the same number of revolutions in a given time as the lead-screw, it is clear the tool will cut six threads per inch. In such a case the gear *D* on the spindle stud *J*, and gear *E* on the lead-screw, are alike. If the spindle makes twice the number of revolutions of the lead-screw, the spindle revolves twelve times while the tool moves one inch, and consequently twelve threads per inch will be cut. But in order to make the spindle revolve twice as fast as the lead-screw, it is necessary that a gear be put on the spindle stud of only half the number of teeth of the gear on the lead-screw, so that when the lead-screw revolves once the spindle stud gear makes two revolutions. The conditions governing the relationship between the number of teeth in gears and the number of revolutions of the studs on which they are mounted were, as we remember, explained in the last chapter.

Simple Gearing.

Suppose we wish to cut nine threads per inch with a lead-screw of six threads per inch, as referred to above. Then the six threads of the lead-screw correspond to nine threads on the piece to be threaded, which is the same as to say that six revolutions of the lead-screw correspond to nine revolutions of the spindle; or in other words, one revolution of the lead-screw corresponds to 1½ of the spindle. From this it is evident that the gear on the lead-screw must make only one revolution while the spindle stud gear makes 1½. Thus, if the lead-screw gear has, for instance, 36 teeth the gear on the spindle stud should have only 24; the smaller gear, of course, revolving faster than the larger. If we express what has been previously said in a formula we have:

$$\frac{\text{threads per inch of lead-screw}}{\text{threads per inch to be cut}} = \frac{\text{teeth in gear on spindle stud}}{\text{teeth in gear of lead-screw}}$$

Applying this to the case above, we have:

$$\frac{6}{9}=\frac{24}{36}$$

The values 24 and 36 are obtained by multiplying 6 and 9, respectively, by 4. By multiplying both the numerator and the denominator by the same number, we do not change the proportion. As a general rule we may then say that the change gears necessary to cut a certain number of threads per inch are found by placing the number of threads in the lead-screw in the numerator, the number of threads to be cut in the denominator, and then multiply numerator as well as denominator by the *same* number, by trial, until two gears are obtained, the number of teeth of which are both to be found in the set of gears accompanying the lathe. The gear with the number of teeth designated by the new numerator is to be placed on the spindle stud (at *J*, Fig. 32), and the gear with the number of teeth corresponding to the denominator on the lead-screw *B*.

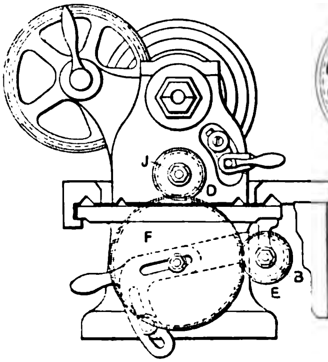
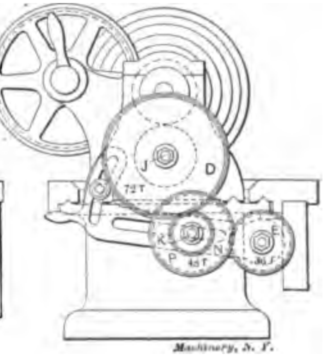


Fig. 32.



Machinery, N. Y.

Fig. 33.

A few examples of this will more clearly explain the rule. Suppose the number of teeth of the change gears of a lathe are 24, 28, 32, 36, and so forth, increasing by 4 teeth up to 100. Assume that the lead-screw is provided with 6 threads per inch, and that 10 threads per inch are to be cut. Then,

$$\frac{6}{10}=\frac{6\times 4}{10\times 4}=\frac{24}{40}$$

By multiplying both numerator and denominator by 4, we obtain two available gears with 24 and 40 teeth, respectively. The 24-tooth gear goes on the spindle stud, and the 40-tooth gear on the lead-screw. Assuming the same lathe and gears, let us find the gears for cutting 11½ threads per inch, this being the standard number of threads for certain sizes of pipe thread. Then,

$$\frac{6}{11\frac{1}{2}}=\frac{6\times 8}{11\frac{1}{2}\times 8}=\frac{48}{92}$$

It will be found that multiplying with any other number than eight would, in this case, not have given us gears with such number of teeth as we have in our set with this lathe. Until getting accustomed to figuring of this kind, we can, of course, only by trial find out the correct number by which to multiply numerator and denominator. The number of teeth in the *intermediate* gear *F*, Fig. 32, which meshes with both the spindle stud gear and the lead-screw gear, is of no consequence.

Lathes with Reduction Gearing in Head-stock.

In some lathes, however, there is a reduction gearing in the head-stock of the lathe, so that if equal gears are placed on the lead-screw and the spindle stud, the spindle does not make the same number of revolutions as the lead-screw, but a greater number. Usually in such lathes the ratio of the gearing in the head-stock is 2 to 1, so that with equal gears the spindle makes two revolutions to one of the lead-screw. This is particularly common in lathes intended for cutting fine pitches or, in general, in small lathes. In figuring the gears this must, of course, be taken into consideration. As the spindle makes twice as many revolutions as the lead-screw with equal gears, if the ratio of the gears be 2 to 1, that means that if the head-stock gearing were eliminated, and the lead-screw instead had twice the number of threads per inch as it has, with equal gears the spindle would still revolve the same as before for each inch of travel along the piece to be threaded. In other words, the gearing in the head-stock may be *disregarded, if the number of threads of the lead-screw is multiplied by the ratio of this gearing.* Suppose, for instance, that in a lathe the lead-screw has eight threads per inch, that the lathe is geared in the head-stock with a ratio of 2 to 1, and that 20 threads are to be cut. Then

$$\frac{2 \times 8}{20} = \frac{16}{20} = \frac{16 \times 4}{20 \times 4} = \frac{64}{80}$$

which two last values signify the number of teeth in the gears to use.

Sometimes the ratio of the gearing in the head-stock cannot be determined by counting the teeth in the gears, because the gears are so placed that they cannot be plainly seen. In such a case, equal gears are placed on the lead-screw and the spindle stud, and a thread cut on a piece in the lathe. The number of threads per inch of this piece should be used for the numerator in our calculation instead of the actual number of threads of the lead-screw. The ratio of the gearing in the head-stock is equal to the ratio between the number of threads cut on the piece in the lathe and the actual number of threads per inch of the lead-screw.

Compound Gearing.

The cases with only two gears in a train referred to are termed simple gearing. Sometimes it is not possible to obtain the correct ratio excepting by introducing two more gears in the train, which, as has previously been said, is termed compound gearing. This class of gearing is shown in Fig. 33. The rules for figuring compound gearing are exactly the same as for simple gearing, excepting that we must divide both our numerator and denominator into two factors, each of which are multiplied with the same number in order to obtain the change gears.

Suppose a lathe has a lead-screw with six threads per inch, that the number of the teeth in the gears available are 30, 35, 40, and so forth, increasing by 5 up to 100. Assume that it is desired to cut 24 threads per inch. We have then,

$$\frac{6}{24} = \text{ratio.}$$

By dividing up the numerator and denominator in factors, and multiplying *each pair of factors by the same number*, we find the gears:

$$\frac{6}{24} = \frac{2 \times 3}{4 \times 6} = \frac{(2 \times 20) \times (3 \times 10)}{(4 \times 20) \times (6 \times 10)} = \frac{40 \times 30}{80 \times 60}$$

The last four numbers indicate the gears which should be used. The upper two, 40 and 30, are driving gears, the lower two, with 80 and 60 teeth, are driven gears. Driving gears are, of course, the gear *D*, Fig 33, on the spindle stud, and the gear *P* on the intermediate stud *K*, meshing with the lead-screw gear. Driven gears are the lead-screw gear, *E*, and the gear *N* on the intermediate stud meshing with the spindle stud gear. It makes no difference which of the driving gears is placed on the spindle stud, or which of the driven is placed on the lead-screw.

Suppose, for a final example that we wish to cut 1¾ threads per inch on a lathe with a lead-screw having six threads per inch, and that the gears run from 24 and up to 100 teeth, increasing by 4. Proceeding as before, we have

$$\frac{6}{1\frac{3}{4}} = \frac{2 \times 3}{1 \times 1\frac{3}{4}} = \frac{(2 \times 36) \times (3 \times 16)}{(1 \times 36) \times (1\frac{3}{4} \times 16)} = \frac{72 \times 48}{36 \times 28}$$

This is the case directly illustrated in Fig. 32. The gear with 72 teeth is placed on the spindle stud *J*, the one with 48 on the intermediate stud *K*, meshing with the lead-screw gear. These two gears (72- and 48-teeth) are the *driving* gears. The gears with 36 and 28 teeth are placed on the lead-screw, and on the intermediate stud, as shown, and are the *driven* gears.

Fractional Threads.

Sometimes the lead of the thread is expressed by a fraction of an inch, instead of stating the number of threads per inch. For instance, a thread may be required to be cut having a ⅜-inch lead. In such a case the expression "⅜-inch lead" should first be transformed to "number of threads per inch," after which we can proceed in the same way as has already been explained. To find how many threads per inch there is when the lead is stated, we simply find how many times the lead is contained in one inch, or, in other words, we divide *one* by the given lead. Thus *one* divided by 3/8 gives us 2 2/3, which is the number of threads per inch of a thread having ⅜-inch lead. To find change gears to cut such a thread we would proceed as follows:

Assume that the lead-screw has 6 threads per inch, and that the change gears run from 24 up to 100 teeth, increasing by 4. Proceeding to find the gears as before, we have:

$$\frac{6}{2\frac{2}{3}}=\frac{2\times 3}{1\times 2\frac{2}{3}}=\frac{(2\times 36)\times(3\times 24)}{(1\times 36)\times(2\frac{2}{3}\times 24)}=\frac{72\times 72}{36\times 64}.$$

The rule for finding the number of threads per inch, when the lead is given, may be expressed by the formula:

$$\text{number of threads per inch}=\frac{1}{\text{lead of thread}}$$

which is simply a reversal of the formula given on page 25.

What has been said in the foregoing in regard to the figuring of change gears for the lathe may be summed up in the following rules:

1. To find the number of threads per inch, if the lead of a thread is given, *divide one by the lead.*

2. To find the change gears used in simple gearing, when the number of threads per inch on the lead-screw, and the number of threads per inch to be cut are given, *place the number of threads on the lead-screw as numerator and the number of threads to be cut as denominator in a fraction, and multiply numerator and denominator with the same number until a new fraction results representing suitable number of teeth for the change gears.* In the new fraction, the numerator represents the number of teeth on the spindle stud, and the denominator, the number of teeth in the gear on the lead-screw.

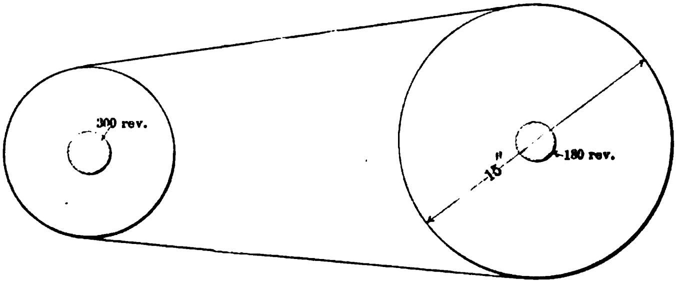
3. To find the change gears used in compound gearing, *place the number of threads per inch on the lead-screw as numerator, and the number of threads per inch to be cut as denominator in a fraction, divide up both numerator and denominator in two factors each, and multiply each pair of factors* (one factor in the numerator and one in the denominator making "a pair") *by the same number, until new fractions result representing suitable number of teeth for the change gears.* The gears represented by the numbers in the new numerators are driving gears, and those in the denominators are driven gears.

CHAPTER VI.

SPEED OF PULLEYS.

The principle applied to gearing in regard to the ratio between the speeds of two shafts, may be directly applied to the question of sizes of pulleys, with the only difference that we here deal with the number of inches to the diameter of the pulley instead of the number of teeth in the gear. Suppose that a shaft is required to make 300 revolutions per minute, and that this shaft is driven from a line-shaft making 180 revolutions per minute, as shown in Fig. 34. The pulley on the line-shaft is already in place, and is 15 inches in diameter. What diameter should the pulley on the shaft making 300 revolutions per minute be made?

As the belt over the two pulleys runs at the same speed as the circumference of the pulleys, it is clear that the circumferences of both pulleys run at the same speed. The pulley running a fewer number of revolutions consequently must be larger, in order that its circumference may run at the same speed as the circumference of the pulley running faster in regard to number of revolutions. The circumference depends directly upon the length of the diameter, because we know that the circumference equals the diameter times 3.1416. But as the factor 3.1416 would enter in the case of both pulleys, it is not necessary to carry it along in the calculation. We can figure with the diameter directly, substituting the diameter for the circumference, so to say. *For the obtaining of our ratio of speed between the pulleys, the diameters serve the same purpose in figuring as the circumferences*, because the ratio between the diameters is the same as the ratio between the circumferences. Now, the circumference of the pulley making 180 revolutions, and having a diameter of 15 inches, passes through a distance of 180 times its circumference in one minute, or 180 × 15 × 3.1416. The circumference of the pulley making 300 revolutions must pass through the same distance, for as we have



Machinery, N.Y.

Fig. 34.

said, the belt causes the circumferences to run equally fast. Therefore, for *each* revolution the latter pulley must pass through the distance 180 × 15 × 3.1416 divided by 300. This then would be the circumference of the smaller pulley, or its diameter times 3.1416. We can therefore write

$$\frac{180 \times 15 \times 3.1416}{300} = \text{diameter of small pulley} \times 3.1416.$$

As we said before, 3.1416 enters as a factor in both cases, and we can therefore cancel it. Then we have

$$\frac{180 \times 15}{300} = \text{diameter of small pulley} = 9 \text{ inches.}$$

From this we can formulate a rule for all figuring of pulleys:

The number of revolutions of one shaft multiplied with the diameter of the pulley on the same shaft, divided by the number of revolutions of the second shaft, gives the diameter of the pulley of the second shaft.

We may also write this as a formula, thus:

$$\begin{array}{c}\text{diameter of pulley on}\\ \text{second shaft in inches}\end{array} = \frac{\begin{array}{c}\text{number of revolutions}\\ \text{of first shaft}\end{array} \times \begin{array}{c}\text{diameter of pulley on}\\ \text{first shaft in inches}\end{array}}{\text{number of revolutions of second shaft}}$$

If we know the sizes of the pulleys, and the number of revolutions of one shaft, and want to find out the number of revolutions of the other shaft, the method is very similar to that used for figuring change gears. Evidently this should be so, because the diameters of the pulleys simply express the ratio of speeds of the shaft. Suppose one pulley is 12 inches in diameter, and another 20, and the shaft with the 12-inch pulley makes 180 revolutions. How many revolutions does the other pulley make?

Our ratio is $\frac{20}{12}$. Now multiply the numerator, 20, with a number giving 180 as result. This number is 9. Multiply the denominator by the *same* number, and we get 9 times 12 equals 108 as the number of revolutions of the second pulley. In other words, we have

$$\frac{20}{12} = \frac{20 \times 9}{12 \times 9} = \frac{180}{108}$$

It should be understood that the number of revolutions are in a *reverse* ratio to the ratio of the diameters, that is,

$$\frac{\text{diameter of first pulley}}{\text{diameter of second pulley}} = \frac{\text{revolutions of second pulley}}{\text{revolutions of first pulley}}$$

In the first ratio (of diameters), the diameter of the first pulley is the numerator and that of the second pulley is the denominator, but in the second ratio (of revolutions), the number of revolutions of the second pulley is the numerator, and that of the first pulley is the denominator.

A few examples may tend to make this clearer and fix the procedure more firmly in the mind of the reader.

Example 1.—A line-shaft for a few grinding machines is required to run at 320 revolutions per minute, and to be driven from a main line-shaft, running at 200 revolutions per minute. The pulley on the main line-shaft is already in place, and is 24 inches in diameter. What diameter ought the pulley on the grinding machine line-shaft to be?

We find this diameter directly from our rule on page 32 saying that the number of revolutions of one shaft times the diameter of the pulley on this shaft, divided by the number of revolutions of the second shaft, gives us the diameter of the pulley of the second shaft. Thus

$$\frac{200 \times 24}{320} = \frac{4800}{320} = 15.$$

The diameter of the pulley on the grinding machine line-shaft, therefore, should be 15 inches.

Example 2.—The pulley of a machine tool is 8 inches in diameter. The driving pulley on the line-shaft is 34 inches in diameter. It is known that the line-shaft makes 120 revolutions per minute. How many revolutions per minute does the 8-inch pulley on the machine make?

The diameters of the pulleys are in *inverse ratio* to the number of revolutions, that is, as already said,

$$\frac{\text{diameter of first pulley}}{\text{diameter of second pulley}} = \frac{\text{revolutions of second pulley}}{\text{revolutions of first pulley}}$$

From this we have, then, in our present example,

$$\frac{8}{34} = \frac{8 \times 15}{34 \times 15} = \frac{120}{510}.$$

The 8-inch pulley, consequently, makes 510 revolutions per minute.

Example 3.—The largest step of a cone-pulley in the countershaft of a machine is 12 inches in diameter. The smallest step is 6 inches. The largest step of the cone pulley on the machine is 10 inches, the smallest, 4. If the countershaft runs 300 revolutions per minute, which are the highest and lowest speeds of the spindle of the machine on which the cone pulley is mounted?

The largest step on each respective pulley runs with the smallest step of the other. Therefore, proceeding exactly as if we had two sets of pulleys, one set 12 and 4, and one 6 and 10 inches in diameter, we find by figuring exactly as in our previous example:

$$\frac{4}{12} = \frac{4 \times 75}{12 \times 75} = \frac{300}{900},$$

and

$$\frac{10}{6} = \frac{10 \times 30}{6 \times 30} = \frac{300}{180}.$$

The smallest speed is therefore 180 revolutions per minute, and the largest 900 revolutions.

CHAPTER VII.

INDEXING MOVEMENTS FOR MILLING MACHINE.

The figuring of indexing movements for the dividing head of the milling machine is a subject which many mechanics think complicated, although it really is very simple. The index head is constructed with a worm and worm-wheel mechanism, the worm being on the crank turned when indexing, and the worm-wheel being mounted on the index spindle to which the work is attached. The worm-wheel has 40 teeth, so that turning the crank around one full revolution, which also turns the worm one revolution, moves the worm-wheel one tooth, or one-fortieth of its circumference. In the same way, to turn the worm-wheel and the spindle on which it is mounted around one full revolution, we must turn the index-crank 40 revolutions. If we thoroughly understand this, the figuring of indexing movements is very easy. Suppose that we want to mill a hexagon. We then want to turn the work one-sixth of a revolution for each side milled. As it requires 40 revolutions of the index-crank to turn the index-spindle once, it evidently requires only one-sixth of that number to turn the index-spindle one-sixth revolution. Consequently, the index-crank should be turned around $\frac{40}{6} = 6\ 2/3$ revolutions for milling a hexagon. That is, we first turn the crank around 6 times, and then, by means of the index-plate, we turn it 2/3 of a revolution. This would mean 12 holes in an 18-hole circle, for instance, as 12 is two-thirds of 18.

Suppose we should want to mill a piece of work having 8 sides, regularly spaced. The indexing for each space is found by dividing 40 by 8. Then, $\frac{40}{8} = 5$, represents the number of turns of the index-crank for each side indexed. If we want to cut 9 flutes, regularly spaced, in a reamer, we must turn the crank $40/9 = 4\ 4/9$ revolutions to index for each flute. The 4/9 of a revolution would correspond to 8 holes in the 18-hole circle, because $8/18 = 4/9$.

In order to find which index circle to use, and how many holes in the index circle to move for a certain fractional turn of the index-crank, numerator and denominator of the fraction expressing the fractional turn are multiplied by the same number, this number being so selected that the denominator in the new fraction equals the number of holes in some index circle. The new numerator then expresses *how many* holes in this circle the crank is to be moved. Suppose that we want to index for 12 flutes in a large tap. We first divide 40 by 12 to find the number of turns of the index-crank required. Writing out this division as a fraction, and carrying out the calculation, gives us:

$$\frac{40}{12} = 3\frac{4}{12} = 3\frac{1}{3}.$$

The fractional turn required is 1/3 of a revolution. Now multiply, according to the rule given, the numerator and denominator of this fraction by a number so that the denominator equals the number of holes in some index circle. Multiplying with 6 would give us

$$\frac{1 \times 6}{3 \times 6} = \frac{6}{18}$$

in which fraction 18 is a number expressing the number of holes in the index circle to use, and 6 the number of holes the crank has to be moved in this circle to turn one-third of a revolution.

Suppose, that instead of having given a certain number of times which the work is to be indexed to complete one revolution (as 80 for an 80-tooth gear; 6 for a hexagon nut, etc.), we instead had given a certain number of degrees which it is required that the work be turned before taking another cut by the milling cutter.

As there are 360 degrees in a complete circle or turn (one right angle is 90 degrees, and the whole circle, of course, consists of four right angles), and as 40 turns of the index crank are required for one revolution of the work, *one* turn of the index crank must correspond to $360/40 = 9$ degrees. Then, if one complete turn of the index crank equals 9 degrees, it is clear that if we index in the 18-hole circle, 2 holes must correspond to one degree. This is, therefore, the fundamental principle or rule for indexing angular movement. *Two holes in the 18-hole circle equals a movement of one degree of the work.*

Suppose that we wish to index 11 degrees. We first, then, divide the number of degrees by 9 to find how many complete turns the index crank should make; and the number of degrees left to turn when we have completed our full turns are indexed by taking two holes in the 18-hole circle for each degree. In this case $11/9 = 1\ 2/9$, which gives us the answer that we must turn the crank one full revolution, and then index 2 degrees more, or 4 holes in the 18-hole circle.

It is evident that one hole in the 18-hole circle represents 1/2 degree, when two holes represent one degree. Should it be required to index only 1/3 degree, this may be done by using the 27-hole circle. If two holes in the 18-hole circle, which is 1/9 of a turn, equals one degree, three holes in the 27-hole circle, which also is 1/9 of a turn, must also equal one degree; and if a 3-hole movement equals one degree, a 1-hole movement in that circle must equal 1/3 degree. Therefore, if we wish to index the work through an angle of 48 degrees 40 minutes (there are 60 minutes to one degree, so that 40 minutes equals 2/3 degree), we simply turn the crank five complete turns for 45 degrees ($5 \times 9 = 45$), and we have then 3 degrees 40 minutes, or 3 2/3 degrees left. In the 27-hole circle 3 degrees correspond to 9 holes, and 2/3 degree to 2 holes, according to what we have just said. Consequently, we turn the crank 11 holes, in all, further in the 27-hole circle to complete the angular movement of 48 degrees 40 minutes.

CHAPTER VIII.

CHANGE GEARS FOR MILLING SPIRALS.

The method for the figuring of change gears for cutting spirals on the milling machine, is, in principle, exactly the same as that used for figuring change gears for the lathe, but it will be necessary to shortly refer to the construction of the mechanism for connecting the index head spindle and the feed-screw to make perfectly clear the fundamental ideas governing the selection of change gears. In Fig. 35, *A* is the feed-screw of the milling machine; and *B* is the gear placed on this feed-screw, commonly called the feed-screw gear. This gear meshes with the gear *C*, placed on the stud *D*, from which, in turn, motion is imparted to the worm in the index head and from the worm to the worm-wheel and the index spindle. The gear *C* on the stud *D* is called the worm gear, because it directly operates the movement of the worm. This expression "worm gear" should not be confused with the worm-wheel, which is placed on the index spindle. The case shown in Fig. 35 is one of simple gearing. In Fig. 36 is shown a case of compound gearing. Here *B* still represents the feed-screw gear, *E* is a gear on the intermediate stud, meshing with gear *B*, and gear *F* is another gear on the same intermediate stud, meshing with gear *C*, thus transmitting motion from the feed-screw to the stud *D* by compounding the gears.

The figuring of change gears for the milling machine consists simply in the selection of the proper gears, *B* and *C*, used in a simple train, as in Fig. 35, or gears *B*, *E*, *F*, and *C*, as used in a compound train of gears, as shown in Fig. 36.

In order to figure change gears for the lathe we remember that it was necessary to first know the number of threads per inch in the lead-screw. Knowing that, we knew how many revolutions the lead-screw had to make to move the carriage and the thread tool one inch along the work. In the case of the milling machine we must know *how far the table travels while the index spindle makes one complete revolution, when gears B and C, Fig.* 35, *have an equal number of teeth.* This distance is the constant which we use in figuring the change gears, the same as we used the number of threads per inch of the lead-screw in figuring change gears for the lathe. This constant, which may be different for different milling machines, is called the *lead of the milling machine.* We will now see how this constant is found.

Suppose, for instance, that in a milling machine one revolution of the worm gear, *C*, Figs. 35 and 36, will produce exactly one revolution of the shaft on which the worm is placed, that is, one revolution of the index crank, and suppose that equal gears are placed on the feed-screw and on the stud *D*, so that one revolution of the feed-screw produces exactly one revolution of the gear *C* and stud *D*. Then, if the feed-screw revolves one revolution, the milling machine table will advance a distance equal to the lead of the feed-screw, because, as we have said before, the lead of the screw is the distance which it will advance in one turn.

Now, when the table of the milling machine moves forward a distance equal to the lead of the feed-screw, or a distance equal to one thread in the feed-screw, the feed-screw turns one revolution, and gear *C* also turns one revolution, the worm-shaft and the index crank turn one revolution, and, there being 40 teeth in the worm-wheel which is mounted on the index head spindle, this worm-wheel, with its spindle, will turn 1/40 of one revolution. To make one complete revolution of the index head spindle, the feed-screw would have to be revolved as many times as there are teeth in the worm-wheel, each revolution, as we have seen, moving it one tooth. The distance which the milling machine table will advance, when the index head spindle revolves one complete revolution, is, as we have said, the lead of the machine. This distance evidently equals the distance that the feed-screw advances the table in one revolution (which is the lead of the screw) times the number of revolutions made. If we assume that the feed-screw has a lead of ⅛ inch, and that there are 40 teeth in the worm-wheel on the index head spindle, the feed-screw, when the gears *B* and *C* are equal, will have to turn 40 times, in order to move the index head spindle around once, and the distance the table will then advance will be 40 times ⅛ inch, or 5 inches. In this particular case, 5 inches is then the *lead of the machine.* If now a piece of work had been affixed to the index spindle, it is clear that this piece of work would have made one complete revolution, while the milling machine table advanced 5 inches, and that the lead of the spiral cut on the work would have been 5 inches.

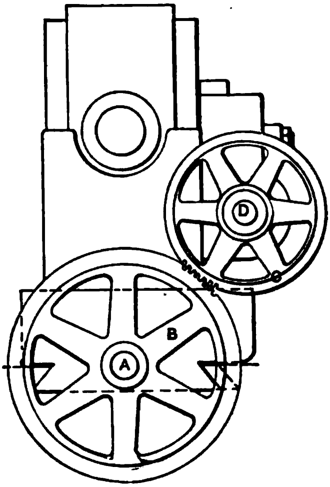
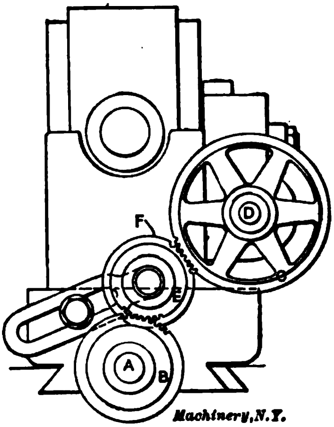


Fig. 35.



Machinery,N.Y.

Fig. 36.

A general rule for finding the lead of a milling machine may be stated as follows: *To find the lead of a milling machine, place equal gears on stud D, Fig. 35, and on the feed-screw, and multiply the number of revolutions made by the feed-screw in order to produce one revolution of the index head spindle, by the lead of the feed-screw.*

Suppose, for an example, that it is necessary to make 40 revolutions of the feed-screw in order to turn the index head spindle one complete revolution, when gears *B* and *C*, Fig. 35, are equal, and that the lead of the feed-screw of the milling machine is ¼ inch, then the lead of the machine equals 40 × ¼ inch, or 10 inches.

The rule just given is general, and will apply even if the number of teeth in the indexing worm-wheel were different from that in standard indexing heads, because, in the rule no consideration is taken of the number of teeth of the worm-wheel directly, but simply the number of turns made by the feed-screw to correspond to one turn of the index spindle itself.

If it is now perfectly clear that the lead of the machine means the distance which the table of the milling machine must move forward in order to turn the work placed on the index head spindle around one complete revolution, with equal gears, then we see that if we want to get a spiral that is twice as long as the lead of the machine, we must place gears on the feed-screw and on the stud *D* of such size that the indexing spindle will only turn half a revolution while the table moves forward a distance equal to the lead of the machine. Suppose, for instance, that we want to cut a spiral, having a lead of 20 inches, that is, making one complete turn in a distance of 20 inches, and that the lead of the milling machine is 10 inches. Then, while the table moves forward 20 inches, we want the indexing spindle to turn once. In order to make the table move forward 20 inches, when the indexing spindle turns around once, the feed-screw evidently must turn twice as many times as it did in the case when the table only moved 10 inches for one turn in the index spindle, in which case we had equal gears on the feed-screw and on stud *D*. Here we then have two studs or shafts, *A* and *D*, where we wish that the one should turn twice as fast as the other. The ratio between the speeds, then, is 2 to 1, which means that the feed-screw, which is required to turn twice while the stud *D* turns once, must have a gear that has only half the number of teeth of the gear placed on stud *D*.

If the lead of the machine be 10 inches, and the lead required to be cut on a piece of work is 30 inches, then it would be necessary to have the ratio between the gears 3 to 1, which, of course, is the same as the ratio between the lead of the machine and the lead of the spiral to be cut (30 to 10 equals 3 to 1). We can therefore express the rule for finding the change gears by a simple formula:

$$\frac{\text{lead of spiral to be cut}}{\text{lead of machine}} = \frac{\text{number of teeth in gear on worm stud } (D, \text{ Fig. 35})}{\text{number of teeth in gear on feed-screw}}$$

Expressed as a rule, this formula would read: *To find the change gears to be used in a simple train of gearing, when cutting spirals on a milling machine, place the lead of the spiral in the numerator and*

the lead of the milling machine in the denominator of a fraction, and multiply the numerator and denominator with the same number, until a new fraction is obtained in which the numerator and denominator give suitable numbers of teeth.

As an example of the above rule, we will take the case of a milling machine in which we have found that there are 4 threads per inch on the feed-screw, and that 20 revolutions of the feed-screw are necessary to make the index spindle turn one complete revolution when having equal gears on *A* and *D*, Fig. 35. On a machine of this kind, assume that it is required to cut a spiral the lead of which is 12 inches. The first thing for us to do is to find the lead of the machine. This, as we have already said, is equal to the revolutions of the feed-screw necessary to turn the index spindle one revolution, multiplied by the lead of the feed-screw. As the feed-screw has 4 threads per inch, the lead of the feed-screw is ¼ inch, and this, multiplied by 20, gives us $\frac{1}{4} \times 20 = \frac{20}{4} = 5$. To find our gears we now place the lead of the spiral in the numerator of a fraction and the lead of the machine in the denominator, and multiply both numerator and denominator with the same number until we get a new fraction in which the numerator and denominator express a suitable number of teeth. Following this rule, we have then:

$$\frac{12}{5} = \frac{12 \times 6}{5 \times 6} = \frac{72}{30}.$$

The gear with 72 teeth is placed on stud *D*, which, of course, is required to revolve slower than the feed-screw, in order to cut a spiral which is 12 inches, when the spiral cut with equal gears is only 5 inches, or equal to the lead of the machine. The gear having 30 teeth is placed on the feed-screw. If it should be necessary to put an intermediate gear between the gear on the feed-screw and the gear on stud *D*, the number of teeth in this intermediate gear would have no influence on the ratio of the speeds of feed-screw *A* and stud *D*, but would simply serve the purpose of transmitting motion from the one gear to the other. In Chapter IV this matter of the intermediate gear was treated at length, and an example given showing that even a series of intermediate gears did not influence the ratio of speeds of two shafts.

Compound Gearing.

If it is not possible to find a set of two gears that will transmit the motion required, it will be necessary to use compound gearing. In this case the manner of getting the compound gears is exactly the same as that of getting compound gearing for the lathe. We have already explained that the lead of the spiral to be cut is placed in the numerator of the fraction, and the lead of the milling machine in the denominator. We then divide up this numerator and denominator in two factors, the same as we did in the case of figuring lathe change gears, and having divided them up in two factors, we multiply each two of these factors by the same number, exactly as before, thus getting the gears we require. As an example, let us suppose that the lead of a certain machine is 10 inches, and that we wish to cut a spiral the lead of which is 60 inches. We then have:

$$\frac{60}{10}=\frac{6\times 10}{2\times 5}=\frac{(6\times 15)\times(10\times 8)}{(2\times 15)\times(5\times 8)}=\frac{90\times 80}{30\times 40}.$$

The gear having 90 teeth is placed on the stud *D*, and meshes with the 30-tooth gear *F* (see Fig. 36) on the intermediate stud; on the same intermediate stud is then also placed the gear having 80 teeth, which is driven by the gear having 40 teeth placed on the feed-screw. This makes the gears having 90 and 80 teeth the driven gears, and the gears having 30 and 40 teeth the driving gears, the whole train of gears being driven from the feed-screw of the table.

In general, for compound gearing it may be well to remember the rule given by the formula:

$$\frac{\text{lead of spiral to be cut}}{\text{lead of machine}}=\frac{\text{product of driven gears}}{\text{product of driving gears}}$$

CHAPTER IX.

SQUARE AND SQUARE ROOT.

The expressions "square" and "square root" often occur in technical formulas, and to one unfamiliar with these names, and the mathematical operations which they signify, as well as the signs by which they are indicated, it may appear that difficult mathematical operations are involved. But this is not the case.

The square of a number is simply the product of that number multiplied by itself. Thus, the square of 2 is $2\times 2=4$, and the square of 3 is $3\times 3=9$. Similarly, the square of 177 is $177\times 177=31{,}329$. Instead of writing 177×177, it is common practice to signify this operation 177^2, which is read "177 square," and simply means that 177 is multiplied by itself. Thus we have $5^2=5\times 5=25$, and $27^2=27\times 27=729$. The "2" at the upper right-hand corner is called *exponent*. Most mechanical and engineering handbooks have tables which give the squares of all numbers up to 1,000, so that by means of such a handbook it is unnecessary to figure out the squares of the numbers there given when required, by actual multiplication. The square of numbers is very much used in solving many of the problems occurring in the machine shop.

The square root of a number is that number which, if multiplied by itself, would give the given quantity. Thus, the square root of 4 is 2, because 2 multiplied by itself equals 4. The square root of 25 is 5, and so forth. It will be noticed at once that the square root is simply the reverse of the square, so that if the square of 25 is 625, then the square root of 625 is 25. The mathematical sign for the square root is $\sqrt{\ }$. Thus, $\sqrt{4}=2$, $\sqrt{16}=4$, and so forth. The engineering handbooks give tables of the square roots of all numbers up to

1,000 or more, so that the process of actually figuring the square root, which is rather complicated, and which space does not permit to deal with here, has been omitted.

In the same way as we write $2^2 = 4$ we can write $2^3 = 8$, in which case $2^3 = 2 \times 2 \times 2 = 8$, the exponent indicating how many times the number given is to be repeated as factor. Similarly, $4^3 = 4 \times 4 \times 4 = 64$. The expression 4^3 is read "4 cube" or "the third power of 4." The expression 2^5 would mean that two is to be repeated as factor five times; thus, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. The expression 2^5 is read "the fifth power of 2."

In the same way as the square root means the reverse of square, so cube root means the reverse of cube (or "third power"), that is, the cube root of a number is the number which, if repeated as factor three times, would give the number given. Thus, the cube root (or "third root") of 27 is 3, because $3 \times 3 \times 3 = 27$. We see, of course, that if the cube of a number, as 5, is 125 ($5 \times 5 \times 5 = 125$), then the cube root of 125 is 5. The sign for the cube root is $\sqrt[3]{\ }$. Thus, $\sqrt[3]{8} = 2$ (because $2 \times 2 \times 2 = 8$), and $\sqrt[3]{27} = 3$ (because $3 \times 3 \times 3 = 27$). Similarly $\sqrt[3]{125} = 5$, and $\sqrt[3]{1728000} = 120$.

If we want the number which repeated as factor four times gives a given number, we must get the "fourth root" or $\sqrt[4]{\ }$. Thus, $\sqrt[4]{81} = 3$, because $3 \times 3 \times 3 \times 3 = 81$. Similarly we write the fifth root, $\sqrt[5]{\ }$; and we write $\sqrt[5]{32} = 2$, because $2 \times 2 \times 2 \times 2 \times 2 = 32$.

After this explanation, formulas reading $\dfrac{3^2 \times \sqrt{4}}{\sqrt{36}}$ do not any longer look so mysterious.

If we simplify, according to the meanings given to squares and square roots, we have:

$$\frac{3^2 \times \sqrt{4}}{\sqrt{36}} = \frac{9 \times 2}{6} = \frac{18}{6} = 3.$$

Similarly,

$$\frac{\sqrt[3]{27} + \sqrt[3]{216}}{3^2} = \frac{3 + 6}{9} = \frac{9}{9} = 1,$$

and

$$\frac{5^2 + 4^2}{5^4} = \frac{25 + 16}{625} = \frac{41}{625}.$$

Right Angle Triangles.

One of the most common applications of the squares of numbers is that of the right angle triangle, as shown in Fig. 37, in which angle *A* is a right, or 90 degree angle. If we assume the lengths of the sides to be 5 inches, 4 inches, and 3 inches, respectively, as indicated on the cut, we find that

$$5^2 = 4^2 + 3^2, \text{ or } 25 = 16 + 9.$$

This holds good for all right angle triangles. The square of the side *opposite* the right angle equals the sum of the squares of the sides *including* the right angle.

Suppose, for instance, that in Fig. 38 we know the length of the two sides including the right angle to be 6 inches and 8 inches, and we want to know the length of the side *BC* opposite the right angle. Then the sum of the squares of the two given sides which include the right angle equals the square of the side opposite the right angle, and the square root of this number must be the length of *BC*. Thus

$$6^2 + 8^2 = 36 + 64 = 100.$$

The square of the length *BC* is then 100, and the length itself is the square root of 100, or $\sqrt{100} = 10$. The side *BC* is therefore 10 inches long.

If we knew that *BC* was 10 inches, and *AB* 8 inches, and wanted to find *AC*, we would subtract the square of *AB* from the square of *BC* to find the square of *AC*, or

$$10^2 - 8^2 = 100 - 64 = 36 = \text{square of } AC.$$
$$\sqrt{36} = 6 = \text{length of } AC.$$

A little thought, of course, at once convinces us that if the sum of two numbers equals a third, then if one of the first numbers is subtracted from the third, the remainder equals the other of the given numbers. Thus, in Fig. 38, we have the square of *AB* plus the square of *AC* equals the square of *BC*; and the square of *BC* minus the square of *AB* equals the square of *AC;* and, finally, the square of *BC* minus the square of *AC* equals the square of *AB*. In this case, where the sides are 10, 8, and 6 inches, respectively, we then have

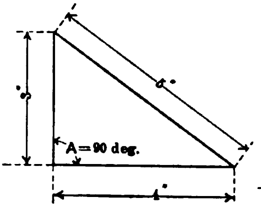


Fig. 37.

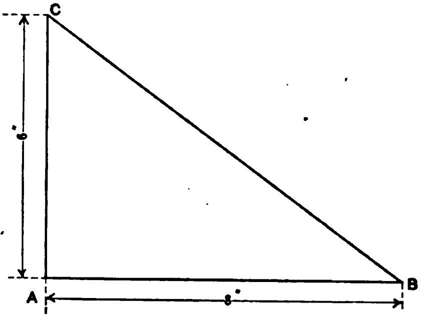


Fig. 38.

Machinery, N.Y.

$$8^2 + 6^2 = 10^2, \text{ or } 64 + 36 = 100.$$
$$10^2 - 8^2 = 6^2, \text{ or } 100 - 64 = 36.$$
$$10^2 - 6^2 = 8^2, \text{ or } 100 - 36 = 64.$$

As general formulas we may write this,

$$AB^2 + AC^2 = BC^2, \text{ or } BC = \sqrt{AB^2 + AC^2}$$
$$BC^2 - AB^2 = AC^2, \text{ or } AC = \sqrt{BC^2 - AB^2}$$
$$BC^2 - AC^2 = AB^2, \text{ or } AB = \sqrt{BC^2 - AC^2}$$

These last formulas say, that no matter what the actual numerical values of the sides *AB*, *AC*, and *BC*, the relationship between their squares, as expressed by the formulas, holds true for any right angle triangle. This carries us directly in on the subject of the use of formulas for expressing mathematical and engineering facts and data.

CHAPTER X.

USE OF FORMULAS.

As we have said before, if the three sides in a right angle triangle are 10, 8, and 6 inches long, respectively, we know that

$$10^2 = 8^2 + 6^2.$$

But if we write the formula in this manner, although it may be true for this particular triangle, there is no indication of that the same relationship holds good for any right angle triangle. If, however, we substitute the numbers giving the lengths of the sides by the letters a, b, and c, we have

$$a^2 = b^2 + c^2,$$

and this formula expresses the rule given for any right angle triangle, where the length of the side opposite the right angle is a, and the length of the sides including the right angle b and c. *The letters simply stand in place of the figures that would be applied in each particular case.*

Formulas serve the purpose of expressing shortly and precisely some general law or relationship in calculations. Each letter stands for a certain quantity, and when we figure any special case we put the figures for this case into our formula, and figure as usual. Some examples will make this clear.

If we return to the question of figuring tapers, we have on page 6 the formula

$$\text{taper per foot} = \frac{\text{large dia.} - \text{small dia.}}{\text{length of work}} \times 12.$$

Now this formula can be expressed far more simply by putting letters in place of the various quantities given. Let, for instance,

$t =$ taper per foot,
$a =$ large diameter of work,
$b =$ small diameter of work, and
$l =$ length of work.

Then our formula would take the form:

$$t = \frac{a - b}{l} \times 12.$$

If we apply this formula to the example shown in Fig. 9, we have directly,

$$t = \frac{1\frac{1}{4} - 1\frac{1}{64}}{7\frac{1}{2}} \times 12 = \frac{\frac{15}{64}}{7\frac{1}{2}} \times 12 = \left(\frac{15}{64} \div 7\frac{1}{2}\right) \times 12 = \frac{15}{64} \times \frac{2}{15} \times 12 = \frac{1}{32} \times 12 = \frac{3}{8}.$$

As t stands for the taper per foot, this, consequently equals ⅜ inch. It is seen that the dimensions for the large and small ends of the work, the length, etc., are simply put *in place of the letters* in the formula, and then simple arithmetic is applied. Formulas are very easy to use if this is understood. It is only necessary to know exactly what quantity each letter stands for, and then put the numerical value of that quantity into the formula.

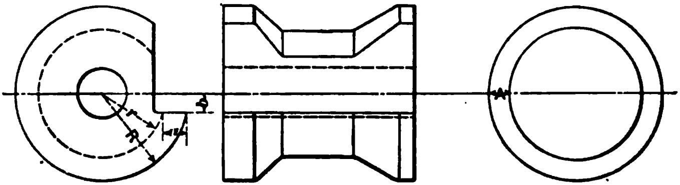
The formula for finding the number of teeth in the change gears for a lathe may, for instance, be written,

$$\frac{l \times a}{c \times a} = \frac{T}{t},$$

in which formula

l = number of threads per inch on lead-screw,
c = number of threads to be cut,
a = the number by which numerator and denominator is multiplied to find the number of teeth in change gears,
T = number of teeth in spindle stud gear, and
t = number of teeth in lead-screw gear.

Suppose now that we want to cut 16 threads per inch in a lathe having a lead-screw with 6 threads per inch, and let the number with which we multiply numerator and denominator be 6. Then, by putting these figures in place of the letters in the formula we have:



Machinery,N.Y.

Fig. 39.

$$\frac{6 \times 6}{16 \times 6} = \frac{36}{96} = \frac{T}{t}.$$

Consequently T, or the number of teeth in the spindle stud gear, is 36, and t, or the number of teeth in the lead-screw gear, is 96.

A useful application both of the use of formulas and of the square and square root of numbers, is found in the problems occurring when figuring forming tools.

Formulas for Circular Forming Tools.

When laying out circular forming tools, such as shown in Fig. 39, the cutting edge, as is well known, must be located a certain amount below the horizontal center line of the tool, in order to provide for sufficient clearance for the cut. On account of this, the actual differences of diameters in the piece of work to be formed cannot be directly copied in the forming tool. The distance A in the piece to be formed must equal the distance a on the forming tool, but as this latter distance is measured in a plane a certain distance b below the horizontal plane through the center of the forming tool, it is evident that the differences of diameters in the tool and the piece to be formed are not the same. A general formula may, however, be deduced, by use of elementary geometry, by means of which the various diameters of the forming tool may be determined if the largest (or smallest) diameter of the tool, the amount that the cutting edge is below the center, and, of course, the diameters of the piece to be formed, are known.

If R = the largest radius of the tool,
a = difference in radii of steps in the work, and
b = amount cutting edge is below center,
then, if r be the radius looked for,

$$r = \sqrt{\left(\sqrt{R^2 - b^2} - a\right)^2 + b^2}$$

If the smaller radius r is given and the larger radius R sought, the formula takes the form:

$$R = \sqrt{\left(\sqrt{r^2 - b^2} + a\right)^2 + b^2}$$

Suppose, for an example, that a tool is to be made to form the piece in Fig. 40. Assume that the largest diameter of the tool is to be 3 inches, and that the cutting edge is to be ¼ inch below the center of the tool. Then the diameter next smaller to 3 inches is found from the formulas given by inserting the given values: R = 1½ inch, b = ¼ inch, and a = ¼ inch (half the difference between 4 and 3½ inches; see Fig. 40).

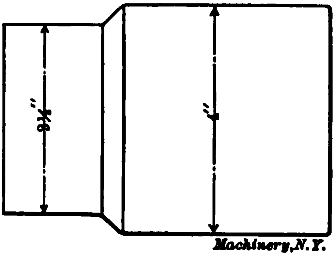


Fig. 40.

Then

$$r = \sqrt{\left(\sqrt{(1\tfrac{1}{2})^2 - (\tfrac{1}{4})^2} - \tfrac{1}{4}\right)^2 + (\tfrac{1}{4})^2} = \sqrt{\left(\sqrt{\tfrac{35}{16}} - \tfrac{1}{4}\right)^2 + \tfrac{1}{16}} = \frac{5.017}{4}$$

= 1.254 inch.

While the formula looks complicated, by means of a table of squares the calculations are easily simplified and can be carried out in three or four minutes. The value of r being 1.254 inch, the diameter to make the smaller step of the forming tool will be 2.508 inches, instead of 2½ inches exact, as would have been the case if the cutting edge had been on the center line.

A parenthesis ( ) used in a formula indicates that all the operations of figuring inside of the parenthesis are to be performed first, before we go further. Thus, for instance,

$$\left(\sqrt{\frac{25}{4}} + \frac{1}{2}\right)^2 + 3 = \left(\frac{5}{2} + \frac{1}{2}\right)^2 + 3 = \left(\ 3\ \right)^2 + 3 = 9 + 3 = 12.$$

It is common in formulas to leave out the multiplication sign (×)

between two letters and write them together without any sign at all. Thus ab means $a \times b$, and the expression

$$\frac{2APLN}{33{,}000} = \frac{2 \times A \times P \times L \times N}{33{,}000}$$

Using Decimal Equivalents Instead of Common Fractions.

It seems to be the general impression that it is easier and quicker to use the decimal equivalent of a fraction, instead of the fraction itself, when multiplying or dividing. There are very few cases, however, where the calculation can be made simpler by this substitution, and the results obtained are invariably less correct, because all the decimals which are necessary to correctly express the value of the fraction are, as a rule, not used, and when multiplying, an original error of only one-half thousandth inch may finally cause serious errors in close work.

An example will plainly illustrate this assertion. In a certain problem the formula reads:

$$R = \frac{a^2 + (b - c)\,b}{2\,(a - \frac{1}{2}c)}.$$

Now suppose that $a = 11/16$, $b = 1\frac{1}{2}$, and $c = \frac{3}{4}$. Then, if the formula is written with decimal equivalents substituted for the letters, and these equivalents are given to the thousandth of an inch, as is most common, the formula would read,

$$R = \frac{0.687^2 + (1.5 - 0.75)\,1.5}{2\,(0.687 - 0.375)}$$

Proceeding we find

$$R = \frac{0.472 + 1.125}{0.624} = \frac{1.597}{0.624} = 2.559.$$

If instead of using decimal equivalents for the fraction originally given in the problem we use the fractions themselves, we would write

$$R = \frac{\left(\frac{11}{16}\right)^2 + \left(1\frac{1}{2} - \frac{3}{4}\right)1\frac{1}{2}}{2\left(\frac{11}{16} - \frac{3}{8}\right)}$$

Simplifying this expression we find

$$R = \frac{\frac{121}{256} + 1\frac{1}{8}}{\frac{5}{8}} = \frac{\frac{121}{32} + 9}{5} = \frac{409}{160} = 2.556.$$

We notice in the first place that the denominator 0.624 ought to have been 0.625 or ⅝, and further, the final result shows a difference of 0.003 inch, which is enough to spoil many a job which may not even be required to be of extreme accuracy. This error is all due to the seemingly small original error of writing 0.687 instead of 0.6875.

CHAPTER XI.

USE OF TABLES OF SINES, COSINES, TANGENTS AND COTANGENTS.

The figuring of angles the average machinist usually looks upon as something above his capacity, and regards the knowledge of this mathematical process with more or less awe. But simple cases of the figuring of angles from given formulas are very easy, indeed; in fact, these cases are often much simpler than many ordinary arithmetical problems in the shop which the machinist tackles with success. All that is necessary is a table of sines, cosines, tangents, and cotangents; after having found the figures corresponding to a given angle from the table, the whole thing resolves itself to a case of simple multiplication or division.

Often, in technical papers, the reader will find himself confronted by such formulas as, for instance,

$$A = \frac{27}{\cos 36 \text{ deg.}}.$$

Of course, it is impossible to figure out how much A is from this formula, unless the expression "cos 36 deg." (read: cosine of 36 degrees) can be transformed and expressed in plain figures. But if we know how much "cos 36 deg." is expressed in plain figures, then we can immediately divide 27 by this value, and thus find the value of A. Suppose that A stands for the length of one side in a triangle, and that the expression "cos 36 deg." equals 0.80901. Then,

$$A = \frac{27}{0.80901} = 33.37.$$

The tables of sines, cosines, tangents, and cotangents simply serve the purpose of giving in figures the values of these expressions for different angles. The angles are given in degrees and minutes. A right angle is 90 degrees (90°), and each degree is further subdivided into 60 minutes (60′). The four expressions: sine, cosine, tangent, and cotangent, which are used to designate certain numerical values, to be found from the tables, are called the *functions of the angle.* These functions or numerical values equal a definite amount for each different angle. On pages 48, 49, 50, and 51 will be found tables giving the values referred to for all degrees and for every ten minutes (1/6 of a degree). The four expressions sine, cosine, tangent, and cotangent are abbreviated "sin," "cos," "tan," and "cot," respectively.

The tables of sines, cosines, etc., are read the same as a railroad time-table. It will be noticed that at the top of the tables on pages 48 and 49 the heading reads "Table of Sines," and at the bottom of the same tables it says "Table of Cosines." At the top of the two

TABLE OF SINES.

Read degrees in left-hand column and minutes at top.
Example: sin 7° 10′ = .12475.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00291	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02326	.02617	.02908	.03199	.03489	88
2	.03489	.03780	.04071	.04361	.04652	.04943	.05233	87
3	.05233	.05524	.05814	.06104	.06395	.06685	.06975	86
4	.06975	.07265	.07555	.07845	.08135	.08425	.08715	85
5	.08715	.09005	.09295	.09584	.09874	.10163	.10452	84
6	.10452	.10742	.11031	.11320	.11609	.11898	.12186	83
7	.12186	.12475	.12764	.13052	.13341	.13629	.13917	82
8	.13917	.14205	.14493	.14780	.15068	.15356	.15643	81
9	.15643	.15930	.16217	.16504	.16791	.17078	.17364	80
10	.17364	.17651	.17937	.18223	.18509	.18795	.19080	79
11	.19080	.19366	.19651	.19936	.20221	.20506	.20791	78
12	.20791	.21075	.21359	.21644	.21927	.22211	.22495	77
13	.22495	.22778	.23061	.23344	.23627	.23909	.24192	76
14	.24192	.24474	.24756	.25038	.25319	.25600	.25881	75
15	.25881	.26162	.26443	.26723	.27004	.27284	.27563	74
16	.27563	.27843	.28122	.28401	.28680	.28958	.29237	73
17	.29237	.29515	.29793	.30070	.30347	.30624	.30901	72
18	.30901	.31178	.31454	.31730	.32006	.32281	.32556	71
19	.32556	.32831	.33106	.33380	.33654	.33928	.34202	70
20	.34202	.34475	.34748	.35020	.35293	.35565	.35836	69
21	.35836	.36108	.36379	.36650	.36920	.37190	.37460	68
22	.37460	.37730	.37999	.38268	.38536	.38805	.39073	67
23	.39073	.39340	.39607	.39874	.40141	.40407	.40673	66
24	.40673	.40939	.41204	.41469	.41733	.41998	.42261	65
25	.42261	.42525	.42788	.43051	.43313	.43575	.43837	64
26	.43837	.44098	.44359	.44619	.44879	.45139	.45399	63
27	.45399	.45658	.45916	.46174	.46432	.46690	.46947	62
28	.46947	.47203	.47460	.47715	.47971	.48226	.48481	61
29	.48481	.48735	.48989	.49242	.49495	.49747	.50000	60
30	.50000	.50251	.50503	.50753	.51004	.51254	.51503	59
31	.51503	.51752	.52001	.52249	.52497	.52745	.52991	58
32	.52991	.53238	.53484	.53730	.53975	.54219	.54463	57
33	.54463	.54707	.54950	.55193	.55436	.55677	.55919	56
34	.55919	.56160	.56400	.56640	.56880	.57119	.57357	55
35	.57357	.57595	.57833	.58070	.58306	.58542	.58778	54
36	.58778	.59013	.59248	.59482	.59715	.59948	.60181	53
37	.60181	.60413	.60645	.60876	.61106	.61336	.61566	52
38	.61566	.61795	.62023	.62251	.62478	.62705	.62932	51
39	.62932	.63157	.63383	.63607	.63832	.64055	.64278	50
40	.64278	.64501	.64723	.64944	.65165	.65386	.65605	49
41	.65605	.65825	.66043	.66262	.66479	.66696	.66913	48
42	.66913	.67128	.67344	.67559	.67773	.67986	.68199	47
43	.68199	.68412	.68624	.68835	.69046	.69256	.69465	46
44	.69465	.69674	.69883	.70090	.70298	.70504	.70710	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COSINES.

Read degrees in right-hand column and minutes at bottom.
Example: cos 56° 20′ = .55436.

TABLE OF SINES.

Read degrees in left-hand column and minutes at top.
Example: sin 56° 20 = .83227.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	.70710	.70916	.71120	.71325	.71528	.71731	.71934	44
46	.71934	.72135	.72336	.72537	.72737	.72936	.73135	43
47	.73135	.73333	.73530	.73727	.73923	.74119	.74314	42
48	.74314	.74508	.74702	.74895	.75088	.75279	.75471	41
49	.75471	.75661	.75851	.76040	.76229	.76417	.76604	40
50	.76604	.76791	.76977	.77102	.77347	.77531	.77714	39
51	.77714	.77897	.78079	.78260	.78441	.78621	.78801	38
52	.78801	.78979	.79157	.79335	.79512	.79688	.79863	37
53	.79863	.80038	.80212	.80385	.80558	.80730	.80901	36
54	.80901	.81072	.81242	.81411	.81580	.81748	.81915	35
55	.81915	.82081	.82247	.82412	.82577	.82740	.82903	34
56	.82903	.83066	.83227	.83388	.83548	.83708	.83867	33
57	.83867	.84025	.84182	.84339	.84495	.84650	.84804	32
58	.84804	.84958	.85111	.85264	.85415	.85566	.85716	31
59	.85716	.85866	.86014	.86162	.86310	.86456	.86602	30
60	.86602	.86747	.86892	.87035	.87178	.87320	.87462	29
61	.87462	.87602	.87742	.87881	.88020	.88157	.88294	28
62	.88294	.88430	.88566	.88701	.88835	.88968	.89100	27
63	.89100	.89232	.89363	.89493	.89622	.89751	.89879	26
64	.89879	.90006	.90132	.90258	.90383	.90507	.90630	25
65	.90630	.90753	.90875	.90996	.91116	.91235	.91354	24
66	.91354	.91472	.91589	.91706	.91821	.91936	.92050	23
67	.92050	.92163	.92276	.92388	.92498	.92609	.92718	22
68	.92718	.92827	.92934	.93041	.93148	.93253	.93358	21
69	.93358	.93461	.93565	.93667	.93768	.93869	.93969	20
70	.93969	.94068	.94166	.94264	.94360	.94456	.94551	19
71	.94551	.94646	.94739	.94832	.94924	.95015	.95105	18
72	.95105	.95195	.95283	.95371	.95458	.95545	.95630	17
73	.95630	.95715	.95799	.95882	.95964	.96045	.96126	16
74	.96126	.96205	.96284	.96363	.96440	.96516	.96592	15
75	.96592	.96667	.96741	.96814	.96887	.96958	.97029	14
76	.97029	.97099	.97168	.97237	.97304	.97371	.97437	13
77	.97437	.97502	.97566	.97629	.97692	.97753	.97814	12
78	.97814	.97874	.97934	.97992	.98050	.98106	.98162	11
79	.98162	.98217	.98272	.98325	.98378	.98429	.98480	10
80	.98480	.98530	.98580	.98628	.98676	.98722	.98768	9
81	.98768	.98813	.98858	.98901	.98944	.98985	.99026	8
82	.99026	.99066	.99106	.99144	.99182	.99218	.99254	7
83	.99254	.99289	.99323	.99357	.99389	.99421	.99452	6
84	.99452	.99482	.99511	.99539	.99567	.99593	.99619	5
85	.99619	.99644	.99668	.99691	.99714	.99735	.99756	4
86	.99756	.99776	.99795	99813	.99830	.99847	.99863	3
87	.99863	.99877	.99891	.99904	.99917	.99928	.99939	2
88	.99939	.99948	.99957	.99965	.99972	.99979	.99984	1
89	.99984	.99989	.99993	.999[illegible]3	.99998	.99999	1.0000	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COSINES.

Read degrees in right-hand column and minutes at bottom.
Example: cos 7° 10′ = .99218.

TABLE OF TANGENTS.

Read degrees in left-hand column and minutes at top.
Example: tan 7° 10′ = .12573.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	.00000	.00290	.00581	.00872	.01163	.01454	.01745	89
1	.01745	.02036	.02327	.02618	.02909	.03200	.03492	88
2	.03492	.03783	.04074	.04366	.04657	.04949	.05240	87
3	.05240	.05532	.05824	.06116	.06408	.06700	.06992	86
4	.06992	.07285	.07577	.07870	.08162	.08455	.08748	85
5	.08748	.09042	.09335	.09628	.09922	.10216	.10510	84
6	.10510	.10804	.11099	.11393	.11688	.11983	.12278	83
7	.12278	.12573	.12869	.13165	.13461	.13757	.14054	82
8	.14054	.14350	.14647	.14945	.15242	.15540	.15838	81
9	.15838	.16136	.16435	.16734	.17033	.17332	.17632	80
10	.17632	.17932	.18233	.18533	.18834	.19136	.19438	79
11	.19438	.19740	.20042	.20345	.20648	.20951	.21255	78
12	.21255	.21559	.21864	.22169	.22474	.22780	.23086	77
13	.23086	.23393	.23700	.24007	.24315	.24624	.24932	76
14	.24932	.25242	.25551	.25861	.26172	.26483	.26794	75
15	.26794	.27106	.27419	.27732	.28046	.28360	.28674	74
16	.28674	.28989	.29305	.29621	.29938	.30255	.30573	73
17	.30573	.30891	.31210	.31529	.31850	.32170	.32492	72
18	.32492	.32813	.33136	.33459	.33783	.34107	.34432	71
19	.34432	.34758	.35084	.35411	.35739	.36067	.36397	70
20	.36397	.36726	.37057	.37388	.37720	.38053	.38386	69
21	.38386	.38720	.39055	.39391	.39727	.40064	.40402	68
22	.40402	.40741	.41080	.41421	.41762	.42104	.42447	67
23	.42447	.42791	.43135	.43481	.43827	.44174	.44522	66
24	.44522	.44871	.45221	.45572	.45924	.46277	.46630	65
25	.46630	.46985	.47341	.47697	.48055	.48413	.48773	64
26	.48773	.49133	.49495	.49858	.50221	.50586	.50952	63
27	.50952	.51319	.51687	.52056	.52427	.52798	.53170	62
28	.53170	.53544	.53919	.54295	.54672	.55051	.55430	61
29	.55430	.55811	.56193	.56577	.56961	.57347	.57735	60
30	.57735	.58123	.58513	.58904	.59297	.59690	.60086	59
31	.60086	.60482	.60880	.61280	.61680	.62083	.62486	58
32	.62486	.62892	.63298	.63707	.64116	.64528	.64940	57
33	.64940	.65355	.65771	.66188	.66607	.67028	.67450	56
34	.67450	.67874	.68300	.68728	.69157	.69588	.70020	55
35	.70020	.70455	.70891	.71329	.71769	.72210	.72654	54
36	.72654	.73099	.73546	.73996	.74447	.74900	.75355	53
37	.75355	.75812	.76271	.76732	.77195	.77661	.78128	52
38	.78128	.78598	.79069	.79543	.80019	.80497	80978	51
39	.80978	.81461	.81946	.82433	.82923	.83415	.83910	50
40	.83910	.84406	.84906	.85408	.85912	.86419	.86928	49
41	.86928	.87440	.87955	.88472	.88992	.89515	.90040	48
42	.90040	.90568	.91099	.91633	.92169	.92709	.93251	47
43	.93251	.93796	.94345	.94896	.95450	.96008	.96568	46
44	.96568	.97132	.97699	.98269	.98843	.99419	1.0000	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COTANGENTS.

Read degrees in right-hand column and minutes at bottom.
Example: cot 56° 20′ = .66607.

TABLE OF TANGENTS.

Read degrees in left-hand column and minutes at top.
Example: tan 56° 20′ = 1.5013.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	1.0000	1.0058	1.0117	1.0176	1.0235	1.0295	1.0355	44
46	1.0355	1.0415	1.0476	1.0537	1.0599	1.0661	1.0723	43
47	1.0723	1.0786	1.0849	1.0913	1.0977	1.1041	1.1106	42
48	1.1106	1.1171	1.1236	1.1302	1.1369	1.1436	1.1503	41
49	1.1503	1.1571	1.1639	1.1708	1.1777	1.1847	1.1917	40
50	1.1917	1.1988	1.2059	1.2131	1.2203	1.2275	1 2349	39
51	1.2349	1.2422	1.2496	1.2571	1.2647	1.2723	1.2799	38
52	1.2799	1.2876	1.2954	1.3032	1.3111	1.3190	1.3270	37
53	1.3270	1.3351	1.3432	1.3514	1.3596	1.3680	1.3763	36
54	1.3763	1.3848	1.3933	1.4019	1.4106	1.4193	1.4281	35
55	1.4281	1.4370	1.4459	1.4550	1.4641	1.4733	1.4825	34
56	1.4825	1.4919	1.5013	1.5108	1.5204	1.5301	1.5398	33
57	1.5398	1.5497	1.5596	1.5696	1.5798	1.5900	1.6003	32
58	1.6003	1.6107	1.6212	1.6318	1.6425	1.6533	1.6642	31
59	1.6642	1.6753	1.6864	1.6976	1.7090	1.7204	1.7320	30
60	1.7320	1.7437	1.7555	1.7674	1.7795	1.7917	1.8040	29
61	1.8040	1.8164	1.8290	1.8417	1.8546	1.8676	1.8807	28
62	1.8807	1.8940	1.9074	1.9209	1.9347	1.9485	1.9626	27
63	1.9626	1.9768	1.9911	2.0056	2.0203	2.0352	2.0503	26
64	2.0503	2.0655	2.0809	2.0965	2.1123	2.1283	2.1445	25
65	2.1445	2.1609	2.1774	2.1943	2.2113	2.2285	2.2460	24
66	2.2460	2.2637	2.2816	2.2998	2.3182	2.3369	2.3558	23
67	2.3558	2.3750	2.3944	2.4142	2.4342	2.4545	2.4750	22
68	2.4750	2.4959	2.5171	2.5386	2.5604	2.5826	2.6050	21
69	2.6050	2.6279	2.6510	2.6746	2.6985	2.7228	2.7474	20
70	2.7474	2.7725	2.7980	2.8239	2.8502	2.8770	2.9042	19
71	2.9042	2.9318	2.9600	2.9886	3.0178	3.0474	3.0776	18
72	3.0776	3.1084	3.1397	3.1715	3.2040	3.2371	3.2708	17
73	3.2708	3.3052	3.3402	3.3759	3.4123	3.4495	3.4874	16
74	3.4874	3.5260	3.5655	3.6058	3.6470	3.6890	3.7320	15
75	3.7320	3.7759	3.8208	3.8667	3.9136	3.9616	4.0107	14
76	4.0107	4.0610	4.1125	4.1653	4.2193	4.2747	4.3314	13
77	4.3314	4.3896	4.4494	4.5107	4.5736	4.6382	4.7046	12
78	4.7046	4.7728	4.8430	4.9151	4.9894	5.0658	5.1445	11
79	5.1445	5.2256	5.3092	5.3955	5.4845	5.5763	5.6712	10
80	5.6712	5.7693	5.8708	5.9757	6.0844	6.1970	6.3137	9
81	6.3137	6.4348	6.5605	6.6911	6.8269	6.9682	7.1153	8
82	7.1153	7.2687	7.4287	7.5957	7.7703	7.9530	8.1443	7
83	8.1443	8.3449	8.5555	8.7768	9.0098	9.2553	9.5143	6
84	9.5143	9.7881	10.078	10.385	10.711	11.059	11.430	5
85	11.430	11.826	12.250	12.706	13.196	13.726	14.300	4
86	14.300	14.924	15.604	16.349	17.169	18.075	19.081	3
87	19.081	20.205	21.470	22.904	24.541	26.431	28.636	2
88	28.636	31.241	34.367	38.188	42.964	49.103	57.290	1
89	57.290	68.750	85.939	114.58	171.88	343.77	∞	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

TABLE OF COTANGENTS.

Read degrees in right-hand column and minutes at bottom.
Example: cot 7° 10′ = 7.9530.

following pages the heading reads "Table of Tangents," and at the bottom of these pages is the legend "Table of Cotangents." At the top of the tables themselves we find that the heading of the extreme left-hand column reads "Deg." and then the following columns are headed 0′, 10′, 20′, and so forth. If we look at the bottom of the pages, we find the same legends *under* the columns, but reading from the extreme right-hand column to the left. The purpose of this will be immediately explained.

When we wish to find the sine or tangent for a given angle, we first find the number of degrees in the extreme left-hand column of the respective tables, and then the number of minutes at the top of the columns. We then follow the column over which the number of minutes is given downward until we come to the figure in line with the given number of degrees. This figure is then the numerical value which we call the sine or the tangent, as the case may be, for the given angle. If the angle is given in even degrees, with no minutes given, the corresponding value will be found opposite the number of the degrees in the column marked 0′ (the sign ′ indicates minutes, just as the sign ° indicates degrees).

The cosines and tangents of angles are found in the same tables as the sines and tangents, but when we want these quantities we read the tables *from the bottom up.* The number of degrees we find in the extreme right-hand column, and the number of minutes at the bottom of the columns. This is the reason that we read the legends "Cosines" and "Cotangents" at the bottom of the pages, because these functions are read from below and up. If the number of minutes given should not be an even number, as 10′, 20′, 30′, etc., but 26′, for instance, it is, for nearly all calculations in the shop, near enough to take the figures given for the nearest given number of minutes, being in this case, then, for 30 minutes.

If we now return to our formula

$$A = \frac{27}{\cos 36 \text{ deg.}},$$

we find in the table on page 49, opposite 36 degrees in the right-hand column and in the column marked 0′ at the bottom, the figures 0.80901. This is the cosine for 36 degrees, and should be placed in the formula instead of the expression "cos 36 deg." We have, then, as we have already shown,

$$A = \frac{27}{0.80901},$$

which we can calculate by simple division.

Suppose the angle had been 36 degrees 20 minutes, instead of 36 degrees even. We then would have read the figures in the column marked 20′ at the bottom, and opposite 36 degrees we would have found 0.80558. A few more examples will tend to make the use of these tables clearer.

In a right angle triangle, as shown in Fig. 41, the side *BC*, which is opposite the right angle, equals the side *AB* divided by the cosine of the angle β, included between these two sides. Expressed as a formula this rule would be:

$$BC = \frac{AB}{\cos \beta}.$$

In a particular case let us assume that AB equals 6 inches and the angle β is 37 degrees. How long is the side BC?

According to our formula, $BC = \frac{6}{\cos 37 \text{ deg.}}$. The cosine for 37 degrees we find from the table to be 0.79863. Then $BC = \frac{6}{0.79863} = 7.51$ inches.

In order to become familiar with the use of the tables, let us find the sine for 56 degrees 20 minutes, or, as it is written in formulas, sin 56° 20′. The "sines" are found by reading the tables from the top. Find 56 in the tables of sines in the left-hand column and read off the value 0.83227 in the column marked 20′. This is the sine 56° 20′.

Find the tangent for 56 degrees 20 minutes, or, as it is written, tan 56° 20′. Exactly in the same way as we found the sine, we find the tangent in the tables headed "Tangents" to be 1.5013.

The cotangent for the same angle (cot 56° 20′) is found by reading the table from the bottom, first finding 56 in the right-hand column, and the required value in line with this figure 56 in the column marked 20′ at the bottom. The cotangent of 56 degrees 20 minutes we thus find to be 0.66607.

These tables are constantly used when figuring triangles. In every triangle, if we know the length of *all the three sides, or of two sides and the measure in degrees of one angle*, or of *one side and two angles*, we can figure the other sides or angles by means of certain formulas into which these angular functions enter.

Suppose, for instance, that we call the three sides in a right angle triangle a, b, and c, as shown in Fig. 42, and the angles opposite those sides A, B, and C. The angle A, of course, is a right or 90-degree angle. Then, for all right angle triangles these formulas hold true:

$$a = \frac{b}{\cos C}; \qquad a = \frac{b}{\sin B};$$

$$a = \frac{c}{\cos B}; \qquad a = \frac{c}{\sin C};$$

$$b = a \cos C; \qquad b = a \sin B;$$

$$b = c \tan B; \qquad b = c \cot C;$$

$$c = a \cos B; \qquad c = a \sin C;$$

$$c = b \tan C; \qquad c = b \cot B.$$

It will be remembered that expressions such as $c \cot C$ mean simply $c \times \cot C$.

By means of the formulas given above, and a table of sines, cosines, etc., either of the sides in a right angle triangle may be found when one side and one angle, besides the 90-degree angle, are known. If two sides are known, but no angle outside of the 90-degree angle, the third side may be found from the formulas on page 42, and the angles found from the formulas:

$$\sin B = \frac{b}{a}; \qquad \cos B = \frac{c}{a};$$

$$\tan B = \frac{b}{c}; \qquad \cot B = \frac{c}{b};$$

$$\sin C = \frac{c}{a}; \qquad \cos C = \frac{b}{a};$$

$$\tan C = \frac{c}{b}; \qquad \cot C = \frac{b}{c}.$$

Thus, suppose that in Fig. 42 $b = 4$ inches and $c = 3$ inches.

Then $a = \sqrt{(4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$ inches.

To find angle b, write

$$\sin B = \frac{b}{a} = \frac{4}{5} = 0.800,$$

and from our table of sines, finding 0.800 in the column marked 10′ at the top of the page, we find that the angle B is 53 degrees 10 minutes, very closely. Angle C is found from formula

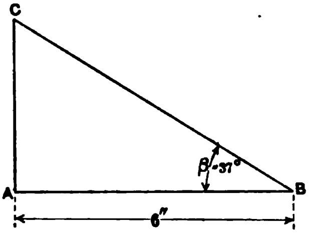
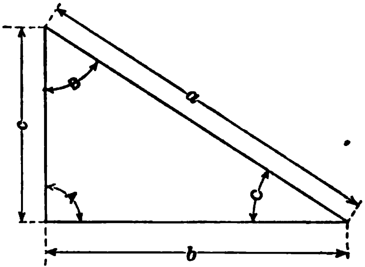


Fig. 41.



Machinery, N.Y.

Fig. 42.

$$\cos C = \frac{b}{a} = \frac{4}{5} = 0.800.$$

This angle is then 36 degrees 50 minutes as read from below in the tables of cosines at the bottom.

For the finding of any angle or side in *any* triangle when the requisite number of sides or angles are given, the chart on page 55, and the tables of angular functions are used. The whole process consists merely in finding the corresponding value of the function of the angle in the table, insert it into the formula, and figure as if the whole were simple arithmetic. This simple process of figuring angles embodies the principles of what is called, in mathematics, *Trigonometry*. But complicated names do not, as we have seen, necessarily imply complicated processes of figuring.

We will figure an example to show the use of the chart on page 55.

CHART FOR SOLUTION OF TRIANGLES.*

Parts. Given	Parts to be found.					
	$a=$	$b=$	$c=$	$\angle A$	$\angle B$	$\angle C$
$a-b-c$				$\frac{b^2+c^2-a^2}{2bc}=\cos A$	$\frac{a^2+c^2-b^2}{2ac}=\cos B$	$\frac{a^2+b^2-c^2}{2ab}=\cos C$
$b-c-\angle A$	$\sqrt{b^2+c^2-2bc\cos A}$				$\frac{b\sin A}{c-b\cos A}=\tan B$	$\frac{c\sin A}{b-c\cos A}=\tan C$
$a-c-\angle B$		$\sqrt{a^2+c^2-2ac\cos B}$		$\frac{a\sin B}{c-a\cos B}=\tan A$		$\frac{c\sin B}{a-c\cos B}=\tan C$
$a-b-\angle C$			$\sqrt{a^2+b^2-2ab\cos C}$	$\frac{a\sin C}{b-a\cos C}=\tan A$	$\frac{b\sin C}{a-b\cos C}=\tan B$	
$a-b-\angle A$			$\frac{a\sin C}{\sin A}$		$\frac{b\sin A}{a}=\sin B$	180°−(A+B)
$a-b-\angle B$			$\frac{b\sin C}{\sin B}$	$\frac{a\sin B}{b}=\sin A$		180°−(A+B)
$a-c-\angle A$		$\frac{a\sin B}{\sin A}$			180°−(A+C)	$\frac{c\sin A}{a}=\sin C$
$a-c-\angle C$		$\frac{c\sin B}{\sin C}$		$\frac{a\sin C}{c}=\sin A$	180°−(A+C)	
$b-c-\angle B$	$\frac{b\sin A}{\sin B}$			180°−(B+C)		$\frac{c\sin B}{b}=\sin C$
$b-c-\angle C$	$\frac{c\sin A}{\sin C}$			180°−(B+C)	$\frac{b\sin C}{c}=\sin B$	
$a-\angle A-\angle B$		$\frac{a\sin B}{\sin A}$	$\frac{a\sin C}{\sin A}$			180°−(A+B)
$a-\angle A-\angle C$		$\frac{a\sin B}{\sin A}$	$\frac{a\sin C}{\sin A}$		180°−(A+C)	
$a-\angle B-\angle C$		$\frac{a\sin B}{\sin A}$	$\frac{a\sin C}{\sin A}$	180°−(B+C)		
$b-\angle A-\angle B$	$\frac{b\sin A}{\sin B}$		$\frac{b\sin C}{\sin B}$			180°−(A+B)
$b-\angle A-\angle C$	$\frac{b\sin A}{\sin B}$		$\frac{b\sin C}{\sin B}$		180°−(A+C)	
$b-\angle B-\angle C$	$\frac{b\sin A}{\sin B}$		$\frac{b\sin C}{\sin B}$	180°−(B+C)		
$c-\angle A-\angle B$	$\frac{c\sin A}{\sin C}$	$\frac{c\sin B}{\sin C}$				180°−(A+B)
$c-\angle A-\angle C$	$\frac{c\sin A}{\sin C}$	$\frac{c\sin B}{\sin C}$			180°−(A+C)	
$c-\angle B-\angle C$	$\frac{c\sin A}{\sin C}$	$\frac{c\sin B}{\sin C}$		180°−(B+C)		

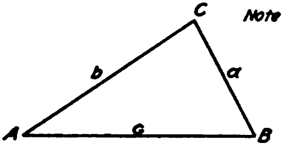


Fig. 1.

Note 1:– By means of the table any part of an oblique triangle may be found when any three other parts are given, with the following exception:

Given two sides and the angle opposite one of them; then, if the side opposite is less than the adjacent × the sine of the angle, the triangle is impossible; or if the side opposite = the adjacent × the sine of the angle, the triangle is a right triangle; or if the side opposite is less than the adjacent but does not come under the above, the triangle is capable of two solutions and can be drawn as in Fig. 2 as well as in Fig. 1.

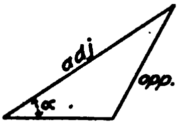


Fig. 2.

Note 2:– In some cases two steps are necessary to solve, as for example, having given sides a and b and angle A, to find c: The formula reads $c=\frac{a\sin C}{\sin A}$ but angle C must first be derived from $C=180°-(A+B)$, and the same applies to other angles in certain cases as is apparent above.

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Assume that in a triangle as shown in Fig. 43, one side c is known to be 6 inches long. The angle A equals 50 degrees and B is 68 degrees. How long are sides a and b, and how many degrees is angle C?

The parts given are side c and angles A and B. We, therefore, first find in our chart the line containing the formulas corresponding to our problem. We find that the extreme left-hand column is headed "Parts given," and following it down we finally find "$c — \angle A — \angle B$,' in which the sign $\angle$ simply stands for "angle." In line with this in the table we find the formulas for finding the sides a and b, and the angle C ($\angle C$), as follows:

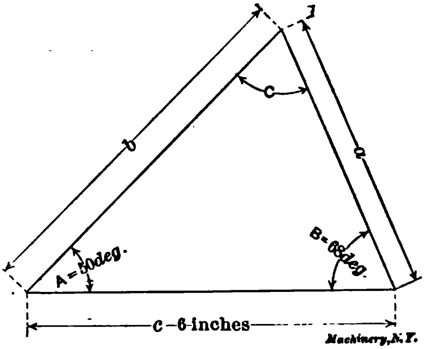

Fig. 43.

$$a = \frac{c \sin A}{\sin C}; \quad b = \frac{c \sin B}{\sin C}; \quad C = 180° - (A + B).$$

Substituting in these formulas the values for c, A, and B, we have:

$$a = \frac{6 \times \sin 50 \text{ deg.}}{\sin C}; \quad b = \frac{6 \times \sin 68 \text{ deg.}}{\sin C}.$$

In these formulas, however, we find that $\sin C$ is required. The angle C is found from the third formula given:

$$C = 180° - (50° + 68°) = 180° - 118° = 62 \text{ degrees.}$$

We have then,

$$a = \frac{6 \times \sin 50 \text{ deg.}}{\sin 62 \text{ deg.}}; \quad b = \frac{6 \times \sin 68 \text{ deg.}}{\sin 62 \text{ deg.}}.$$

We now find, from our tables of sines, the value of the expressions "sin 50 deg.," "sin 62 deg.," and so forth. These values we put in place of these expressions in our formulas, and obtain:

$$a = \frac{6 \times 0.76604}{0.88294} = 5.206,$$

$$b = \frac{6 \times 0.92718}{0.88294} = 6.301.$$

We have thus found that the sides a and b equal 5.206 and 6.301 inches, respectively, and the angle C, 62 degrees.

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No. 19

USE OF FORMULAS IN MECHANICS

CONTENTS

General Remarks on Self-Education, by C. F. BLAKE - 3
Mathematical Signs and Expressions, by C. F. BLAKE 5
Strength of Materials, by C. F. BLAKE - - - 9
Graphical Methods of Solving Problems, by C. F. BLAKE 19
Principle of Moments as Applied to the Lever - - 28
The Center of Gravity, by BENJ. F. LA RUE - - 39

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Principle of Moments as Applied to the Lever - 28

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INTRODUCTION.

GENERAL REMARKS ON SELF-EDUCATION.

There are several ways of obtaining an education: The easiest and, until recent years, the usual way is to begin at the age of seven and continue steadily at school till the age of twenty-four, at father's expense. It is a fortunate fact that education is by no means unattainable otherwise; indeed many of the greatest and most widely useful educations the world has known have been obtained almost without a look at the inside of a school. A second method, quite modern, is the correspondence school—most excellent in many respects, yet not completing the available ways of obtaining an education. The final method is that of self-education. The author has obtained a very large share of his education in this manner, and has, as well, assisted others to do the same, and it is to explain the possibilities of this method, and to plant the seed of self-help elsewhere, that this and the following chapters have been written. They are divided into four heads dealing with the following subjects:

1. Present introduction, explaining general methods to be followed.
2. Technical reading of journals, catalogues and formulas.
3. Strains set up in machine materials, and strength of materials to resist them.
4. Graphical methods in use in machine designing and technical writing.

It is the aim of these chapters to start the ambitious young man of sufficient grit upon a path which, if rightly followed, will in the future surely place him on par with those more fortunate men of his age who have enjoyed a college education, and to leave him in a position to continue to read and study and to understand the technical discussion and articles on design which appear in the technical press.

Engineering education does not consist in knowing things mechanical —very far from it. It consists only in knowing where to find technical literature upon any given subject when it is wanted, and knowing how to read it when it is found. Therefore, the first thing needed by our student is a place to store his newly acquired knowledge, aside from his head. The author's first attempt in this line resulted in a book having black canvas covers and a flexible back. Tapes were provided to lace in the leaves, which were made of fairly heavy cardboard, perforated for the tapes, and having a flexible strip along the perforated edge to enable the leaves to turn back properly. Twenty-six alphabet leaves were made similar to those in dictionaries and memorandum books, and a supply of extra leaves kept on hand.

Clippings from papers and catalogues were pasted on blank leaves and inserted under the proper letter, also notes and formulas received from others were written in, making the book a record of past work and study. The book, finally becoming too large to be convenient and too small to hold everything to be preserved, gave way to the card index and filing case.*

Having provided a systematic way to file our clippings, we are ready to consider the sources of the same. First subscribe for one or two of the leading technical journals devoted to your line of work. Make a practice of sending for catalogues of machinery manufacturers, and file them in the filing box. Many catalogues present, besides the goods manufactured, tables and data of value. If you can clip out these tables and file them in the card index without destroying the catalogue, do so; if not, make an entry in the card index to show where they may be found, before filing the catalogue. Always write your name in the catalogues, for as the file grows, you will find demands upon it from others, and this will aid in keeping the file intact. Remember that a catalogue received implies confidence on the part of the sender that it will eventually prove of use to him by bringing his goods before possible purchasers, and for this reason, as well as for your own convenience, all catalogues received should be listed and filed.

Duplicate clippings, such as tables, may often be exchanged with others, and thus our files are enlarged. This is not meant to encourage a mere mania for collecting—far from it. We should so study all data filed as to understand it at the time, and if found difficult, make such notes as will readily recall the study to our minds in the future.

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* See MACHINERY's Reference Series, No. 2, Drafting-Room Practice, page 44: Card Index for the Draftsman's Individual Records.

CHAPTER I.

MATHEMATICAL SIGNS AND EXPRESSIONS.

The first thing to be done in preparation for study, and for reading the technical papers, is to become familiar with the *engineering language*. The *spoken engineering language* is of course the native tongue of the country, with, however, plenty of new words to master; but the *written engineering language* consists very largely of symbols, so like those of higher mathematics in appearance as often to discourage the beginner from further efforts. In the *written engineering language*, rules, instead of being written in the native tongue, are expressed by combinations of these symbols, and when so expressed are called formulas.

Now, the mathematician, when deriving a formula, uses the same symbols as the engineer when writing a formula, and if we accept the work of the mathematician as correct, we need pay no attention to the use of these symbols in deriving formulas, but give our attention to learning to read the symbolic language of the engineer with sufficient ease to enable us to follow the operations called for by any formula we may wish to use.

The following table exhibits in the first column the symbols most frequently met with; in the second column the arithmetical equivalent of the symbols is given, assuming that $a=2$ and $b=4$; in the third column the symbols are expressed in English to give the proper method of reading the symbols.

TABLE 1. COMMON MATHEMATICAL SIGNS.

$a=2 \quad b=4$		a equals 2 b equals 4
$a+b=c$	$2+4=6$	a plus b equals c
$b-a=d$	$4-2=2$	b minus a equals d
$a\times b=e$ $a.b=e$ $a\,b=e$	$2\times 4=8$	a times b equals e, or ab equals e
$a\,(a+b)=f$	$2\times 6=12$	a times a plus b equals f
$b\div a=h$ $\frac{b}{a}=h$	$\frac{4}{2}=2$	b divided by a equals h, or b over a equals h
$a<b$	$2<4$	a is less than b
$b>a$	$4>2$	b is greater than a
$b:a::f:c$ $\frac{b}{a}=\frac{f}{c}$	$\frac{4}{2}=\frac{12}{6}$	b is to a as f is to c b divided by a equals f divided by c b over a equals f over c

$a^2 = b$	$2 \times 2 = 4$	*a* square equals *b*
$b^3 = k$	$4 \times 4 \times 4 = 64$	*b* cube equals *k*
$\sqrt{b} = a$	$\sqrt{4} = 2$	square root of *b* equals *a*
$\sqrt[3]{e} = a$	$\sqrt[3]{8} = 2$	cube root of *e* equals *a*

Examples of Formulas.

Let us now take the simple case of finding the area of a circle whose diameter we know. Expressed in English the rule is: Multiply the diameter by itself, then multiply the resulting product by 0.7854. The result is the area of the circle. If the diameter is expressed in inches, the area will be expressed in square inches. The corresponding mathematical expression is

$$A = 0.7854\, d^2 \qquad (1)$$

where A = the area in square inches,
d = the diameter in inches.

Now, to solve this expression for a particular case, suppose we wish to know the area of a circle nine inches in diameter. We simply substitute for d^2 its numerical value, and perform the indicated operation, thus:

$$A = 0.7854 \times 9 \times 9 = 0.7854 \times 81 = 63.617 \text{ square inches.}$$

Take as another example the formula for the indicated horse-power of an engine:

$$\text{H.P.} = \frac{PLAN}{33{,}000} \qquad (2)$$

where P = the mean effective pressure in pounds per square inch,
L = the length of stroke in feet,
A = the area of the piston in square inches,
N = the number of strokes per minute.

The whole information as to how to determine the indicated horse-power of an engine is given in the above small space, while to write the same in English would require half a page or more of the space at our disposal.

Take the case of an 8 x 10-inch engine running at 100 revolutions per minute under 125 pounds mean effective pressure; here we have:

$P = 125$ pounds,
$$L = \frac{10 \text{ inches}}{12} = 0.833 \text{ feet,}$$
$A = 0.7854 \times 8 \times 8 = 50.26$ square inches,
$N = 100$ rev. per min. $\times\, 2 = 200$.

Then,

$$\text{H.P.} = \frac{125 \times 0.833 \times 50.26 \times 200}{33{,}000} = 31.7$$

Formulas for the Functions of Angles.

A reader of engineering literature frequently encounters trigonometrical expressions, and must know how to treat them. They are in reality about the easiest of all engineering terms to deal with, although often proving a stumbling block to beginners because of their peculiar names and unfamiliar looks. They are always used in connection with a given angle, and are called the ***trigonometrical functions*** of the angle. They are as follows for angle A in Fig. 1. In the formulas where the sides are given as AC, CB, etc., the length of the side in inches is meant.

TABLE 2. TRIGONOMETRICAL FUNCTIONS.

$CB =$ opposite side, $AB =$ adjacent side, $AC =$ hypotenuse.

Called.	Written.	Rule.
sine of A	$\sin A = \frac{CB}{AC}$	opposite side divided by hypotenuse.
cosine of A	$\cos A = \frac{AB}{AC}$	adjacent side divided by hypotenuse.
tangent of A	$\tan A = \frac{CB}{AB}$	opposite side divided by adjacent side.
cotangent of A	$\cot A = \frac{AB}{CB}$	adjacent side divided by opposite side.
secant of A	$\sec A = \frac{AC}{AB}$	hypotenuse divided by adjacent side.
cosecant of A	$\operatorname{cosec} A = \frac{AC}{CB}$	hypotenuse divided by opposite side.

Tables will be found in the hand books * giving the numerical values for these expressions for all angles. The following is an example of such a table:

Angle.	sine.	cos.	tan.	cot.
10 degrees	0.1736	0.9848	0.1763	5.671
11 degrees	0.1908	0.9816	0.1944	5.144
12 degrees	0.2079	0.9781	0.2125	4.705

Right Angle Triangles.

There is also a proposition in geometry so commonly used as to demand recognition whenever met. It is called the rule of the squares of a right angle triangle. In Fig. 2 let ABC be a right angle triangle, having angle CBA for the right angle. If we draw a square on each side as shown, the rule says that the area of $ADEC$ is equal to the area of $ABHI$ plus the area of $BCFG$. The rule is usually expressed thus: The square on the hypotenuse is equal to the sum of the squares of the other two sides. Given expression as a formula the rule is:

$$AC^2 = AB^2 + BC^2 \tag{3}$$

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* See MACHINERY'S Reference Series, No. 18, Shop Arithmetic for the Machinist, pages 48-51: Tables of Sines and Tangents.

With the help of this formula and the trigonometrical functions of the angles, we can solve any problems of the right angle triangle, as illustrated in the following example. We have an inclined plane or approach like Fig. 3 to make, and wish to know the angle and the length of the incline, knowing the height $B\,C$ to be 23¼ inches, and the length $A\,B$ to be 10 feet.

$$A\,B = 10 \text{ feet} = 120 \text{ inches},$$

$$\tan a = \frac{C\,B}{A\,B} = \frac{23.25 \text{ inches}}{120 \text{ inches}} = 0.194$$

Looking in the table of functions of angles we find opposite 0.194 in the column of tangents that the angle a is 11 degrees. From (3) we have

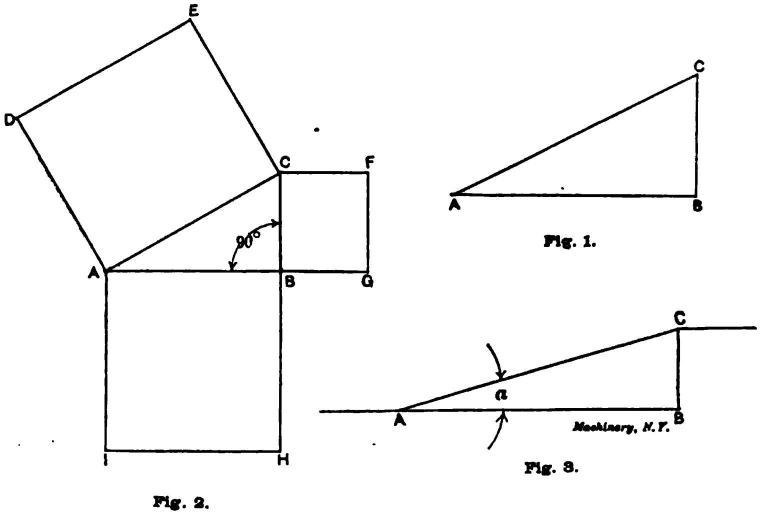


Fig. 1.

Fig. 2.

Fig. 3.

Figs. 1 to 3.

$$A\,C^2 = A\,B^2 + B\,C^2 = 120^2 + 23.25^2 = 14400 + 540.56 = 14940.56$$

then,

$$A\,C = \sqrt{14940.56} = 122.2 \text{ inches.}$$

Thus we find the angle of inclination is 11 degrees, and the length of the incline is 122.2 inches. A table of square roots will be found in any of the handbooks.

CHAPTER II.

STRENGTH OF MATERIALS.

Having mastered the written engineering language sufficiently to deal successfully with formulas, the next step is to make the acquaintance of such engineering terms as are most frequently met with. Foremost among these are the terms relating to the strength of materials.

If a bar is laid across two supports as in Fig. 4, and a weight placed in the center of it, we shall, if the bar be limber, witness the bending of the bar as shown, or as expressed in engineering terms, the deflection of the bar. It is obvious that the stiffer the bar, the less the deflection, and that a bar might be so lacking in stiffness as to actually break when the weight is placed upon it. Now the bar may lack stiffness from one of two causes; it may be that its dimensions are not well proportioned, or it may be made of soft and pliable materials. Sometimes both these causes are combined in the same bar. If the bar does not break when the weight is placed upon it, we must admit three facts; first, that the weight bends the bar; second, that the bar resists the bending; third, that the bar is able to resist the bending because it is large enough and made of stiff enough material.

Important Definitions.

The bending effect that the weight has upon the bar is called the *bending moment* upon the bar due to the weight. The ability of the bar to resist the bending is called the *moment of resistance* of the bar. How these names first came into use the author does not know; perhaps there is no explanation, but the reader must not confuse the terms with any period of time because of the word moment. Time has nothing whatever to do with the strength of the bar, or the effect of the load upon it, except for such materials as wood, when loaded near to the limit of endurance.

In Fig. 4, the point at which the greatest bending occurs is directly under the weight, and we say the bending moment is maximum at this point, and the moment of resistance of the bar must equal the *maximum bending moment* at this point in the bar. In using the term bending moment, the engineer usually means the maximum bending moment, because this has the greatest bending effect upon the bar, and we shall hereafter drop the word maximum.

Relation between Bending Moment and Moment of Resistance.

If now we let $M =$ the bending moment on the bar, and $R =$ the moment of resistance of the bar, we can express the relation of the two as given above thus:

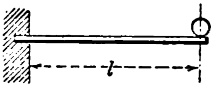
$$M = R. \qquad (4)$$

We said that the maximum bending moment was under the weight, and if the weight is placed further along on the bar, nearer one support than the other, the maximum bending moment will move with the weight. Also, if the bar is differently supported, the maximum bending moment will be at another point. For all cases of frequent occurrence engineers have tables of formulas giving the position and amount of the maximum bending moment, so that it is only necessary to find in the tables the same case as the one we are considering, and taking the formula there given, substitute for the letters the corresponding dimensions in our case, and we have a numerical expression for the bending moment. The formulas given in these tables consist of combinations of dimensions measured along the bar, and weights of the loads on the bar. If, when substituting values for letters in the formula, loads are taken in tons, and distances in feet, the bending moment will be expressed in foot-tons, while if loads are taken in pounds and distances in inches, the usual custom, the bending moment will be expressed in inch-pounds.

TABLE 3. BENDING MOMENT OF BEAMS UNDER VARIOUS SYSTEMS OF LOADING.

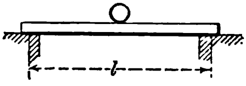
W = total load.
l = length of beam in inches.
I = moment of inertia.
Z = section factor.

Case I.



Beam fixed at one end and loaded at the other.
Max. bending moment at point of support = Wl.

Case II.



Machinery,N.Y.

Beam supported at both ends. Single load in middle.
Max. bending moment at middle of beam $\frac{Wl}{4}$.

Table 3 is a small portion of such a table as may be found in any book on machine design in any drafting room or factory, as well as in all the hand books issued by the steel mills.

So much for the first member of our equation, the bending moment on the bar. We have already seen that the bar offers resistance to bending by reason of two things: its dimensions, and the character of its material, and we should expect to find both dimensions and materials accounted for in the formula for the moment of resistance of any bar. This is just what the formula for the moment of resistance does. It is composed of two parts or terms, one of which expresses the resisting effect of the material of the bar, and the other expressing the resisting effect possessed by the bar because of its shape and size. Let us investigate each term by itself, taking first the resisting effect of the material.

Tension and Compression Stresses.

Let the reader take an ordinary rubber eraser of the form shown in Fig. 5, and bend it as shown in Fig. 6. While holding the eraser in the best position, draw a sharp knife across the top side. The cut immediately spreads out in the form of a V as shown at *a*. Draw the knife a second time through the same cut and the V spreads a little more. Now draw the knife across the bottom. The cut immediately closes up as at *b*. Draw the knife a second time across the same cut and it will still close up completely. In making the second cut on this side it may be necessary to release the eraser from the bent position, because the closing cut grips the knife blade and makes cutting difficult, but the cut will close, upon again bending the rubber.

Having made the two cuts *a* and *b*, reverse the bend in the eraser and witness the closing of cut *a* and the opening of cut *b*. Now if you are a careful experimenter, you can start two such cuts as *a* and *b* directly opposite each other, and by cutting each one the same amount each time, you can succeed in bringing them nearly together in the center as shown at *c*. Of course, it will be impossible to bring them quite together, because that would cut the eraser apart, but by a little care you can satisfy yourself of these facts: that the portion of the eraser above the center line *x x* separates when cut; and that the portion below the line closes when cut. Reversing the bend of the eraser as before reverses the behavior of the cuts, but observe that whichever way the eraser is bent, the opening cuts are to be found on the convex side, and the closing cuts on the concave side.

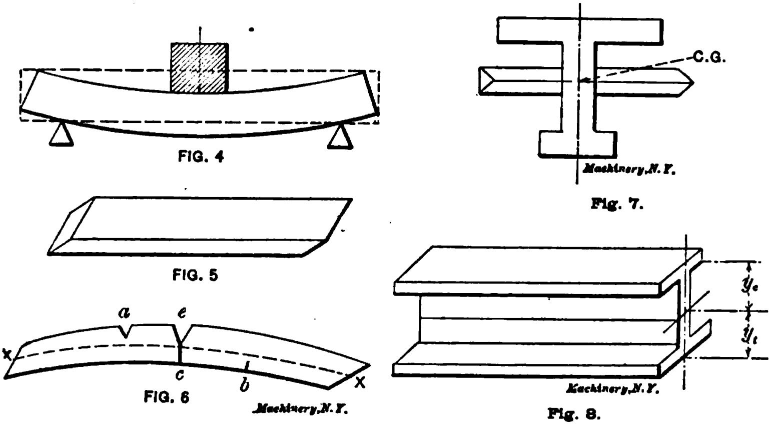


FIG. 4

Fig. 7.

FIG. 5

FIG. 6

Fig. 8.

Figs. 4 to 8.

We know that all material (engineering and building material at least) is composed of fibers, and we must conclude from the behavior of our eraser that all the fibers on the convex side of the line *x x* are streached when the eraser is bent, while the fibers on the concave side of *x x* are compressed. Since the cut through the stretched fibers opens like a V, we may conclude that those fibers lying at the top of the V are stretched more before the cutting than those lying at the point of the V. A careful examination of the cut made through the compressed fibers will show that at the outer portion of the cut, the edges are raised slightly, while at the inner portion, near the center of the eraser, the edges are not raised. We can account for this only by assuming that the fibers at the outer portion are more compressed than those near the center of the eraser.

Having performed these experiments and noted the results, we must admit the following facts: 1st, that half the fibers of a bent bar are in compression while the other half are stretched, or, as engineers say, are in tension; 2nd, that the amount of compression or tension is greatest at the outer portion of the bar, and diminishes towards the center of the bar; 3rd, that it follows from this, as well as from experiments with cut *c*, that there must be a line through the center of the bar where the fibers are neither in compression nor tension.

Now the fibers resist any change in their condition, either stretching or compressing, the amount of resistance differing in different materials. Iron fibers resist more than rubber fibers for instance. When a bar is bent, engineers speak of the fibers as being under stress, some being under compressive stress, and others under tensile stress, as we have seen, and they speak of the bar as being subjected to fiber stress. Now, fiber stress is expressed in pounds per square inch, and it is the duty of the engineer when designing a beam or other structure to keep the fiber stress within safe limits, and these safe limits are given in hand books for a great variety of materials, in tables of which Table 4 is a sample.

TABLE 4. STRENGTH OF MATERIALS—POUNDS PER SQUARE INCH.

MATERALS.	Ultimate Strength.	Safe Working Strength.	Factor of Safety.	Kind of Stress.
Cast Iron.	88,640	16,000	5	Compression
	15,620	8,000	5	Tension
Steel.	82,500	16.000	5	Compression
	80,000	16,000	5	Tension

Factors Determining the Moment of Resistance.

Since the material composing the bar derives its ability to resist bending by reason of the resistance of its fibers to changes, the fiber stress must be one of the terms expressing the moment of resistance of the bar. The fiber stress is denoted by the symbol f. The second term of the moment of resistance of the bar takes into consideration the strength the bar derives from its dimensions.

Bend the eraser in the direction of its greater thickness. We note it takes a much greater force to bend it thus than to bend it as we did at first, in the direction of its least thickness. If we repeat the experiments with the cuts while bending the eraser thus, we shall find that everything witnessed before holds good for this case also. If we look for a reason for the greater force required to bend the eraser in the direction of its greater thickness, we shall find it in the fact previously observed, that the fibers are more stretched or compressed the further they are from the center line *x x*, and thus they present greater resistance to bending. The line *x x* is called the ***neutral axis***, because on it the fibers are neutral, being neither stretched nor compressed, and the fibers at the outer portion of the bar are called the extreme fibers, because they are furthest removed from the neutral axis *x x*.

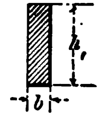
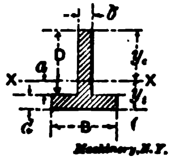
The second term of the moment of resistance, taking account of the shape and size of the bar, is called the section factor, sometimes also called the section modulus, *Z*, and is given in all hand books in the shape of tables for different shapes of beams, in the style shown in Table 5.

The neutral *x x* is not always in the center of the bar, but it always passes through the center of gravity of the cross section of the bar.

Center of Gravity.

Here we shall have to digress for a moment, since it is the intention to leave no term unexplained. The reader may best become acquainted with the center of gravity in the following manner: Cut out of stiff cardboard the shape of the cross section of the bar, and balance it over a sharp edge, in a manner as shown in Fig. 7. Draw a line across the card corresponding to the edge over which it is balanced. Repeat the experiment, turning the card around on the edge, and, balancing it a second time, draw another line. The intersection of these two lines will be the center of gravity of the section of the beam. If the experiment has been done with sufficient care, the card may be balanced upon a sharp point placed at the intersection of the two lines, just as if the entire material of the card were placed vertically above the point. A definition frequently met with is: The center of gravity is that point at which the entire weight of a body may be considered as concentrated.

TABLE 5. VALUES OF I (MOMENT OF INERTIA) AND Z (SECTION FACTOR) FOR VARIOUS SECTIONS.

	Section.	*I*	*Z*	Area.
Case I.		$\frac{bh^3}{12}$	$\frac{bh^2}{6}$	bh
Case II.	 Machinery, N.Y.	$\frac{by_c^3+By_t^3-(B-b)a^3}{3}$	$\frac{I}{y_t}$ $\frac{I}{y_c}$	$Bd+bD$

Another way of finding the center of gravity is to suspend the card by a fine thread alongside of a plumb line, and when the card and line have come to rest, mark the position of the plumb line on the card. Turn the card around, and suspend a second time from a different point, and mark the position of the plumb line again. Where the two marks of the plumb line cross will be the center of gravity of the figure. No matter from how many points the card may be suspended, the plumb line will always be found to pass through the center of gravity. A line in the center of the beam directly opposite the center of gravity thus found will be the neutral axis.

Equation for Bending Moment.

If we now take the equation expressing the relation of the bending moment on the bar to the moment of resistance of the bar, and use the symbols for the two parts of the moment of resistance, we shall have

$$M = R = f\,Z.$$

Some tables do not give the section factors Z for all sections directly, but say it is $\frac{I}{y}$, and therefore we must understand this expression.

The denominator y of the fraction is the distance from the neutral axis $x\,x$ to the extreme fiber of the bar, see Fig. 8, and the numerator I is what is called by engineers the *moment of inertia* of the section of the bar. Here again there is a chance for confusion because of the use of the word inertia.

Moment of Inertia.

The term moment of inertia was originally employed when comparing the energies of rotating bodies. We know that a moving body possesses energy due to that property of matter which engineers call ineria. Inertia is not a force; it is simply resistance, and is due to the incapability of a dead body to move, or of a moving body to change its velocity or direction without the application of some external force. Now the number of foot-pounds of energy possessed by a moving body is equal to $\frac{1}{2} M V^2$, where M is the mass of the body, and V its velocity in feet per second. A moving body then, must be acted upon by an external force before it can be brought to rest. A rotating body is simply a very large number of particles moving in circular paths about an axis called the axis of rotation. Each moving particle, therefore, possesses energy due to its inertia, and the energy of each particle is equal to $\frac{1}{2} m\,v$, where m is the mass of the particle, and v its velocity in feet per second. But the energy varies as $m\,v^2$, because simply dividing by 2 does not change the relative values. It is also obvious that the circumferential velocity of each particle varies as the distance from the axis of rotation, which distance or radius we call r. Hence, substituting r for v, the energy of each particle varies as $m\,r^2$. Suppose we imagine that the whole mass of the rotating piece, that is the sum of all the small particles m, is concentrated in a circle that is of such diameter that the energy possessed by the entire mass is the same as before. The radius of this imaginary circle is called the radius of gyration, and is usually designated by the letter r. Now we may say that $M\,r^2$, where M stands for the whole mass, is a measure of the energy of the rotating piece. This expression $M\,r^2$ is given the name moment of inertia, each particle of which the rotating body is composed possessing a turning moment about the axis of rotation, due to its motion and inertia.

When it was discovered that the flexture of a beam depended upon the value $a r^2$ (where a is the area of the cross section of the bar, and r^2 is the mean of the squares of the distances of the infinite number of small areas into which the area of the section may be supposed to be divided, from the center of gravity of the section) $a r^2$ was seen to be similar to the expression $M r^2$, which, in connection with the rotating bodies, had already become known as the moment of inertia; so, very carelessly on the part of those who first committed the error, it was said that the flexure of a beam varied as its moment of inertia, not because inertia has anything to do with it, for, of course, it has not, but because $a r^2$, the expression for the moment of resistance to flexure, happened to vary in the same way as the moment of inertia $M r^2$ of the same body when rotating about its center of gravity.

The moment of inertia of a bar may be calculated by several methods, but the tables in hand-books give it for all usual shapes of sections, and we will not attempt the calculation here.

Universal Formula for Bending Strength of Beam.

Since we are sometimes able to find in tables only the moment of inertia of a bar, and not the section factor, we must bring our formula one step further, thus:

$$M = R = f Z = f \frac{I}{y} \qquad (5)$$

or

$$Z = \frac{I}{y} = \frac{M}{f} \qquad (6)$$

and here we have the formula for determining the size required for any beam.

For beams in which the center of gravity is not the center of the beam, there will be two values of y, one of which we will denote as y_c, being the distance from the neutral axis to the extreme fibers in compression, and the other as y_t, being the distance from the neutral axis to the extreme fibers in tension, see Fig. 8.

In some materials the ability of the fibers to resist tension is about equal to their ability to resist compression, while in other materials there may be great inequality in this direction, some being much stronger in tension than in compression, while others are stronger in compression than in tension. In such a material we shall have two values of f, which we will denote as f_c and f_t for compression and tension, respectively.

Ultimate and Safe Stresses.

Some tables on the strength of materials give what is called the ultimate or breaking strength of the materials, while other tables give the safe working strength of materials.

When using the latter tables, the values given are to be substituted directly for f_c and f_t in the formula. Since it would not do to have the material of which a beam is made strained up to the breaking point, we must, when using the former tables, make use of a factor of safety. This factor of safety is a divisor by which the breaking strength of a beam is divided to allow a margin of strength in the beam. The divisor varies from 2 to 10, and the proper use of different divisors is given in the text books.

To illustrate, the breaking strength of steel may be given as 80,000 pounds per square inch, and $\frac{80,000}{5} = 16,000$. If we substitute 16,000 for f in the formula, we shall be working out our results with a factor of safety of 5, and the beam should not actually break until loaded with five times the load designed for. As a matter of fact, the beam would become badly bent long before five times the load could be placed upon it.

Limit of Elasticity.

We have seen that all material deflects under the influence of a load, and we suppose that the elasticity of the material causes it to spring back to its original condition when the load is removed. This is true within limits, but there is a point somewhere between the safe load and the breaking load at which, when the load is gradually increased, the beam becomes strained beyond its power to return to its original condition upon the load being removed. This point is variously called the *limit of elasticity*, the *yield point*, the *point of permanent set*.

Practical Examples.

Let us now take up two examples illustrating the ground we have just passed over, and the use of the tables.

Example 1. A rectangular steel bar, 2 inches thick, is built into a wall as in Fig. 9, and is to hold a load of 3,000 pounds at its outer end, 36 inches from the wall. We wish to know the required depth to make the beam.

1st, consider the bending moment on the beam. According to case 1, Table 3, the bending moment is

$$M = W l.$$

For our case we know W and we know l, and substituting these for the letters in the formula gives us

$$M = 36 \times 3,000 = 108,000 \text{ inch-pounds.}$$

2d, consider the permissible fiber stress in the steel bar. Table 4 gives the safe working strength of steel as 16,000 pounds per square inch.

3d, using formula (6) we can find the value of the section factor for our beam. We know the bending moment and we know the fiber stress, so substituting these for the letters in the formula we get

$$Z = \frac{M}{f} = \frac{108,000}{16,000} = 6.75.$$

4th, find the section of our beam in Table 5, case 1, where we find that the section factor is

$$Z = \frac{b\,h^2}{6}.$$

We know Z and we know b, so substituting these values for the letters, we get

$$6.75 = \frac{2 \times h^2}{6}.$$

If we multiply both sides of this equation by 6, we shall not change its value, but shall get

$$6 \times 6.75 = 2 \times h^2.$$

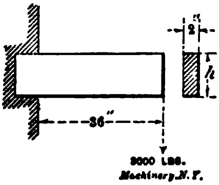


Fig. 9.

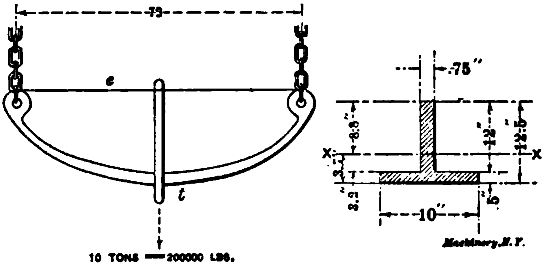


Fig. 10.

If we now divide both sides by 2, we shall not change its value, but shall get

$$\frac{6 \times 6.75}{2} = h^2 = 20.25.$$

5th, we can most conveniently find the square root of 20.25 from a table of squares and roots which may be found in any hand book. This square root is 4.5, and we thus find that

$$h = 4.5 \text{ inches.}$$

If we make the beam 2 inches thick by 4.5 inches deep by 36 inches long, it will support a load of 3,000 pounds at its free end, and the fibers will be strained to 16,000 pounds per square inch.

Example 2. Let us undertake to design a suspension beam like Fig. 10 to carry ten tons, the material to be cast-iron. The proposed section of the beam is more complicated than that of the previous example, and we cannot obtain a result quite so directly.

1st, inspect the proposed beam to locate the compression and tension flanges. We find the compression flange is on top and the tension flange on the bottom, and we mark them c and t respectively.

2d, Table 4 shows us that cast-iron is stronger in compression than in tension, hence we conclude that we should have more metal on the tension side than on the compression side, and accordingly we place the section with the heavy side down.

3d, assume a section by making the best guess possible as to the dimensions shown heavy in the figure. Cut out this section of cardboard, and find the location of the neutral axis $x\,x$ as previously explained. Now fill in the figures shown light by measuring the cardboard section.

4th, find the section in Table 5. Here we find that before we can get the section factor of the beam we must get the moment of inertia of the beam. Substitute the dimensions of our section for the letters of the formula given in Table 5, and we shall get

$$I = \frac{(0.75 \times 8.8^3) + (10 \times 3.7^3) - (10 - 0.75)\,3.2^3}{3}$$

$$= \frac{511.1 + 506.5 - (9.25 \times 3.2^3)}{3}$$

$$= \frac{1017.6 - 303.12}{3} = \frac{714.48}{3} = 238$$

5th, now divide the moment of inertia just found by the distances of the extreme fibers from the neutral axis, that is, by y_c and y_t, and we get

$$Z_c = \frac{I}{y_c} = \frac{238}{8.8} = 27, \text{ the section factor for the compression side.}$$

$$Z_t = \frac{I}{y_t} = \frac{238}{3.7} = 64.3, \text{ the section factor for the tension side.}$$

6th, inspect Table 3 and find the bending moment on the beam according to case 2; substituting the dimensions of the beam, and the load to be carried, in the formula given, we have

$$M = \frac{W\,l}{4} = \frac{20{,}000 \times 72}{4} = 360{,}000 \text{ inch-pounds.}$$

7th, dividing the bending moment just found by the section factors found in the 5th step, will give the fiber stress on the beam according to formula (6), thus

$$\frac{360{,}000}{27} = 13{,}333 \text{ pounds per square inch on the compression side,}$$

$$\frac{360{,}000}{64.3} = 5{,}600 \text{ pounds per square inch on the tension side.}$$

The latter is too high, so another guess must be made, making the section heavier on the tension side. Then the steps 3, 4, 5 and 7 must be repeated, and if the fiber stress then comes below 3,000 pounds per square inch, the section will be right.

CHAPTER III.

GRAPHICAL METHODS OF SOLVING PROBLEMS.

The reader of technical literature will often encounter problems solved by what are known as graphical methods. All arithmetical calculations may be represented by drawings, and this is precisely what the many charts in the technical papers are—drawings of formulas.

To draw a formula, we must first be able to draw the four arithmetical processes, addition, subtraction, multiplication, and division.

Addition and Subtraction.

Let it be required to add graphically 2 and 3, that is, to draw the formula $x = 2 + 3$ and find the value of x from the drawing. In Fig. 11 draw the line *a-b* two inches long, then commencing at *b* draw the line *b-c* three inches long, which shall be a continuation of line *a-b*. Line *a-c* now measures five inches, therefore $x = 5$, and *a-c* is the drawing of the formula.

It is evident that subtraction will be the reverse of addition. Let it be required to draw the formula $x = 5 - 2$. Using the same figure as above, draw line *a-c* five inches long, and from one end, *a*, lay off *a-b* two inches long. Then *b-c* measures three inches, and $x = 3$, and *a-c* is the drawing of the formula. The unit of measurement, here taken as the inch, may be any length convenient so long as all measurements are made to the same unit. The drawing of the addition and subtraction formula as given above applies only to an individual problem, and since the number of problems is infinite, and even within the range of every-day practice is very great, some form of drawing which may be used for a great number of problems, is needed. In Fig. 12 draw two lines at right angles, and draw a diagonal line at 45 degrees. Let the horizontal line represent one unit, from 0 to 1, and let the vertical line also represent one unit, but let this unit be that from 1 to 2. Then the horizontal and vertical lines will be of the same length. Mark the horizontal scale 0 to 1, the vertical scale 1 to 2, and the diagonal line 1-A.

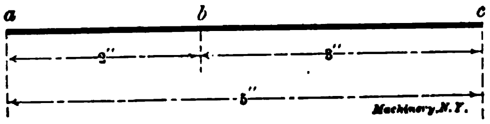


Fig. 11.

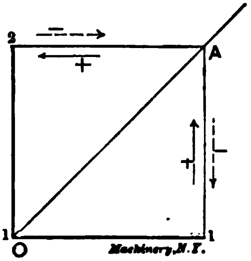


Fig. 12.

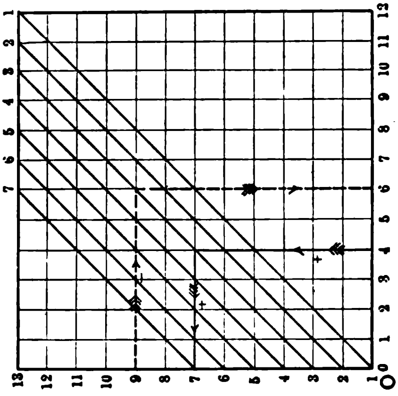


FIG.13

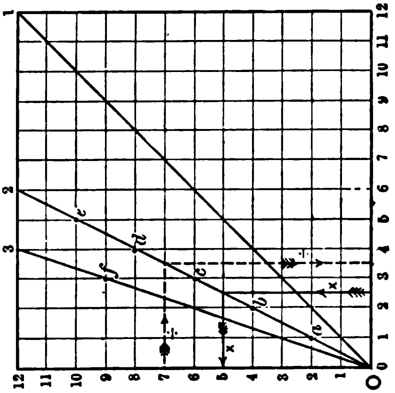


FIG.14

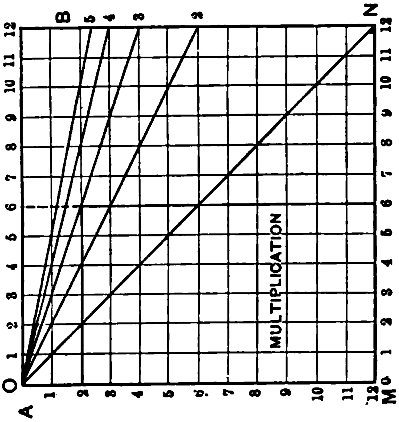


FIG.15

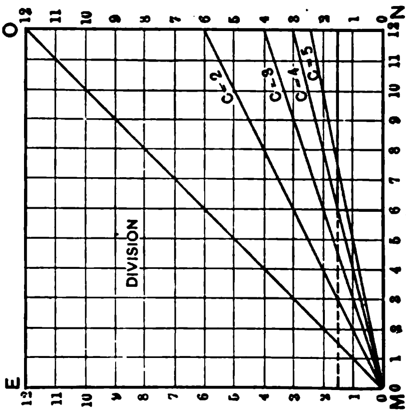


FIG.16

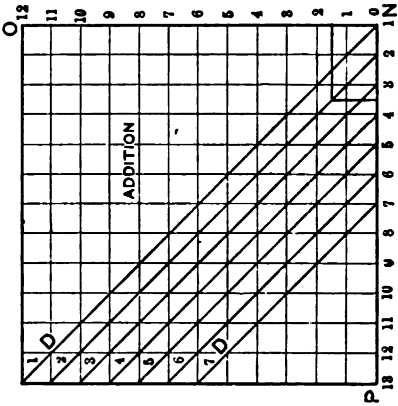
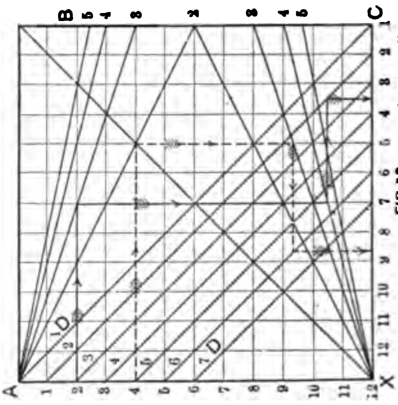


FIG.17



Machinery, N.Y.

FIG.18

Figs. 13 to 18. Graphical Representation of Formulas.

If now we begin at 1 on the horizontal scale, and travel vertically to the diagonal line at A, we have gone one unit, and if from here we travel horizontally to the vertical scale, we will travel another unit, or two units altogether. Then the line 1-A-2 represents the drawing of the formula

$$x = 1 + 1,$$

but since the line has been bent from the form of a straight line, we are no longer able to measure anything to obtain the answer. We do not need to do this, however, since we arrived at the figure 2, which is the answer. The operation has been made plain by the full arrows marked +. The reverse operation, subtraction, is shown with dotted arrows marked —, and the line 2-A-1 is the drawing of the subtraction formula,

$$x = 2 - 1,$$

and beginning at 2 we end at the end figure 1, which is the answer.

More diagonal lines may be drawn from points representing more units in the vertical direction, and thus we arrive at an addition and subtraction diagram, which may be made to cover any desired range of problems. Fig. 13 represents such a diagram. The full line and arrows represent the problem,

$$4 + 3 = 7,$$

and the dotted line and arrows represent the problem

$$9 - 3 = 6.$$

Multiplication and Division.

A diagram similar to the above may be made for multiplication and division, as shown in Fig. 14. Lay out the horizontal and vertical scales equal, each starting from the zero point (observe the difference between the two diagrams in this respect). Draw a 45-degree line from the zero point, and mark the line 1. If we start from either scale and travel to this diagonal line, and then to the other scale, we arrive at the same figure we started from, or in other words, we have either multiplied or divided our starting figure by 1. Start now at 1 in the horizontal scale and travel vertically till opposite 2 in the vertical scale, marking this point a. Start again at 2 and travel till opposite 4, and mark the point b. Start at 3 and travel to 6, at 4 and travel to 8, and at 5 and travel to 10, marking the points c, d, and e. We find that the points a, b, c, d, e, and the zero point are all in line; also that in starting at 2 and going to 4, etc., we have multiplied each of our starting points by 2. Draw a line through these points and mark it 2. Since the points are all in line, it is unnecessary to locate more than one to locate the line. Start at 3 and travel till opposite 9, and mark the point f. Connect f and 0, and mark the line 3, because $3 \times 3 = 9$, and we have multiplied our starting figure by 3 to locate f opposite 9. In this way as many other lines may be located as desired.

The full line marked × is the drawing of the multiplication formula,

$$x = 2\tfrac{1}{2} \times 2 = 5,$$

and the dotted line marked ÷ is the drawing of the division formula,

$$x = \frac{7}{2} = 3\frac{1}{2}.$$

Producing Charts Representing Complex Formulas.

A set of charts may be made for any formula, no matter how complicated, by making a chart for each step or arithmetical process in the formula. These separate charts may usually be combined into one with advantage. Figs. 15, 16, 17 and 18 represent such a series of charts, and their final combination into one chart, for the formula,

$$x = \frac{A\,B}{C} + D.$$

Let $A = 2$
$B = 3$
$C = 4$
$D = 2$

The first chart, Fig. 15, solves the first step, the multiplication of A and B. Values of A are first laid off vertically on the right, and at the top and bottom horizontal scales are laid off. The B lines are then drawn in, as in the previous multiplication chart, by assuming various values of A and B, and locating their proper points in the chart, as $A = 2$ and $B = 3$, $A \times B = 6$; through the intersection of lines from 2 and 6 drawn 0-3, which is the line for $B = 3$. Draw lines for other values of B in the same manner. We find it possible to read the answer either at the top or bottom scale, and for the present we leave this question open.

The next step is to divide the product of A and B by C. In Fig. 16 lay off scale $M\,N$ exactly like scale $M\,N$ in Fig. 15. Lay off the two vertical scales, starting with zero at the bottom, otherwise exactly like the vertical scale in Fig. 15. Assume values for $A \times B$ and for C, and lay off C lines as the B lines were laid off in Fig. 15. We find we can read the answer either on the right or left, and leave the decision till later. By multiplying A and B in the first chart, and dividing the product by C in the second chart, we have left only to add to the result, D, for the final step.

In Fig. 17 lay off scale $O\,N$ exactly like $O\,N$ in Fig. 16, and lay off scale $P\,N$ exactly like scale $M\,N$ in Fig. 15, but beginning with 1 at the right. Now draw an addition chart by assuming values for $A \times B \div C$, on scale $O\,N$, and values for D, and draw the D lines as described for the previous addition diagram. By these diagrams we can solve the equation,

$$x = \frac{A\,B}{C} + D,$$

if we have given values of A, B, C, and D. The process will be as follows: Find the value of A on scale $A\,M$, Fig. 15, and proceed horizontally to value of B, thence vertically, and read product of $A \times B$;

find this product on scale MN in Fig. 16, proceed vertically to value of C, thence to the right and read value of $A \times B \div C$; find this value on scale ON, Fig. 17, and trace to left to value of D, thence down to value of

$$\frac{AB}{C} + D = x$$

and read the answer.

Combining the Detail Charts into a General Chart.

It is a great inconvenience to have to jump from chart to chart, and it may be easily avoided. If these charts were drawn on tracing paper, and placed one over the other, we should find that their lines are all clear and distinct, and do not interfere with each other to the extent of losing their identity. This point was kept in mind when drawing the small charts; for example, in Fig. 16 the scales could be arranged so that the C lines would radiate from corner E, but they would then become confused with the B lines of Fig. 15, when the two are superposed; also in Fig. 17 the D lines could be drawn in the upper half by rearranging the scales, but they would then interfere with the B lines of Fig. 15 when superposed, while as drawn they appear in the opposite half of the chart from the B lines, and they cross the C lines at such a great angle as not to become confused with them.

Fig. 18 shows the three charts drawn superposed one on another, and by inspection we see that it is no longer necessary to read the value of each step in order to take the next step, but that we can proceed from line to line, reading the value of each letter in turn, till we read the value of x at the bottom. The question left open as to which scales to read the answers on in Figs. 15 and 16 has solved itself in the superposed diagram. The full line with arrows shows the working out of the same problem as the lines in the separate diagrams, namely, the value of x in the formula

$$x = \frac{AB}{C} + D$$

when $A = 2$, $B = 3$, $C = 4$, and $D = 2$.

Such problems as the following, and others, may be solved on this chart, and the reader should practice till he can solve such problems easily and quickly. One of these is shown in the diagram in dotted lines.

$$\left.\begin{array}{r} A = 4 \\ B = 2 \\ C = 3 \\ D = 6 \end{array}\right\} x = \frac{4 \times 2}{3} + 6 = 8.7$$

$$\left.\begin{array}{r} A = 3 \\ B = 3 \\ C = 4 \\ D = 5 \end{array}\right\} x = \frac{3 \times 3}{4} + 5 = 7.25$$

It is in this manner that the charts we see in the technical journals, giving the strength of gears, shafts, etc., are worked out. It often happens that it is necessary to vary the scales, also that in plotting out points, as *a*, *b*, *c*, *etc.*, in Fig. 14, the resulting line is a curve.

Graphical Statics.

Such graphical processes as we have just been considering, and the laying out of charts, is called graphical arithmetic. There is another branch of graphics, called graphical statics, so frequently used as to demand a knowledge of its fundamental principles of any reader of technical publications.

In Fig. 19 let *A* be a small ball, and let *E* be a force acting on the ball, and just sufficient to drive the ball to *B*. Also let *F* be a similar force just sufficient to drive the ball to *C*. If the force *E* acts alone, the ball will start and travel *A-B* and come to rest at *B*, while if the force *F* acts alone, the ball will travel *A-C*, and come to rest at *C*. Now, suppose that both forces act at the same instant. We would expect the ball to partake of motion resulting from each force, and to travel to the right at a distance *A-C*, and also to travel downward on the page a distance *A-B*, and this lands the ball at *D*. It cannot, however, travel either the path *A-C-D* or *A-B-D*, because it partakes instantly of motion from each force, and it can only travel *A-B* or *A-C* under the influence of one force acting alone. We will also assume that the ball must travel in a straight line, because to cause it to travel in a curved line we should have to introduce a third force in the shape of guides or other device, and we are supposing the ball to be acted on only by the forces *E* and *F*. Therefore, since it must travel along a straight line, must land at *D*, and cannot travel either along *A-C* or *A-B*, it must travel along *A-D*.

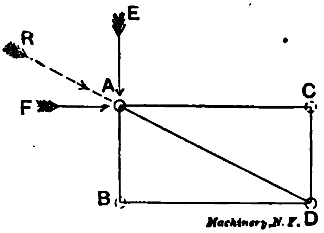


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Fig. 19.

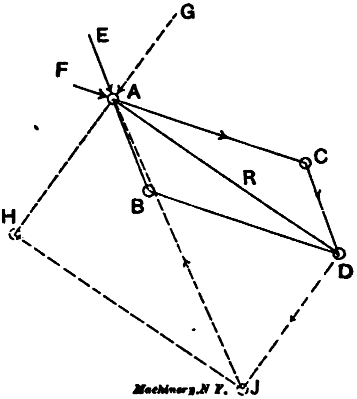

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Fig. 20.

It is obvious that we could easily move the ball from *A* to *D* by a single force acting in the direction of *A-D*, and just strong enough to move the ball and bring it to rest at *D*. Such a force would be the force *R*, shown in dotted lines in Fig. 19. There are formulas involving the trigonometrical functions, by which, knowing the force and direction of both *E* and *F*, we may find *R*, but it is more simply done by the following method.

Adopt some convenient scale of force, as one inch equals ten pounds. Along *A-C* lay off by this scale the force *F;* that is, if *F* is fifteen pounds, make *A-C* one and a half inches for the above scale. Lay off on *A-B* the force *E* in the same manner and to the same scale. Now complete the parallelogram by drawing *B-D* parallel to *A-C*, and *C-D* parallel to *A-B*. This is called the parallelogram of forces. If, now, we measure *A-D* by the force scale adopted, we find *R*. Thus, if with the above scale, *A-D* measures two inches, then force *R* is twenty pounds.

This force *R* is called the resultant of forces *E* and *F*, and the latter are called the components of *R*. By a little study of the figure it will be seen that the problem may be worked backwards, or that a given force *R* may be divided into two other forces at will. This process we call the resolution of forces, and we say that any force may be resolved into its components.

Three Forces Acting on a Body.

Now, it has been proved that the same thing is true when the forces act at any angle, as in Fig. 20. Now, suppose the ball acted upon by three forces, *E*, *F*, and *G*, the latter being shown in dotted lines in Fig. 20, and being such a force as acting alone would move the ball from *A* to *H*. Now, we know that *E* and *F*, acting together, move the ball to *D*, for we can locate *D* by drawing the parallelogram of forces, *A-B-D-C*, and *A-D* will be the resultant of *E* and *F*. Now, since any two forces may be replaced by their resultant, we may consider that the ball is acted on by only two forces, *R* and *G*. We may now draw another parallelogram of forces for *G* and *R*, being *A-H-J-D*, and the final resting place of the ball acted upon by the three forces *E*, *F*, and *G* will be *J*; also, *A-J* will be the resultant of the three forces *E*, *F*, and *G*.

Polygon of Forces.

The observant reader will notice that it is not necessary to draw the entire parallelogram of forces in order to locate the ball, or the magnitude or direction of the resultant. Thus, we could draw *A-C* parallel to *F*, *C-D* parallel and equal to *E*, and *D-J* parallel and equal to *G*, thus locating *J*. Then, drawing *J-A*, we would complete a polygon, *A-C-D-J*, called the polygon of forces, and *J-A* is called the closing line, because it completes or closes the polygon.

Let us now suppose that, instead of being moved by the three forces, the ball remains at rest, but is still acted upon by the three forces, *E*, *F*, and *G*. We then say that the ball is in equilibrium, and this can only be true when some force acts counter or opposite to the resultant of the three forces. Such a counter force is called a reaction, and is represented by *J-A*.

Let us now place upon each line, *A-C*, *C-D*, and *D-J*, an arrow corresponding to the forces *F*, *E*, and *G*, respectively, and we see that the arrows follow each other around the polygon; this must be so, or the polygon is incorrectly drawn. Following this rule, and placing an arrow on *J-A*, we see the direction of the reaction that would keep the ball in equilibrium.

Since the three forces may be replaced by the resultant, *A-J*, and the reaction, *J-A*, keeps the ball at rest, we would expect: 1st, the line of the reaction to coincide with the line of the resultant; 2d, that the magnitude of the reaction and the resultant would be the same; 3d, that their directions would be opposite. This is precisely what the diagram shows because, 1st, the closing line of the polygon, *J-A*, has fallen upon the line of the resultant, *A-J*, of the three forces *E*, *F*, and *G*; 2d, the points *A* and *J* mark the ends of both the lines of the resultant and reaction; 3d, because the arrow placed upon the closing line, in conformity with the other arrows, points opposite to an arrow placed upon the line representing the direction of motion of the ball if no force prevents its motion.

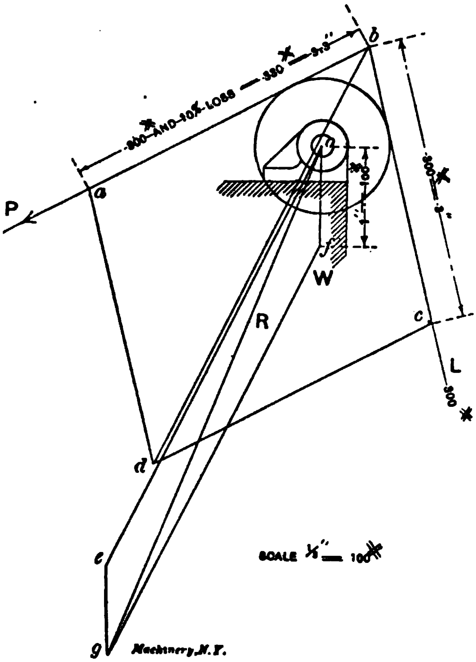

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Fig. 21.

Practical Application of Graphical Statics.

The use of these principles of graphical statics may be well illustrated by the following example, Fig. 21. This represents a pulley or sheave, over which is passed a rope, on one end of which is a load, *L*, of 300 pounds. We know that, owing to the stiffness of the rope and the friction of the sheave pin, we must pull with more than 300 pounds on the other end of the rope in order to lift the load. Suppose we lose 10 per cent of our pull. We must then pull on the rope with a force, *P*, of 330 pounds to raise the load, *L*, of 300 pounds. Continue the lines of the rope to meet at *b*. Adopting a scale of 1 inch to 100 pounds, lay off *b-c* three inches long, representing the load of 300 pounds, and *b-a* 3.3 inches long, representing the pull of 330 pounds. Draw *c-d* parallel to *b-a*, and *a-d* parallel to *b-c*. Draw *b-d*, the resultant of *P* and *L*.

There is a third force acting on the sheave pin, namely, the weight of the sheave and rope, which we will call *W*. Let this weight be 100 pounds. From *o* lay off *o-e* parallel and equal to *b-d*, and in a vertical direction measure from *o* one inch, representing 100 pounds by our scale, locating point *f*. From *f* draw a line parallel to *o-e*, and from *e* draw a line parallel to *o-f*, thus locating point *g*. Then *o-g* will be the resultant, *R*, of the load and pull on the rope and the weight of the sheave, and the reaction of the support must be in this direction. Thus we know on which side to cap the bracket, to make the base, etc. If we measure the line *o-g*, we find it is 5.83 inches long, and multiplying by 100, since one inch equals 100 pounds, we find the resultant force on the sheave pin is 583 pounds. If we had not taken account of the stiffness of the rope and friction of the pin, and had assumed the pull, *P*, equal to the load, *L*, which, of course, it is not, the line *b-d* would have passed through the center, *o*.

CHAPTER IV.

PRINCIPLE OF MOMENTS AS APPLIED TO THE LEVER.

The lever is the simplest element of a machine, and the principles of its action are of a simple nature. There is no reason why anyone who chooses to devote a little time to study should not be able to master these principles, and having done this, he will have gone a long way toward mastering the principles of all the elements that make up a machine.

Webster defines a lever as "a bar of metal, wood or other substance, used to exert a pressure or to sustain a weight, at one point at its length, by receiving a force or power at a second, and turning at a third on a fixed point called a fulcrum. It is of three kinds, according as either the fulcrum *F*, the weight *W*, or the power *P*, respectively, is situated between the other two." This is the usual definition of a lever as it is found in most books upon mechanics and physics, and attention should be called to certain points about it that could easily lead a beginner astray and cause confusion at the outset. It is always best to start with a clear idea of a subject, so that there will be no uncertainty to begin with.

In Fig. 22 is a lever, in which, according to the definition, *W* is a weight acting at one point, *P* is the power or force acting at another point to raise the weight *W*, as indicated by the arrow, and *F* is the fulcrum on which the lever turns. That part of the lever between the weight and the fulcrum is called the "weight arm," and that part between the fulcrum and the power is called the "power arm." It will be noted that the fulcrum in Fig. 22 is located between the weight and power. In Figs. 23 and 24, however, are two levers in which the arrangement is different, the weight being placed between the power and fulcrum in Fig. 23, and the power placed between the weight and fulcrum in Fig. 24. These three figures illustrate the first, second, and third kinds of lever, as above defined.

The objections to this definition of the lever are, in the first place, the use of the word "power" for the force applied at the end of the lever to raise the weight. "Power" has a totally different meaning from "force," and takes into account not only force, but time and distance. A force is merely a push or pull, such as is exercised by the hand, and this is the kind of effort that is always required to raise a weight or overcome any other resistance. In the reference letters of the illustrations, therefore, we will let *P* stand for a push or a pull, as the case may be, instead of for the word "power." Hereafter, also, instead of calling the resistance to be overcome the "weight," we will call it the "resistance" and represent it by the letter *R*. A lever may have to overcome a number of resistances besides that of raising a weight, such as the resistance of friction, of a coiled spring, or of the pressure of steam, and the term "resistance" implies this better than the term "weight."

Finally, regarding the three kinds of levers mentioned above, there is no necessity for trying to separate levers into any number of classes, or for trying to remember to which class they belong in the solution of examples. All levers depend upon the same principles, which are simple and easily understood, and all that is necessary is to first master these principles without regard to the relative position of the applied force, the resistance, or the fulcrum.

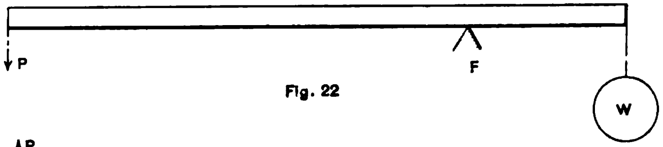

Fig. 22

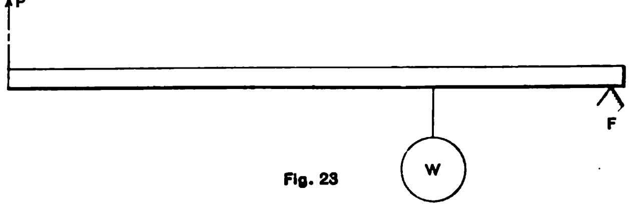

Fig. 23

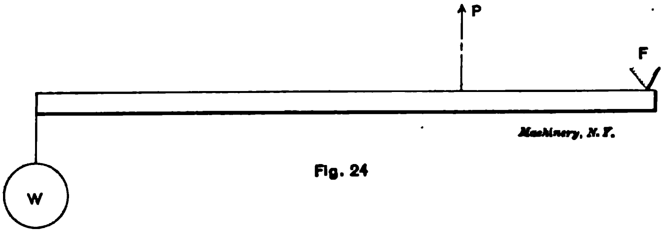

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Fig. 24

Figs. 22 to 24.

The Moment of a Force.

We have seen what is meant by the term "force," and the next thing to learn is what the moment of a force is. When a force acts at a point on a lever, that is, when that point is given a push or a pull, the tendency is to cause the lever to turn about its fulcrum. This tendency depends first upon the strength of the force acting, and second upon the perpendicular distance from the line of action of the force to the fulcrum. If either the strength of the push or pull exerted by the force, or the perpendicular distance of its line of action from the fulcrum, is changed, the tendency of the force to rotate the lever will be greater or less, as the case may be. The rotative effect of any force thus depends upon both the strength and the distance, and is measured by their product, this product being called the moment of the force.

The moment of a force, therefore, is the measure of the turning effect of that force, and is found by multiplying the force by the perpendicular distance from its line of action to the fulcrum. If the force be measured in pounds and the distance in feet, the moment will be in foot-pounds; if the force be in pounds and the distance in inches, the moment will be inch-pounds; if the force be in tons and the distance in feet, the moment will be in foot-tons, etc. The foot-pounds measurement is the most commonly used, however.

This subject of moments is important—in fact, the most important in the whole subject of levers—and in order to fix it firmly in the mind, it will be helpful to have some common fact or operation that will illustrate it, and that can be referred to in solving complicated examples in which the application of the principle may not be entirely clear.

There is one kind of lever that is very familiar to every mechanic, and that is the wrench. We will select the wrench, therefore, to illustrate the subject of moments, and having once grasped the principle as applied to the wrench, no mechanic will be likely to have trouble with its other applications.

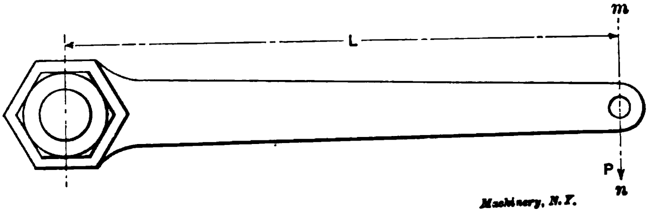

Fig. 25.

Fig. 25 represents a box wrench, and, as is often done in work of a heavy character, a hole is punched in the outer end of the handle, into which a chain or rope can be hooked or fastened to assist in screwing the bolt or nut "home." Suppose the wrench is being used to screw up a nut, as shown in Fig. 25, and that the pull P on the rope is in the direction shown by the arrow, or in the direction of the line $m\,n$. The tendency of this pull to turn the wrench and nut will then be measured by the pull P in pounds, multiplied by the distance L in feet measured from the fulcrum at the center of the bolt, to the line $m\,n$, the distance being taken in the direction of a line at right angles or perpendicular to the line $m\,n$. This product gives the effect of the pull P in foot-pounds, and is called the moment of this force. Thus, if the pull P is 300 pounds, and the length L is 4 feet, the moment of the force P is $300 \times 4 = 1{,}200$ foot-pounds, and this is the measure of the turning effect of this force.

The reason why this is so will be evident if we consider another case shown in Fig. 26. Here the wrench has been placed in a new position, ready for another turn, and the pull P acts in the same direction as before, along the line $m\,n$. Now, anybody who has used a wrench knows that with the same pull a greater effect will be produced with the wrench as placed in Fig. 25 than as placed in Fig. 26, although in each case the hook is at the same distance (4 feet) from the fulcrum F. The direct distance, however, of the point of application of the force from the fulcrum does not necessarily have any influence on the effectiveness of this force in moving the lever. The only distance that can be considered is the perpendicular distance from the line along which the force acts to the fulcrum, and this distance is greater in Fig. 25 than in Fig. 26, and in the former the force of 300 pounds has a greater leverage than in the latter. In Fig. 26 the measure of the rotative effect is the pull P, which is 300 pounds, times the distance L, which in this case measures 2 feet, or $300 \times 2 = 600$ foot-pounds. The distance L, as before, is measured at right angles to the line $m n$, and if the rope had extended along the line $c d$, instead of the line $m n$, L would have been measured at right angles to the line $c d$, as indicated by the line L_1.

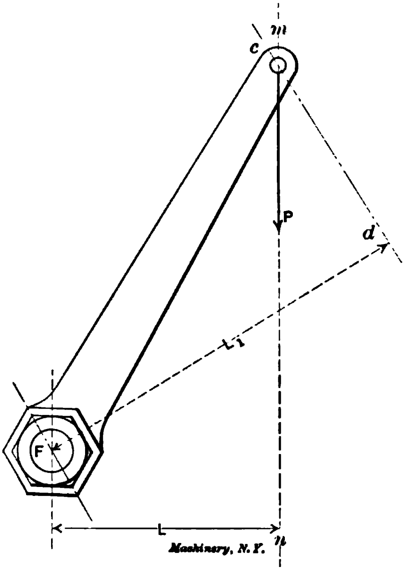

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Fig. 26.

The True Lever Arm.

The distance L in Figs. 25 and 26 is called the lever arm. Ordinarily the arm of a lever is understood to mean that part of the lever that lies between the fulcrum and the point where the force is applied, or between the fulcrum and the point where the resistance takes place; and such it is in a strict sense if the lever arm is straight and the force acts at right angles to the lever. But in Fig. 26 the true length of the lever arm is the distance L, and not the length of the handle of the wrench, because L is the effective length acting, in the position shown. ***The true lever arm***, therefore, ***is the perpendicular distance from the line of action of the force to the fulcrum.***

A familiar example of the moment of a force is to be had in the action of the foot in pedaling a bicycle. When the crank has passed the upper center, and the foot is ready for the downward push, it will require a much greater effort to drive the wheel ahead than when the crank is at right angles to the direction of the motion of the foot. The crank, of course, is of the same length whatever its position; but considered as a lever, the length of its arm varies from nothing at the upper center, to the full length of the crank at the extreme forward movement of the foot. The moment of the force exerted by the foot, therefore, gradually increases from nothing at the upper part of the stroke to the greatest amount at the forward position.

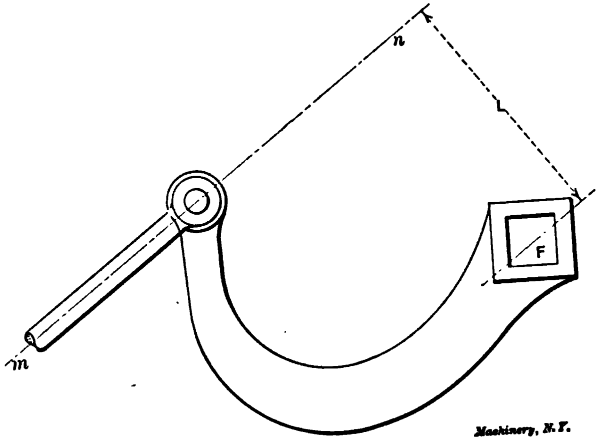

Fig. 27.

Still another illustration is to be had in the curved crank shown in Fig. 27. The crank turns about the point *F*, and a rod is attached at the outer end which pushes in the direction shown by line *m n*. Drawing this dotted line *m n* through the point at which the push is applied and in the direction in which the push is exerted, we have *L*, which is drawn at right angles to *m n*, as the length of the lever arm, and the moment of the force is the length *L* multiplied by the force *P*.

The Principle of Moment.

Thus far the illustrations that have been used have pertained to what might be called single-armed levers. We have considered only the forces acting without regard to the resistance that had to be overcome, and the levers themselves have been more of the nature of a crank than of a lever, though it is not always easy to make a distinction between the two. It is evident, however, that wherever a force is exerted, there must also be a resistance, as otherwise no initial force would be required to create motion. In the case of the wrench, the resistance was the friction between the threads of the bolt and nut acting at the end of a lever arm equal to the radius of the bolt; and in the case of the bicycle crank, the resistance was at the rim of the bicycle wheel, the lever arm in this case being more complicated because of the sprockets and chain.

In Fig. 28 is shown a bell-crank lever pivoted at the fulcrum *F*. A pull *P* is exerted along the rod at the left, and this is balanced by another pull along the rod at the right, which acts as a resistance to the force *P*. To determine the relative rotative effects of the pull *P* and the resistance *R*, we must determine the moments of these two forces. To find the moment of *P*, draw a line *m n* through the point of the lever at which *P* takes effect, and in the direction of the line in which it acts. Then draw the line *L* from the fulcrum *F* and at right angles to the line *m n*. This will be the true lever arm, and the moment of *P* will be the product of *P* and the length *L*. To find the moment of *R*, draw the line *c d* through the point of application of *R* and in the direction of *R*. Then draw the line *D* of a length equal to the perpendicular distance from *F* to line *c d*. This will be the true lever arm for *R*, and the moment of *R* will be the product of *R* and the distance *D*.

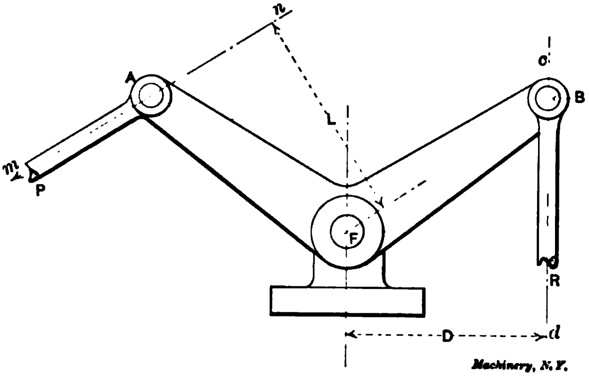

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Fig. 28.

Since the moment of *P* measures the rotative effect of this force and the moment of *R* measures the rotative effect of the resistance, it is clear that if the lever is to balance, these two moments must be equal. If *L* is longer than *D*, as it is in this case, then *R* must be enough greater than *P* to make up for this, or otherwise the lever would begin to turn about *F*. This, in substance, is all there is to the principle of moments. The principle states that, if a body is to be in equilibrium, the sum of the moments of the forces which tend to turn it in one direction about a point is equal to the sum of the moments that tend to turn it in the opposite direction about the same point. In other words, if a body is to balance about a point, the opposing moments must be equal.

Calculation of Simple Levers.

We will now be ready to solve examples of the lever by the aid of the principle of moments, and we will first consider that the weight of the lever may be neglected, and that there are only two forces acting—the push or pull—which is applied to the lever, and the resistance overcome, these being balanced, of course, by the pressure at the fulcrum, which, in reality, is another force, but which need not be considered for the present, at least.

In Fig. 29 is shown a lever supported on the fulcrum *F*. At one end a push, *P*, of 10 pounds, is exerted, and at the other end is a resistance *R*, in the shape of a 100-pound weight. The distance from *F* to *P* is 40 inches, and from *F* to *R*, 4 inches. The principle of moments states that when a lever is in balance, the moment of the force tending to turn it in one direction must equal the moment of the force tending to turn it in the opposite direction. In Fig. 29 the moment of force *P* about fulcrum *F*, tending to depress the left-hand end of the lever, is $10 \times 40 = 400$ inch-pounds, and the moment of force *R* is $100 \times 4 = 400$ inch-pounds also, so that the lever is in balance.

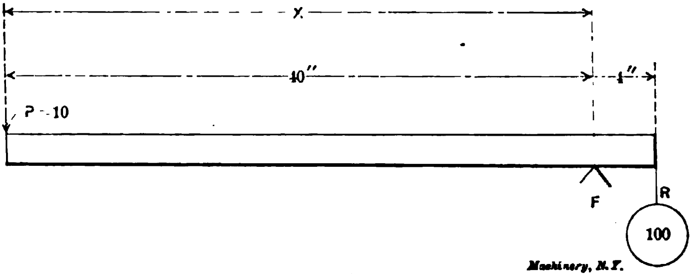

Fig. 29.

Now, suppose that we had *P*, *R*, and the distance from *F* to *R* given in Fig. 29, and that we wanted to find the distance from *F* to *P*, which we will call x. By the principle of moments we have,

Moment of $P = 10 \times x$,

Moment of $R = 100 \times 4 = 400$.

But these moments are equal; hence, $10 \times x = 400$, and what we have to do is to find the value of x. It is clear that, if ten times the distance $x = 400$, the distance x must be 1/10 of 400, and all we have to do is to divide 400 by 10, giving 40 inches as the distance x.

Again, suppose it were desired to find the resistance *R*, the other quantities being known. For convenience we will take the moment of *R* first, because this contains an undetermined value. This is always a good rule to follow.

Moment of $R = 4 \times R$. (It makes no difference whether the 4 or the *R* is written first, but it is better to write the figure first.)

Moment of $P = 10 \times 40 = 400$,

Then $4 \times R = 400$ and $R = \frac{400}{4} = 100$ pounds.

These simple examples contain all that need be known to solve lever problems where there are only two forces acting; but to make the subject still clearer, a more general example will be taken.

In Fig. 30 the lever shown is pivoted at *F*, which serves as the fulcrum. A push *P* is exerted by the rod at the right, which receives its motion from the cam and roller, as indicated. This push acts to overcome a resistance *R*, which acts along the rod seen at the left, and which may be supposed to consist of the resistance of the spring coiled around the rod, and of any piece of mechanism that this rod may have to operate. Let it be required to find how great a push, *P*, is necessary to overcome a resistance, *R*, of 250 pounds. The first thing is to find the length of the true lever arms, since without these the moments cannot be determined. To do this, first draw lines through the points on the lever at which the forces act, and in the direction in which they act. Thus, the force *P* acts at the point *C*, and the line *D H* indicates the position and direction of this force. Likewise the force *R* acts at point *B*, and line *A B* indicates the position and direction of force *R*.

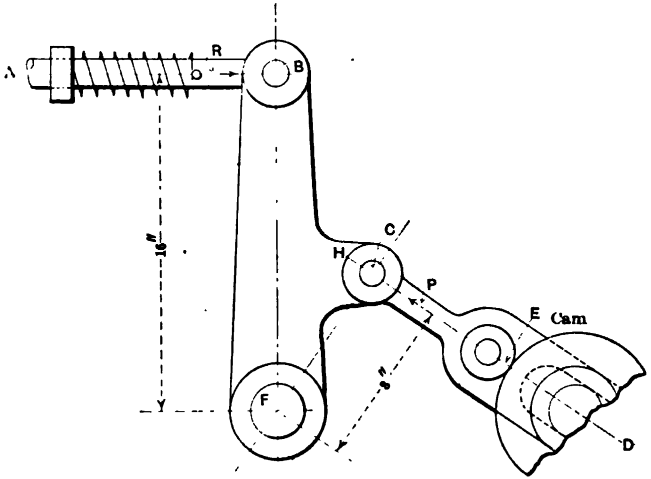

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Fig. 30.

Now, the lever arm of force *P* is the perpendicular distance from *F* to line *D H*, and the lever arm of force *R* is the perpendicular distance from *F* to line *A B*. Assume that these distances measure 8 and 16 inches, respectively. Then,

Moment of $P = 8 \times P$.

Moment of $R = 250 \times 16 = 4{,}000$.

$$8 \times P = 4{,}000; \text{ and } P = \frac{4000}{8} = 500 \text{ pounds.}$$

Example.—Suppose $P = 400$, $R = 150$, and the short arm $= 6$ inches. What is the length of the long arm? Answer—16 inches.

The safety valve in Fig. 31 is an example of a lever in which there are three forces to be considered, if we take into account the weight of the lever, which is quite essential to do. The valve at *V* is acted upon by the pressure of the steam, tending to raise it. This pressure constitutes the push *P* upon the lever, which is resisted by the suspended weight *R*, and the weight of the lever, which we will call R_1. The weight of the lever is effective at the point *G*, the center of gravity of the lever. This point can be found by balancing the lever on a knife edge, the center of gravity being directly over the knife edge. The fulcrum of the lever is at *F*, and the lever arms for *R*, R_1 and *P* are marked *A*, *B*, and *C*, respectively.

Example 1.—Assume that $A = 30$ inches, $B = 14$ inches, $C = 3$ inches,

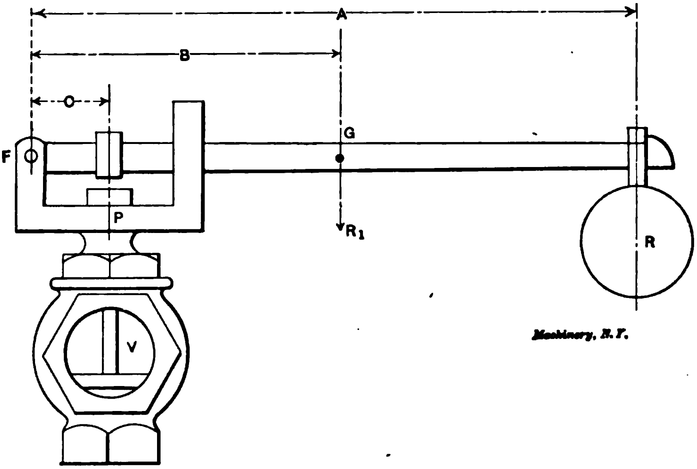

Fig 31.

$R = 20$ pounds, and $R_1 = 8$ pounds. Find what pressure of steam the valve will carry.

Moment of $P = 3 \times P$,
Moment of $R = 20 \times 30 = 600$,
Moment of $R_1 = 8 \times 14 = 112$.

For the valve to balance, the moment of *P* must be equal to the sum of the moments of *R* and R_1, for the moment of *P* tends to raise the lever, and the other moments tend to hold it down. Adding the moments of *R* and R_1, therefore, we have $600 + 112 = 712$, and this must balance the moment of *P* or $3 \times P$. Hence, $3 \times P = 712$, and $P = \frac{712}{3}$ $= 237\ 1/3$ pounds. This last part of the operation is like the work of the previous examples. The 237 1/3 pounds is the total pressure upon the valve, and to obtain the pressure per square inch that can be carried, we have have simply to divide 237 1/3 by the area of the valve.

To be theoretically exact, the weight of the valve and stem should be added to the figure 237 1/3.

Example 2.—Suppose it were desired to carry a total pressure upon the valve of 300 pounds. With the other dimensions remaining as before, how heavy a weight R would have to be provided? Again, taking moments, we have,

$$\begin{aligned}\text{Moment of } R &= 30 \times R,\\ \text{Moment of } R_1 &= 8 \times 14 = 112,\\ \text{Moment of } P &= 300 \times 3 = 900.\end{aligned}$$

The sum of the first two moments must equal the last one, but we cannot add them as they stand, because we do not yet know what the first one is. Hence we will indicate the addition as follows:

$$30 \times R + 112 = 900.$$

Those who have had a little practice with formulas will have no trouble with finding the value of R; but for the benefit of those who have not, it can be said that we subtract the 112 from 900 and proceed as in the other examples. Thus, $900 - 112 = 788$, whence

$$R = \frac{788}{.30} = 26\ 4/15 \text{ pounds.}$$

The following explanation will make the reason for subtracting 112 from 900 clear. We have found that the moment of R is 788; of R_1, 112; and of P, 900. Now, if 788 added to 112 equals 900, 900 must be 112 greater than 788, and 788 must be equal to 900 with 112 subtracted from it. Again, taking the formula as we have it, if $30 \times R$ plus 112 equals 900, $30 \times R$ must equal 900 with 112 subtracted from it.

Calculation of Compound Levers.

It often happens that it is necessary to use two or more levers connected one to the other in a series, where it would not be convenient to obtain the desired multiplication with a single lever, or where it is necessary to distribute the forces acting. In such cases the levers are called compound levers, and their application is found in testing machines, car brakes, printing presses, and many other machines and devices. Probably the most familiar example is that of a pair of scales, and we will take this to illustrate the method of making the calculations for compound levers.

In Fig. 32 is a diagram showing an arrangement of levers that might be used for platform scales. The fulcrum of the various levers are in each case marked F. The scale platform is at E, bearing at each end on levers C and D, and loaded at the center with 1,000 pounds. A pressure of 500 pounds, therefore, is transmitted to lever C at a point 6 inches from the fulcrum, and 500 to lever D. As lever D is proportioned exactly the same as that part of lever C to the left of the center line of the weight—that is, as the distance from F to L in each case is exactly 4 feet, and the short arms are each 6 inches long—it follows that the final effect is the same as though the whole 1,000 pounds acted at a point 6 inches from the fulcrum F of the lever C.

Continuing through the various connections, the right-hand end of *C* pulls down on the lever *B* at a point 8 inches from its fulcrum, and this in turn pulls down on the scale beam *A* at a point 4 inches to the left of its fulcrum, and lifts the weight *R*. Question: What weight at *R* is required to balance the 1,000 pounds on the platform, assuming that the system of levers is in balance so that there is no unbalanced weight to be considered? This is always provided for by a counterpoise on the scale beam.

The best way to solve any example of compound levers is to first determine the number of multiplications of each lever. Lever *A* has arms 40 and 4 inches long, and multiplies 10 times; lever *B* multiplies

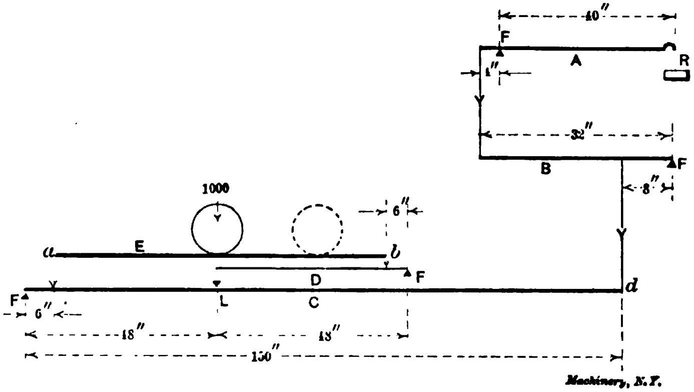

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Fig. 32.

4 times; and lever *C*, 25 times. Each lever multiplies in the same direction; that is, it tends to increase the force acting when we start at point *R*. Hence, the total multiplication is 10 × 4 × 25 = 1,000, and thus one pound at *R* would balance the 1,000 pounds on the platform.

It may be asked whether with this arrangement the weighing of the scale would not be altered should the weight be moved to the dotted position shown in Fig. 32. A little thought will show that it would not. We have seen that the reduction from both points *a* and *b* to point *d* is 25 to 1, and it can make no difference whether 500 pounds acts at both *a* and *b*, or whether, for example, 300 pounds acts at *a* and 700 at *b*, the total 1,000 pounds being reduced 25 to 1 in either case.

CHAPTER V.

THE CENTER OF GRAVITY.

The force of gravity is exerted upon every one of the particles composing a body. The number of gravity forces acting upon a body may therefore be considered equal to the number of particles composing it. The sum or resultant of these individual forces constitutes the aggregate gravity of the body; and that point in the body at which may be applied a single resultant force that will have an effect the same as that of all the gravity forces acting upon its separate particles, is the center of gravity of the body. The center of gravity of a body will, therefore, be given by the position of the resultant of all the gravity forces acting upon its particles. If a body is supported upon its center of gravity, it will be in equilibrium in any position, and will have no tendency to rotate. This is, in substance, a definition that is sometimes given for the center of gravity.

Each one of the gravity forces acting upon the particles of a body, except those forces whose lines of action pass through its center of gravity, is producing a moment, and has a rotative effect. The lever arm of each moment is the perpendicular distance between the line of action of the force and the center of gravity of the body. Every such moment tends to produce rotation in the body, and as rotation is not produced when the body is supported upon its center of gravity, it follows that the center of gravity of a body is that point at which the moments of all the gravity forces acting upon its particles balance each other, or, in other words, at which the resultant moment of all the gravity forces is zero. This fact may be made use of in determining the position of the center of gravity. Different methods are employed for finding the center of gravity, according to the form of the body, or the arrangement of the system of bodies, for which it is to be found. Some of these methods will now be explained.

Center of Gravity of Lines.

The word line, as here used, means a material line; that is, a homogeneous body of given length, having a uniform and very small transverse section, such as a fine wire. A theoretical line would, of course, have no width or thickness, and consequently, no mass and no gravity.

Single, Straight Line.

The center of gravity of a straight line is at its middle point. If we conceive the line to be composed of uniform individual particles, the gravity of each particle will be the same; and the distance of each particle on one side of the middle point, from that point, will be the same as that of the corresponding particle on the opposite side.

Hence, the moments of all the gravity forces acting upon the particles, taken about the middle point of the line, will balance, and that point will, therefore, be the center of gravity of the line. A straight line will balance upon its middle point; if supported upon that point, it will be in equilibrium in any position, and will have no tendency to rotate.

Two Straight Lines of Different Length.

Let AB and CD, Fig. 33, be two straight lines of any lengths and having any positions with respect to each other. The center of gravity of each line is àt its middle point, as O and O_1. If these two centers of gravity be connected by the straight line OO_1 the center at gravity of the system will be somewhere on this line. Draw the line OB_1 equal and parallel to $O_1B = \frac{1}{2}AB$; on the opposite side of OO_1 lay off on the line BA, a length O_1C_1 equal to $OC = \frac{1}{2}CD$, and draw B_1C_1. The point g, where the lines OO_1 and B_1O_1 intersect, will be the center of gravity of lines AB and CD. If the given lines are parallel, OB_1 is simply laid off on OD prolonged. The distance O_1g may also be calculated; it is given by the equation:

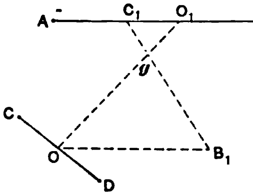

Fig. 33.

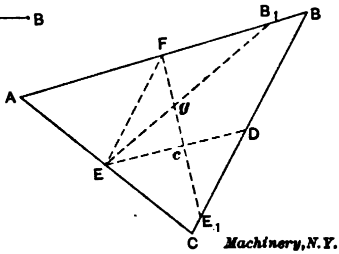

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Fig. 34.

$$O_1g = \frac{CD \times OO_1}{AB + CD}$$

Perimeter of the Triangle.

Let ABC, Fig. 34, be any plane triangle, in which D, E and F are the centers of gravity of the three respective sides. Join any two of these centers, as D and E, and on this line determine, by the method just explained, the center of gravity c of the two sides joined. To do this, join E and F; the line EF will be equal and parallel to CD; then lay off DE_1 equal to CE; the intersection c of the lines DE and E_1F will be the center of gravity of the sides BC and CA. Now lay off $FB_1 = \frac{1}{2}AE + \frac{1}{2}BD$ and draw EB_1; the intersection g of the lines EB_1 and cF will be the center of gravity of the three sides, or perimeter, of the triangle.

Circular Arc.

Let ABC, Fig. 35, be the arc of a circle whose center is at O; AC is the chord and B is the middle point of the arc. The center of gravity of the arc will be at some point g on the radius OB, at such distance from O that

$$Og = \frac{AC \times BO}{ABC}$$

Center of Gravity of Plane Surfaces.

A theoretical surface has no thickness, and, therefore, no mass and no gravity. In mechanical problems, however, it is often necessary to find the center of gravity of a plane figure, or, more correctly, that point in its surface corresponding to what would be the center of gravity of the figure, were it a material body of uniform thickness. As here used, therefore, the word surface may be taken to mean a material surface, such as a very thin, homogeneous plate of a piece of cardboard.

Axis of Symmetry.

If a plane figure can be divided by a straight line in such manner that the two parts of the figure will exactly coincide when folded together along the line, the line so dividing the figure is called an axis of symmetry. The diameter of a circle and the diagonal of a square are axes of symmetry for those figures.

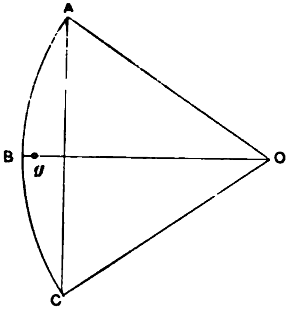

Fig. 35.

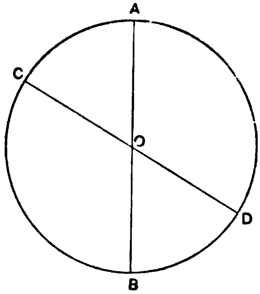

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Fig. 36.

The center of gravity of a plane figure having an axis of symmetry, must lie on such axis; if the figure has more than one axis of symmetry, the center of gravity must be at the intersection of the axes. Let *A B*, Fig. 36, be a diameter of a circle whose center is at *O*; it is also an axis of symmetry, for, if folded along this diameter, the two parts of the circle will exactly coincide. If, now, we consider the area of the circle to be composed of straight lines perpendicular to *A B*, which are not shown in the figure, the diameter *A B* will bisect each line; in other words, it will pass through the center of gravity of each line composing the area of the circle. Hence, the center of gravity of the entire system of lines composing the area of the circle, which will be the center of gravity of the circle itself, must be some point on the diameter *A B*. In like manner it can be shown that the center of gravity of the circle must lie on any other diameter, as the diameter *C D*. Consequently, the center of gravity of the circle must be at the center *O*, the only point common to all diameters. That the center of gravity of the circle is at the geometrical center of the figure is so evident as to scarcely require proof; but the circle serves as a very simple example to illustrate the process of reasoning, which applies to any plane figure having two axes of symmetry, such as a circle, ellipse, rectangle, rhombus, equilateral triangle, square, or any regular polygon, and also to the perimeters of such figures.

Center of Gravity of Parts of Circles.

Semicircle. The center of gravity is located on its axis of symmetry, at a distance of $0.4244r$ from the center of the circle, r being the radius of the circle.

Sector of a Circle. The center of gravity is located on its axis of symmetry, at a distance x from the center of the circle, the value of x being given by the equation:

$$x = \frac{2\,c\,r}{3\,l},$$

in which c is the chord and r the radius of the circle, and l the length of the arc.

Quadrant of a Circle. The center of gravity is located on its axis of symmetry, at a distance of $0.4244r$ from each radial side, or $0.6002r$ from the center of the circle, r being the radius of the circle.

Segment of a Circle. The center of gravity is located on its axis of symmetry, at a distance x from the center of the circle, the value of x being given by the equation:

$$x = \frac{c^3}{12\,a},$$

in which c is the chord and a the area of the segment.

Other Surfaces with Curved Outlines.

Parabolic Surface. The center of gravity is located on its axis of symmetry, at 2/5 the length of the axis from the base.

Semi-Parabolic Surface. The center of gravity is located at 2/5 of the length of the axis of the parabola from the base, and 3/8 the length of the semi-base from the axis.

Surface of a Hemisphere. The center of gravity is located at the middle of its axis or center radius.

Gravity Axis.

It is not necessary, however, for a plane figure to have two, or even one, axis of symmetry, in order that its center of gravity may be determined. Any plane figure can be balanced upon a knife edge. The position of the knife edge will be defined by a straight line in such position that the moments of all the gravity forces acting upon the particles composing the surface on one side of the line will just balance the moments of those on the other side. This line, about which the moments of the gravity forces balance, will here be called a gravity axis. By a process of reasoning analogous to that employed in finding the center of gravity of the circle, it can be shown that every gravity axis of a plane figure contains the center of gravity of the figure. Consequently, the intersection of any two gravity axis determines the position of its center of gravity. It should be noticed that in many practical problems it is necessary to find the position of a gravity axis only, the exact center of gravity not being required.

Triangle.

Let *A B C*, Fig. 37, be any triangle; the line *C D* extends from the vertex *C* to the middle of the opposite side. If we consider the area of the triangle to be composed of straight lines parallel to the base *A B*, each of these parallel lines will be bisected by the line *C D*; that is, the line *C D* will pass through the center of gravity of each of the parallel lines. Every line composing the area of the triangle, and, consequently, the triangle as a whole, will just balance upon the line *C D*, which will be a gravity axis of the triangle. If, also, a line be drawn from any other vertex of the triangle to the middle of the opposite side, as the line *A E*, it will also be a gravity axis. As the center of gravity must lie on both these gravity axes, it must be at their intersection *g*. It is not necessary, however, to draw more than one gravity axis, in order to determine the position of the center of gravity of a triangle. If a line be drawn from any vertex to the middle of the opposite side, the center of gravity of the triangle will be on this line and at two-thirds the length of the line from the vertex. Thus, the center of gravity *g*, Fig. 37, is at two-thirds the length of *A E* from *A*, two-thirds the length of *B F* from *B*, and two-thirds the length of *C D* from *C*; its position may be located on any one of the lines.

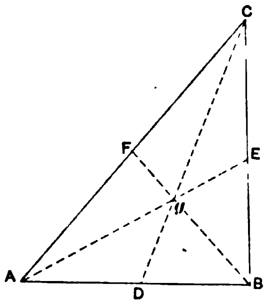

Fig. 37.

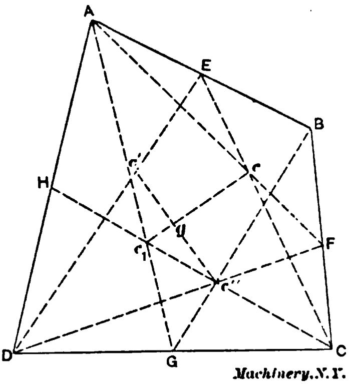

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Fig 38.

Trapezium.

There are several quite satisfactory methods for finding the center of gravity of a trapezium. The following simple method is probably as expeditious as any, and, as it depends upon the method just explained for finding the center of gravity of a triangle, and is readily connected with that method, it has the advantage of being easily remembered.

Let *A B C D*, Fig. 38, be any four-sided plane figure. Consider it first to be divided into the two triangles *A B C* and *A D C*. The points *E*, *F*, *G*, and *H* are the centers of the respective sides, the common side *A C* not being drawn. The intersection *c* of the lines *A F* and *C E* is the center of gravity of the triangle *A B C*, and, similarly, the intersection c_1 of the lines *A G* and *C H* is the center of gravity of the triangle *A D C*. The line $c\,c_1$ connecting these two centers of gravity, will be a gravity axis of the entire figure. The trapezium is then considered to be divided into the triangles *B A D* and *B C D*, and, by a similar construction, the position of the gravity axis $c'c''$ is determined. The intersection *g* of these two gravity axes will be the center of gravity of the trapezium.

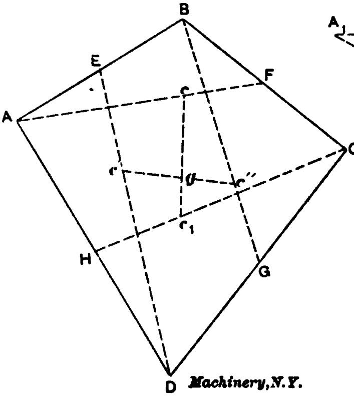

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Fig. 39.

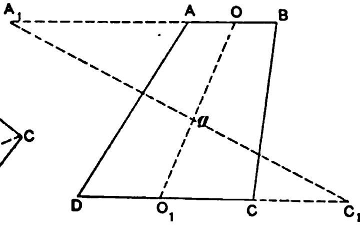

Fig. 40.

For this construction, it is not necessary to draw the entire portion of each constructional line, as shown in the figure, but only such portions of the lines as are necessary to locate their intersections. Some may prefer the construction shown in Fig. 39; it is the same as that shown in Fig. 38, except that only one gravity axis is drawn for each triangle, and the center of gravity of the triangle located at two-thirds the length of the axis from its vertex.

Trapezoid.

If the figure is a trapezoid, the following construction, taken from "Trautwine's Engineer's Pocket Book," is a very simple method of finding its center of gravity. Let *A B C D*, Fig. 40, be any trapezoid for which the center of gravity is to be found. Prolong the two parallel sides in opposite directions, making each prolongation equal to the other side, and join the extremities of the prolongations by a straight line; also join the centers of the parallel sides. The intersections of these lines will be the center of gravity of the figure. Thus, in the figure, $A\,A_1$ is made equal to *D C*, and $C\,C_1$ equal to *A B*, and the exteremities of the prolongations joined by the line A_1C_1, while the line $O\,O_1$ joins the centers of the parallel sides; the intersection g of the lines A_1C_1 and $O\,O_1$ is the center of gravity of the trapezoid.

Irregular Figure.

The center of gravity of any irregular figure bounded by straight lines may be found by dividing it into triangles, finding the center of gravity of each triangle, and then finding the center of gravity of the system of triangles, the area of each being considered to be concentrated at its center of gravity. For finding the center of gravity of the system of triangles, the method of rectangular co-ordinates may be employed. Let $A\,B\,C\,D\,E\,F$, Fig. 41, be any irregular figure bounded by straight lines. By the lines $A\,C$, $A\,D$, and $A\,E$ the figure can be divided into the four triangles $A\,B\,C$, $A\,C\,D$, $A\,D\,E$, and $A\,E\,F$, whose centers of gravity, g_1, g_2, g_3, and g_4 may be found by the method explained for triangles. Draw the vertical and horizontal axes $O\,Y$ and $O\,X$, intersecting at O; these may be any vertical and horizontal lines, but it is generally convenient to draw them through the left-hand and lower extremities of the figure, as shown; $O\,X$ is the axis of

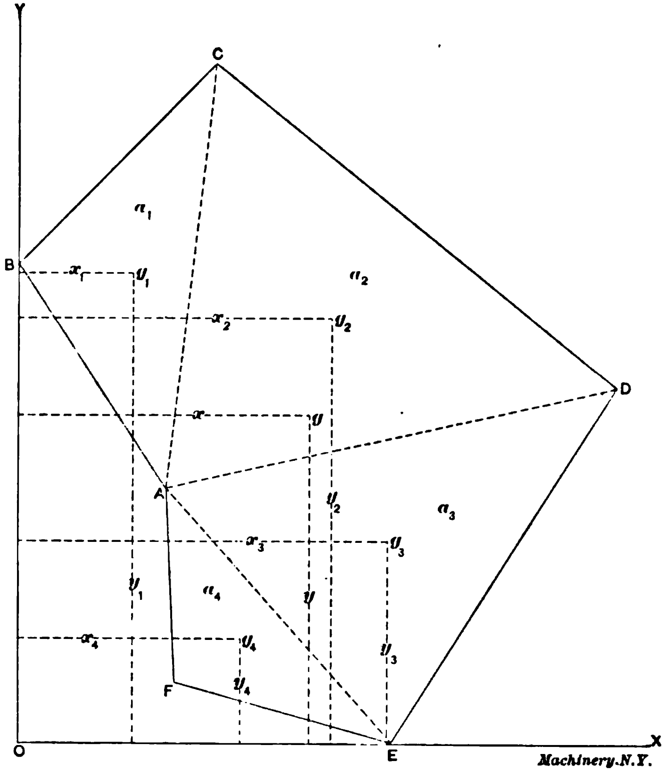

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Fig. 41.

abscissas and $O\,Y$ is the axis of ordinates. The lines x_1, x_2, x_3, and x_4 are, respectively, the abscissas of the centers of gravity from the axis of ordinates; and the lines y_1, y_2, y_3, and y_4 are, respectively, the ordinates of the same points, or their perpendicular distances from the axis of abscissas. If a_1, a_2, a_3, and a_4 represent the areas of the four respective triangles, then the abscissa x to the center of gravity g of the entire figure will be given by the equation:

$$x = \frac{a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4}{a_1 + a_2 + a_3 + a_4},$$

and the ordinate y to the same point will be given by the equation:

$$y = \frac{a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4}{a_1 + a_2 + a_3 + a_4}.$$

This method applies to any figure or system of figures, either separate or joined, that can be divided into triangles or other simple figures such that their centers of gravity and areas can be determined. It will also apply to a system of weights or solid bodies.

The gravity axes, and, consequently, the center of gravity, of a very irregular figure can be determined by the equilibrium polygon, as constructed for determining the position of the resultant of any system of forces. Let $M\,N$, Fig. 42, be any plane figure whose center of gravity is to be determined; the following construction may be employed:

By means of parallel lines, divide the figure into a sufficient number of parts of equal width, so that each part will be approximately a trapezoid, as shown by full lines in the figure; also draw lines midway between these dividing lines and parallel to them, as shown in the figure by dotted lines; as the trapezoids are of equal width, these dotted lines will be proportional to their areas. On a vertical line, as the line 0-6 (commonly called the load line), lay off in order downwards the amounts 0-1, 1-2, etc., by any scale, equal, respectively, to these latter lines, that is, proportional to the areas of the respective trapezoids. Choose any point P as a pole and draw the radial lines P-0, and P-1, etc. Find the centers of gravity, a, b, c, etc., of the respective trapezoids, and from these centers draw lines vertically downward, as the lines $a\,a'$, $b\,b'$, etc. Through any point on the line drawn downward from a, as the point a', draw the line $a'g'$, of indefinite length and parallel to P-0; from the same point also draw the line $a'b'$, parallel to P-1, to intersect the vertical through b; from this intersection draw the line $b'c'$, parallel to P-2, to intersect the vertical through c, etc. The line $e'f'$ drawn parallel to P-5, intersects the last vertical at f'; from this intersection draw the line $f'g'$, parallel to P-6, to intersect $a'g'$; the vertical line $g\,g'$ drawn through the intersection g', will be a gravity axis of the figure. In a similar manner the figure may be divided into another set of trapezoids of uniform width, by parallel lines extending in a different direction, and the position of another gravity axis determined; the intersection of the two gravity axes will be the center of gravity of the figure.

The method of construction just described is generally the most expeditious, but it is not necessary that the lines dividing the figure be drawn at equal distances apart, nor that they be parallel; they may divide the figure into smaller figures of any form, of which the centers of gravity and areas can be easily determined. It is necessary only that each of the vertical lines *a a′*, *b b′*, etc., be drawn through the center of gravity of the corresponding part, and that the amounts

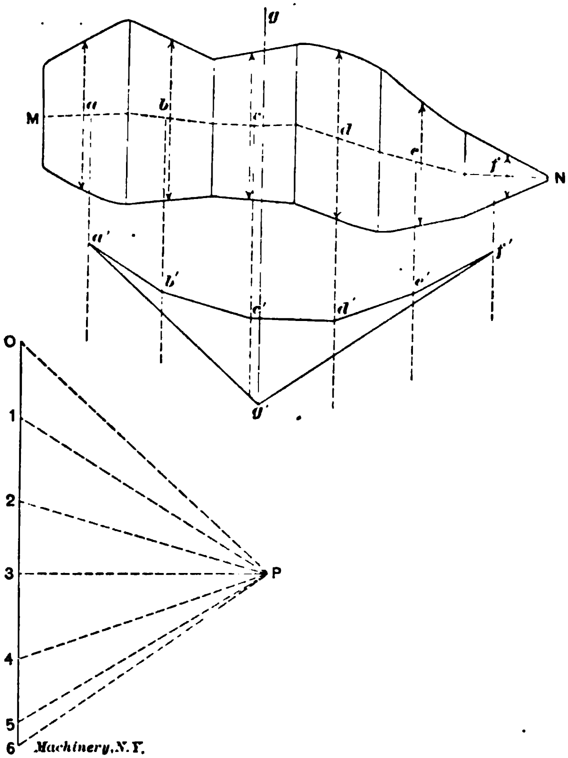

Fig. 42.

0-1, 1-2, etc., laid off on the vertical load line, be proportional to the areas of the respective parts.

Experimental Methods.

To find the center of gravity of any plane figure experimentally, proceed as follows:

First Method. Cut out the figure of cardboard or very thin metal plate. Balance it across a perfectly straight knife edge and mark the line of the knife edge across the figure; then turn the figure about 90 degrees and balance it across the knife edge again, marking the line of the knife edge. Each line defining the position of the knife edge will be a gravity axis, and the intersection of the two axes will be the center of the figure.

Second Method. Cut out the figure of cardboard or thin metal, as in the preceding method. Suspend it by a point near one edge in such manner that the figure will be free to turn about the point of suspension; suspend a plumb line from the same point of suspension and mark its direction across the figure. Then suspend the figure from some other point, and again mark the direction of the plumb line. The two lines defining the positions of the plumb line will be gravity axes, and their intersection will be the center of gravity of the figure. The figure may be suspended by a pin driven through the cardboard into a vertical wall; but, in order that the figure may be free to turn, it will generally be better to suspend it by a thread. These methods have already been explained in Chapter II, in connection with the moment of resistance.

Center of Gravity of Solid Bodies.

The center of gravity of a sphere, spheroid, cylinder, cylindrical ring, cube, prism, parallelopipedon or any polyhedron is at the geometrical center of each body.

The center of gravity of a cylinder or prism is at the middle point of a line joining the centers of gravity of its parallel surfaces.

The center of gravity of a hemisphere is on its axis, or radius perpendicular to its base, at ⅜ length of the radius from the center of the sphere.

The center of gravity of a right cone or right pyramid is in the line joining the vertex with the center of gravity of the base, at ¼ the length of the line from the base.

If a body be suspended freely at a point other than its center of gravity, its center of gravity will be vertically below the point of suspension. This principle affords an easy method of finding the center of gravity of any body, as described in the second method for finding experimentally the center of gravity of any plane figure.

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No. 20

SPIRAL GEARING

CONTENTS

Calculating Spiral Gears, by E. H. FISH - - - - 3

Rules and Formulas for Designing Spiral Gears, by RALPH E. FLANDERS - - - - - - - - - 14

Diagrams for Designing Spiral Gears, by FRANCIS I. BOSTOCK - - - - - - - - - - 25

Comparison of Efficiency of Spiral Gears, by RALPH E. FLANDERS - - - - - - - - - - 32

Setting the Table when Milling Spiral Gears, by WALTER GRIBBEN - - - - - - - - - - 37

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CHAPTER I.

CALCULATING SPIRAL GEARS.

In taking up the subject of spiral gears with students at the Worcester Polytechnic Institute, some difficulty was experienced over the formulas relating to their construction. If these trouble men accustomed to the use of trigonometry, they must certainly be confusing to shop men. As the explanation of the method of figuring these gears, which has been arrived at for the students' use, involves a minimum amount of mathematics, it may be of value to others. At the same time, the formulas given will be thoroughly explained and mathematically proved for the benefit of those who wish to fully study the subject.

As it is most convenient to adapt these gears to the standard diametral pitch cutters used for spur gears, we will consider the subject only from that point of view. This gives us at once the normal pitch, that is, the pitch measured perpendicular to the face of the tooth, and also the shape and depth of the tooth.

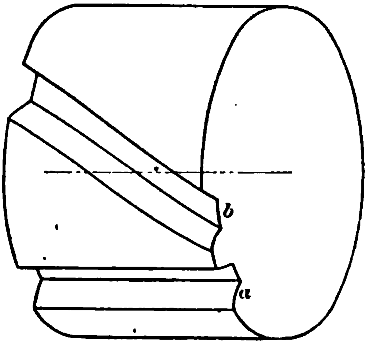

Fig. 1.

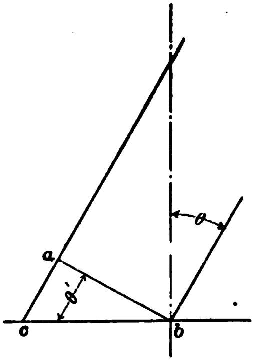

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Fig. 2.

In the case of a spur gear, we cut the teeth at right angles with the base of the cylinder on which the gear is cut, as at *a* in Fig. 1. The space appears in its true size and shape on the base of the blank. If, now, we cut the teeth at some other angle, say at 30 degrees with a line parallel to the axis of the gear blank, as at *b* in Fig. 1, we see that the width of the space measured on the base is greater, and it will be greater still if the angle is increased. It is thus evident that the number of teeth that can be cut on a given cylinder decreases as the angle of the teeth with a line parallel with the axis of the gear increases.

Number of Teeth and Diameter of Blank.

Referring to Fig. 2, suppose the line $b\,c$ to be a part of the base of the cylinder, and the two parallel lines making the angle θ with a line parallel with the axis to represent the center lines of two adjacent teeth. Then, $a\,b$ will represent the normal pitch, and $b\,c$ the circumferential pitch; but, as $\theta' = \theta$,

$$a\,b = b\,c \cos\theta, \text{ whence}$$

$$b\,c = \frac{a\,b}{\cos\theta}.$$

The number of spur-gear teeth that can be cut in a blank of pitch radius r is expressed by the formula:

$$N_c = 2\,r\,P, \qquad (1)$$

where P is the diametral pitch and N_c the number of teeth.

From this we see that the number of teeth in a spiral gear of this pitch and pitch radius, and of angle θ, will be

$$N = 2\,r\,P \cos\theta. \qquad (2)$$

Take as an example a gear to be cut 6-pitch, with teeth at an angle of 60 degrees to a line parallel with the axis of the gear, and a pitch diameter of about 2½ inches. Then, $r = 1\frac{1}{4}$; $P = 6$; $\cos\theta = 0.5$. Hence,

$$N = 2 \times 1\tfrac{1}{4} \times 6 \times 0.5 = 7\tfrac{1}{2}.$$

As a gear of 7½ teeth is impossible for continuous rotation, we must make the number of teeth either 7 or 8. Suppose we make it 8. Then, to find the pitch diameter of the gear we use the same formula, but transposed as follows:

$$r = \frac{N}{2\,P \cos\theta},$$

from which we get, after substituting 8 for N,

$$r = \frac{8}{2 \times 6 \times 0.5} = 1\ 1/3.$$

The pitch diameter of our blank must, therefore, be 2 2/3 inches, the same as for a spur gear of 16 teeth. As we are using diametral pitch cutters, the addendum will be the same as for a spur gear of the same pitch. Adding $\frac{1}{P}$ to the pitch diameter on each side will make the whole diameter 3 inches in this case.

Milling Spiral Teeth.

In order to mill the teeth, we must be able to set up the machine so as to make, approximately, the correct advance per revolution of the work. This advance will be equal to the circumference of the blank, measured on the pitch line, multiplied by the cotangent of the angle of the teeth. As this usually presents no difficulty, we pass it over with simply saying that gears run together quite nicely, even if the lead as figured is not exactly obtainable on the milling machine.

After having set our machine to cut the desired spiral, we next wish to select the proper cutter. This will be, unless the angle θ is very small, quite a different cutter from that used for a spur gear of the same diameter, or of the same number of teeth. Brown & Sharpe Co. advises turning up a blank of the size of the pitch diameter and laying out on it a helix at right angles to the helix of the teeth of the gear to be cut, as in Fig. 3, fitting a cardboard templet to the face of the cylinder along this curve, and then finding the diameter of the circle corresponding to this templet.

The cutter should be such as will be suitable for a gear of this diameter and the given normal pitch. This is a sufficiently close method for gears of a large number of teeth, but requires considerable care for gears of 12 or less teeth. Moreover, we require a method that can be worked out entirely in the drafting room. Grant says that the cutter should be right for a spur gear having a number of teeth equal to the number of teeth in the spiral gear, divided by the cube of the cosine of the angle of the teeth. This gives an exact result, but he offers no explanation of his statement. The following, we hope, will seem a clear demonstration.

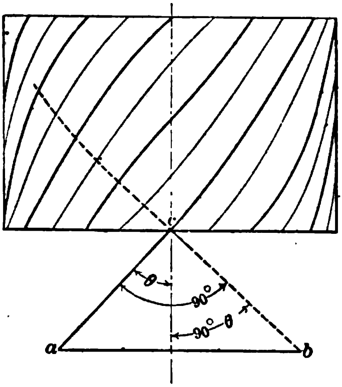

Fig. 3.

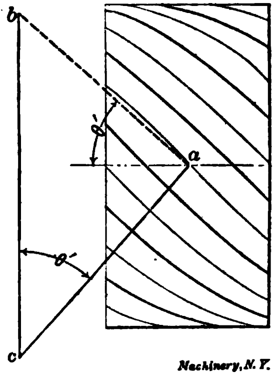

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Fig. 4.

Demonstration of Grant's Formula.

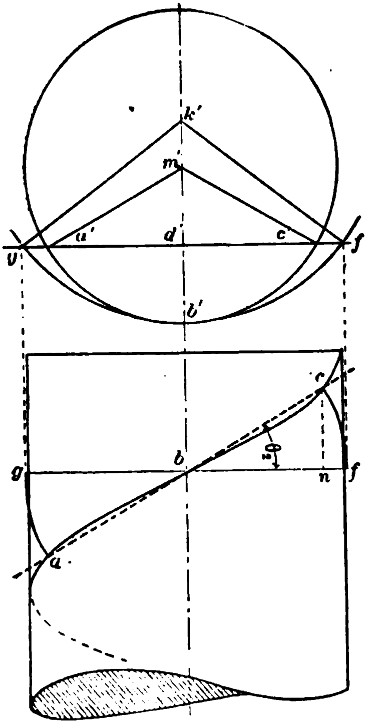
It will be seen that what we wish to find at the start is a circle having the same radius as the helix which is drawn on our pitch cylinder perpendicular to the teeth, as in Fig. 3. The angle of this helix will be $90 - \theta$ degrees. If $R =$ radius of curvature of this helix, then from the well-known formula in analytic geometry for the radius of curvature of a helix, we have

$$R = \frac{r}{\sin^2(90 - \theta)} = \frac{r}{\cos^2 \theta}. \qquad (3)$$

The demonstration of this formula is as follows:

Assume that $r =$ radius of the cylinder on which the helix is drawn, and θ is the angle of the helix with a line parallel to the axis of the cylinder. In Fig. 5 is a cylinder of radius $m'c' = r$, on which is drawn a helix. We have assumed three points, *a*, *b*, and *c* equidistant on the helix, the middle point, *b*, being taken at the extreme front of the helix, for convenience only.

We wish to draw a circle passing through the three points, *a*, *b*, and *c*. To do this we have revolved the two outside points into the same horizontal plane as *b*, placing *a* at *g* and *c* at *f*. We represent these points in the top view by *g′* and *f′*. Through *g′*, *b′*, and *f′* we draw a circle having its center at *k′* and radius *k′ f′*, which we will call R_2.



Machinery, N.Y.

Fig. 5.

This circle will be represented in the front view by the horizontal line *g* to *f*. The original position of this circle in the front view is represented by the straight line *a* to *c*. The angle between these two lines we call θ_2. Remember that this is not the angle of the helix with the base, but is the angle of the original plane of the circle through *a*, *b*, and *c* with the horizontal. Now,

$$b\,n = d'\,c' = b\,c \cos\theta_2 \tag{4}$$

$$b\,c = b\,f = d'\,f'. \tag{5}$$

Then,

$$d'\,c' = d'\,f' \cos\theta_2. \tag{6}$$

Squaring,

$$(d'c')^2 = (d'f')^2 \cos^2 \theta_2 \qquad (7)$$

$$(d'c')^2 = (m'c')^2 - (m'd')^2 = r^2 - (m'd')^2 \qquad (8)$$

$$(d'f')^2 = (k'f')^2 - (k'd')^2 = R^2_2 - (k'd')^2. \qquad (9)$$

Substituting (8) and (9) in (7), we have,

$$r^2 - (m'd')^2 = [R^2_2 - (k'd')^2] \cos^2 \theta_2 \qquad (10)$$

$$m'd' = r - d'b'$$

$$(m'd')^2 = r^2 - 2\,r\,(d'b') + (d'b')^2 \qquad (11)$$

$$k'd' = R_2 - d'b'$$

$$(k'd')^2 = R_2^2 - 2\,R_2\,(d'b') + (d'b')^2. \qquad (12)$$

Substituting from (11) and (12) in (10) we get:

$$r^2 - r^2 + 2\,r\,(d'b') - (d'b')^2 =$$
$$[R^2_2 - R^2_2 + 2\,R_2\,(d'b') - (d'b')^2] \cos^2 \theta_2. \qquad (13)$$

Cancelling we have,

$$2\,r - d'b' = (2\,R_2 - d'b') \cos^2 \theta_2. \qquad (14)$$

This expression is true for any three points equidistant on the helix. Let us remember that the radius of curvature for any curve is the radius of the circle passing through any three *consecutive* points. We will accordingly consider points a and c moved up so that they become consecutive points with b, and see what the effect is on equation (14). Then

r will remain constant,

$d'b'$ will become practically zero on each side of the equation and may be neglected,

R_2 becomes R. the radius of curvature of the helix, and

θ_2 becomes θ, the angle of the helix.

Substituting these values in (14), we have,

$$2\,r = 2\,R \cos^2 \theta \qquad (15)$$

$$\text{or, } R = \frac{r}{\cos^2 \theta}$$

Referring, now, to formula (1), and applying it to a gear of radius R, we have

$$N_c = 2\,R \times P = \frac{2\,r}{\cos^2 \theta} \times P. \qquad (16)$$

For our spiral gear we found, by formula (2), that:

$$N = 2\,r\,P \cos \theta.$$

Dividing (16) by (2), we have

$$\frac{N_c}{N} = \frac{2\,r\,P}{\cos^2 \theta} \times \frac{1}{2\,r\,P \cos \theta} = \frac{1}{\cos^3 \theta}$$

or,

$$N_c = \frac{N}{\cos^3 \theta}.$$

Another derivation of the same formula, which may be of interest to some, was presented by H. W. Henes in MACHINERY, April, 1908. The following notation is used in this derivation:

N_c = number of teeth in spur gear for which cutter is intended.*
N = number of teeth in the desired spiral gear.
a = the angle which the direction of the spiral makes with the axis of the gear.

Let P_n be the perpendicular distance between two consecutive teeth on the spiral gear, and let D_1 be the diameter of the spiral gear. Let the gear be represented as in Fig. 6, and pass a plane through it perpendicular to the direction of the teeth. The section will be an ellipse as shown in *CEDF*. Designate the semi-major and semi-minor axes by a and b respectively.

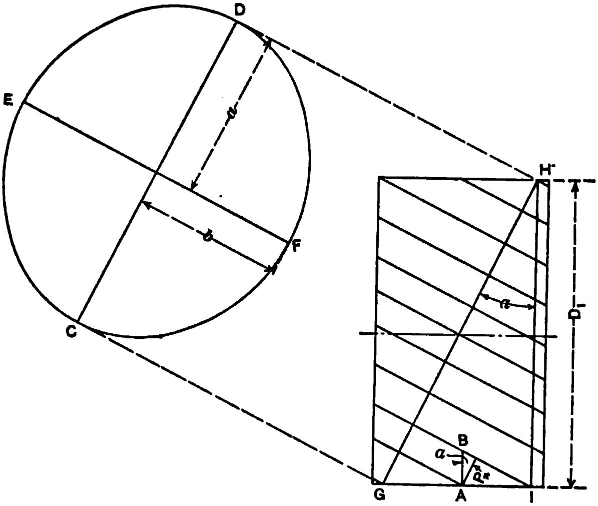


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Fig. 6. Diagram for Deriving the Formula for Determining Spur Gear Cutter for Cutting Spiral Gears.

Now N_c is the number of teeth which a spur gear would have if its radius were equal to the radius of curvature of the ellipse at *E*. Therefore, it is required to determine the radius of this curvature of the ellipse. This is done as follows:

From the figure we have:

$$2b = \text{axis } EF = D_1 \qquad (17)$$

$$2a = \text{axis } CD = GH = \frac{HI}{\cos a} = \frac{D_1}{\cos a} \qquad (18)$$

From (17) and (18) we have for a and b,

$$b = \frac{D_1}{2} \qquad (19)$$

and

$$a = \frac{D_1}{2 \cos a} \qquad (20)$$

It is known, and shown by the methods of calculus, that the minimum curvature of an ellipse, that is, the curvature at *E* or *F*, equals

$\frac{b}{a^2}$. Taking the values of a and b found in (19) and (20), we have the curvature at E:

$$\text{Curvature} = \frac{b}{a^2} = \frac{\frac{D_1}{2}}{\frac{D_1^2}{4 \cos^2 a}} = \frac{4 D_1 \cos^2 a}{2 D_1^2} = \frac{2 \cos^2 a}{D_1} \qquad (21)$$

It is also shown in calculus that the *curvature* is equal to $\frac{1}{R}$, where R is the *radius of curvature* at the point E. Therefore from (5) we have:

$$\frac{1}{R} = \frac{2 \cos^2 a}{D_1}, \text{ whence } R = \frac{D_1}{2 \cos^2 a} \qquad (22)$$

Formula (22) can also be arrived at directly, without reference to the minimum curvature of the ellipse, by introducing the formula for the radius of curvature in the first place. The curvature is simply the reciprocal value of the radius of curvature, and is only a comparative means of measurement. The radius of curvature of an ellipse at the end of its short axis is $\frac{a^2}{b}$, from which formula (22) may be derived directly by introducing the values of a and b from equations (19) and (20).

Having now found the radius of curvature of the ellipse at E, we proceed to find the number of teeth which a spur gear of that radius would have. From Fig. 6 we have:

$$A B = \frac{P_n}{\cos a} \qquad (23)$$

Now, if $A B$ be multiplied by the number of teeth of the spiral gear, we shall obtain a quantity equal to the circumference of the gear; that is:

$$A B \times N = \pi D_1, \text{ and since } A B = \frac{P_n}{\cos a} \text{ from (23)}$$

$$\frac{P_n}{\cos a} \times N = \pi D_1 \qquad (24)$$

Since N_c is the number of teeth which a spur gear of radius R would have, then,

$$N_c = \frac{2 \pi R}{P_n} \qquad (25)$$

In equation (25) the numerator of the fraction is the circumference of the spur gear whose radius is R, and the denominator is the circular pitch corresponding to the cutter.

From equation (22) we have:

$$R = \frac{D_1}{2\cos^2 a}$$

Substituting this value of R in (25), we have:

$$N_c = \frac{2\pi D_1}{P_n \times 2\cos^3 a} \qquad (26)$$

From equation (24) we have:

$$D_1 = \frac{N P_n}{\pi \cos a} \qquad (27)$$

Substitute this value of D_1 in equation (26) and we have:

$$N_c = \frac{2\pi N P_n}{2 P_n \pi \cos^3 a}$$

or

$$N_c = \frac{N}{\cos^3 a} \qquad (28)$$

Since N is the number of teeth in our spiral gear and N_c is the number of teeth in a spur gear which has the same radius as the radius of curvature of the helix above referred to, this is the equivalent of saying that the cutter to be used should be correct for a number of teeth which can be obtained by dividing the actual number of teeth in the gear by the cube of the cosine of the tooth angle. Since the cosine of angle θ (or a, as it was denoted in the derivation of the cutter formula last given) is always less than unity, its cube will be still less, so N_c is certain to be greater than N, which will account for the fact that spiral gears of less than 12 teeth can be cut with the standard cutters. The getting of the cube of $\cos\theta$ may bother some, as the cubing of any fraction is apt to do, but a graphical method is given later in this chapter which, even if roughly laid out, will give sufficiently accurate results for this purpose. For the other uses of this graphical method, care must be used, or the results are not to be depended on.

Calculation of Velocity Ratio.

Now we are able to cut the gear, once having decided on the number of teeth, pitch (or pitch diameter), and angle of teeth, but in designing we almost always wish to transmit motion with some definite velocity ratio. If we were dealing with spur gears we would know that the ratio of speeds would be inversely proportional to the pitch diameters or the number of teeth. If the teeth were twisted or cut spiral on the surface and the axes still were left parallel, this same velocity ratio would obtain, but the moment we move the axes out of the same plane, this convenient ratio ceases to exist. Then there can be but one point of contact of the pitch cylinders, consequently all motion must be transmitted as if through this one point, if smooth running is to be attained. The actual motion of the tooth at this point must be at right angles to the axis of the gear, but it may be considered as the resultant of two motions, one of sliding parallel to the teeth, which we can see must happen since the two gears do not run in the same plane, and the other perpendicular to the teeth, which is the effective, or driving motion. This latter motion normal to the teeth must be the same for both gears.

In the case of a driving gear of radius r and angle θ, the velocity of this point in a plane perpendicular to the axis will be $2 \pi r n$, where n is the number of revolutions per minute of the driving gear. Let us consider the point c of the gear in Fig. 3. Assume the line $a b$ to represent the linear velocity and to be equal to $2 \pi r n$. The line $c b$ is perpendicular to the tooth, and $a c$ parallel to the tooth. These three lines complete the triangle $a b c$, and therefore $a c$ will represent the sliding component of the point c, and c b the motion perpendicular to the tooth. Then,

$$b c = 2 \pi r n \cos \theta,$$

since in the triangle $a b c$ the angle $a b c$ is equal to θ.

This, also, is the velocity of the contact point of the driven gear in the same direction, or in a direction normal to the teeth of the driven gear. We will assume this gear to have a radius r' and angle θ'. Considering the gear in Fig. 4 to be the driven gear, with axis at right angles to the axis of the driving gear, we have

$$a c \text{ (Fig. 4)} = b c \text{ (Fig. 3)} = 2 \pi r n \cos \theta.$$

The resulting motion perpendicular to the axis of the gear will then be

$$c b \text{ (Fig. 4)} = \frac{a c}{\cos \theta'} = \frac{2 \pi r n \cos \theta}{\cos \theta'}.$$

This is the linear velocity of the point a; to get the number of revolutions of the driven gear we divide by the circumference of the driven gear, which is $2 \pi r'$, giving

$$n' = \frac{2 \pi r n \cos \theta}{2 \pi r' \cos \theta'},$$

$$\text{whence } \frac{n'}{n} = \frac{r \cos \theta}{r' \cos \theta'}.$$

That is, the relative motion of the two gears is inversely proportional to the product of their diameters and the cosines of the angles of their teeth.

If both are 45-degree gears, this last factor becomes inoperative, and the gears produce motion in the same ratio as spur gears of the same sizes. The same is also true if the axes are parallel, for θ and θ' then become equal.

If the axes are at right angles, $\theta - 90 = \theta'$, and $\frac{\cos \theta}{\cos \theta'} = \frac{\cos \theta}{\sin \theta} =$ $\cot \theta$, whence: $\frac{n'}{n} = \frac{r}{r'} \times \cot \theta$.

This property of spiral gears, of having a varying velocity ratio for both size and angle, is valuable, in that it enables one to obtain varying velocity ratios with the same size gear. For example, suppose we have two gears, one of 8 teeth and one of 16 teeth, both 45-degree gears, on axes at right angles. The velocity ratio is 2 to 1. If, now, we want a velocity ratio of 3 to 1 on the same axes with the same size gears, we use the formula last arrived at,

$$\frac{n'}{n} = \frac{r}{r'} \cot \theta, \text{ or, } \frac{1}{3} = \frac{1}{2} \cot \theta,$$

$$\cot \theta = \frac{2}{3} = 0.6666.$$

θ will then be 56° 19′ and θ' will be 33° 41′.

If we use cutters of the same pitch as before, the number of teeth become fractional numbers, thus making impossible conditions for practical use. It will, then, be necessary to use a fractional pitch cutter. To find what this cutter should be, decide on the number of teeth to be used in each of the two gears to give the desired new velocity ratio of 3 to 1; then solve formula (2) for P, substituting the required data from either of the two gears.

The relation between the angles of the shafts and gear teeth will be readily understood by a little thought. In gears whose axes are at right angles we have seen that the sum of the angles of the gear teeth is equal to 90 degrees, the angle of the shafts. This is true for any gears whose spirals are both right-hand or both left-hand. Carrying this to an extreme, we find that if the tooth angles become zero degrees (as in spur gears), the shaft angle becomes 180 degrees, or the shafts are parallel. If one gear is right-hand and the other left-hand, then the angle of the shafts will be equal to the difference of the tooth angles. If the gears have their teeth at equal angles, but one right-hand and one left-hand, then the shaft angle will be zero; that is, the shafts are parallel and the gears are twisted gears, or Hooke's gears.

Fig. 7, while it is innocent looking enough, contains a solution of all the bothersome points of the figuring of the spiral gears to be cut with the usual diametral pitch cutters.

To illustrate the use of the figure, we will take as an illustration a 24-tooth gear of 30-degree spiral angle, to be cut with an 8-pitch spur-gear cutter.

Lay off $a\ b = 3$ inches, the diameter of a spur gear of 24 teeth, 8-pitch. Lay off the angle θ 30 degrees as shown, and erect a perpendicular at b to $a\ b$, intersecting at $a\ c$ at c. The line $a\ c$ will be the pitch diameter of the required spiral gear (3.46 inches). The outside diameter will be equal to this diameter plus $\frac{2}{P}$, as in spur gears (3.71 inches). The depth of tooth will be the same as for a spur gear of the same pitch.

Extend *a b* and *a c*. At *c* erect a perpendicular to *a c*, meeting *a b* at *d*. At *d*, in turn, erect a perpendicular to *a d*, meeting *a c* at *e*; *a e* will be the diameter of a spur gear having the correct number of teeth from which to choose a cutter to cut our spiral. In this case the diameter is 4⅝ inches, corresponding to a 37-tooth gear. So we will use the same cutter to cut our 24-tooth spiral gear as that we would use to cut a 37-tooth spur gear.

Extend, in turn, *a d* and *a e* till a line, of length equal to *a c*, drawn perpendicular to *a d* will meet *a e*, as *f g*; then $f a \times \pi$ will be the lead of the spiral to which we should set the milling machine (in this case 18.85 inches). The diagram depends on the following facts relating to spiral gears.

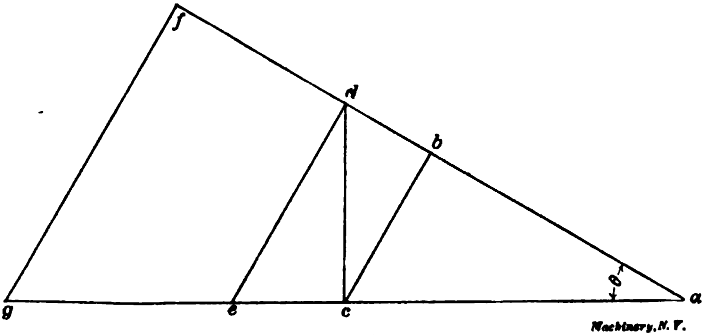

Fig. 7. Graphical Solution of Spiral Gear Problems.

$$\frac{b\,a}{a\,c} = \cos\theta = \frac{\text{diam. spur gear}}{\text{diam. spiral gear}}$$

$$a\,c = \frac{b\,a}{\cos\theta}$$

$$a\,d = \frac{a\,c}{\cos\theta} = \frac{b\,a}{\cos^2\theta}$$

$$e\,a = \frac{a\,d}{\cos\theta} = \frac{b\,a}{\cos^3\theta}.$$

$$\frac{\text{Lead of helix}}{\text{Circumference of pitch cylinder}} = \tan\ (90^\circ - \theta)$$

Divide by π and transpose

$$\frac{\text{Lead}}{\pi} = \text{diameter of pitch cylinder} \times \frac{f\,a}{f\,g}$$

But pitch diameter $= a\,c = f\,g$; therefore

$$\text{Lead} = \frac{f\,a}{f\,g} \times f\,g \times \pi = f\,a \times \pi$$

Therefore lead of helix $= f\,a \times \pi$.

CHAPTER II.

RULES AND FORMULAS FOR DESIGNING SPIRAL GEARS.

In accordance with time-honored custom, this contribution to the art of designing helical or "spiral" gears opens with an apology. The subject is one which, from its very nature, can be approached from any one of a number of different ways, and it has been approached from so many of these possible different ways that perhaps the subject has become quite confused in the minds of many readers of technical literature. The writer does not offer the excuse of novelty in the methods presented in the following paragraphs, since some of the details which were independently worked out by him have been described by others. His reason for adding one more to the series of solutions of helical gear problems is that the method described appears to reduce the more serious of this class of problems to its most simple elements. The method of procedure will be described without proof or comment.

Two terms will be used which may require some explanation. In using the expression "tooth angle," the angle made by the teeth with the axis of the gear is meant, not the angle of the tooth with the face of the gear, an unfortunate use sanctioned by some writers. Fig. 8 shows α_a as the tooth angle of gear a, and α_b as the tooth angle of gear b, used in the sense in which we will use them. The angle between the shafts, γ, is 90 degrees in all the examples which will be considered in this chapter. The first rule to be used in the design of helical gears relates to the tooth angles.

Rule 1. *The sum of the tooth angles of a pair of mating helical gears is equal to the shaft angle.*

That is to say, in Fig. 8, angle α_a added to angle α_b equals angle γ, as is self-evident from the cut.

The second term which requires explanation is the "equivalent diameter." The quotient obtained by dividing the number of teeth in a helical gear by the diametral pitch of the cutter used gives us a very useful factor for figuring out the dimensions of helical gears, so the writer has ventured to give it this name "equivalent diameter," an abbreviation of the words "diameter of equivalent spur gears," which more accurately describe it. This quantity cannot be measured on the finished gear with a rule, being only an imaginary unit of measurement. The next rule deals with this term.

Rule 2. *The equivalent diameter of a helical gear is found by dividing the number of teeth in the gear by the diametral pitch of the cutter with which it is cut.*

For instance, in a 20-tooth gear, cut with an 8 diametral pitch cutter, the equivalent diameter will be 2½ inches. The actual diameter of the gear will vary widely from this, depending on the tooth angle.

The process of locating a railway line over a mountain range is divided into two parts; the preliminary survey or period of exploration, and the final determination of the grade line. The problem of designing a pair of helical gears resembles this engineering problem in having many possible solutions, from which it is the business of the designer to select the most feasible. For the exploration or preliminary survey the diagram shown in Fig. 9 will be found a great convenience. The materials required are a ruler with a good straight edge, and a piece of accurately ruled, or, preferably, engraved, cross-section paper. If a point, *O*, be so located on the paper that *B O*, the distance to one margin line, be equal to the equivalent diameter of gear *a*, while *B′ O*, the distance to the other margin line, be equal to the equivalent diameter of gear *b*, then (when the rule is laid diagonally across the paper in any position that cuts the margin lines and passes through point *O*) *D O* will be the pitch diameter of gear *a*, *D′ O* the pitch diameter of gear *b*, angle *B O D* the tooth angle of gear *a* and angle *B′ O D′* the tooth angle of gear *b*. This simple diagram presents instantly to the eye all possible combinations for any given problem. It is, of course, understood that in the shape shown it can only be used for shafts making an angle of 90 degrees with each other.

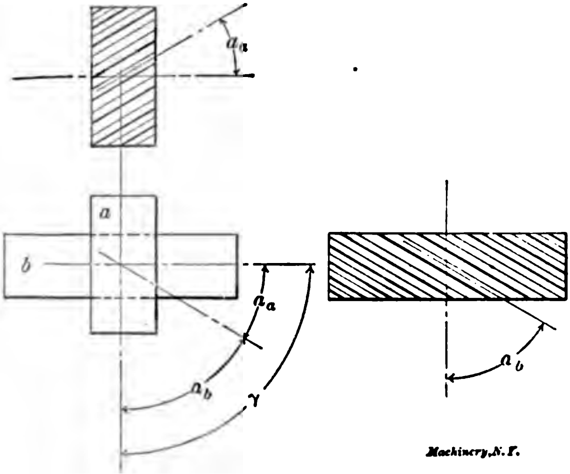

Fig. 8. Diagram Showing Notation used for Tooth Angles.

The diagram as illustrated shows that a pair of helical gears having 12 and 21 teeth each, cut with a 5-pitch cutter, and having shafts at 90 degrees from each other and 5 inches apart, may have tooth angles of 36° 52′ and 53° 8′ respectively, and pitch diameters of 3 inches and 7 inches.

Suppose it were required to figure out the essential data for three sets of helical gears with shafts at right angles, as follows:

1st. Velocity ratio 2 to 1, center distance between shafts 2¼ inches.
2d. Velocity ratio 2 to 1, center distance between shafts 3⅜ inches.
3d. Velocity ratio 2 to 1, center distance between shafts 4 inches.

We will take the first of these to illustrate the method of procedure about to be described.

We have a center distance of 2¼ inches and a speed ratio between driver and driven shafts of 2 to 1. The first thing to determine is the pitch of the cutter we wish to use. The designer selects this according to his best judgment, taking into consideration the cutters on hand and the work the gearing will have to do. Suppose he decides that 12-pitch will be about right. In Fig. 9 it will be remembered that *D O* was the pitch diameter of gear *a*, while *D′ O* was the pitch diameter of gear *b*. That being the case *D O D′* is equal to twice the distance between the shafts. In the problem under consideration this will be equal to 2 × 2¼, or 4½ inches. Fig. 10 is a skeleton outline showing the operation of making the preliminary survey with rule and cross-section paper. *A G* and *A G′* represent the margin lines of the sheet, while *D D′* represents the graduated straight edge. By the conditions of the problem the distance between points *D* and *D′*, where the ruler crosses the margin lines, must be equal to 4½ inches. There has next to be determined at what angle of inclination the ruler shall be placed in locating this line. To do this, we will first find our "ratio line." Select any point *C* such that *C F′* is to *C F* as 2 is to 1, which is the required ratio of our gears. Draw through point *C*, so

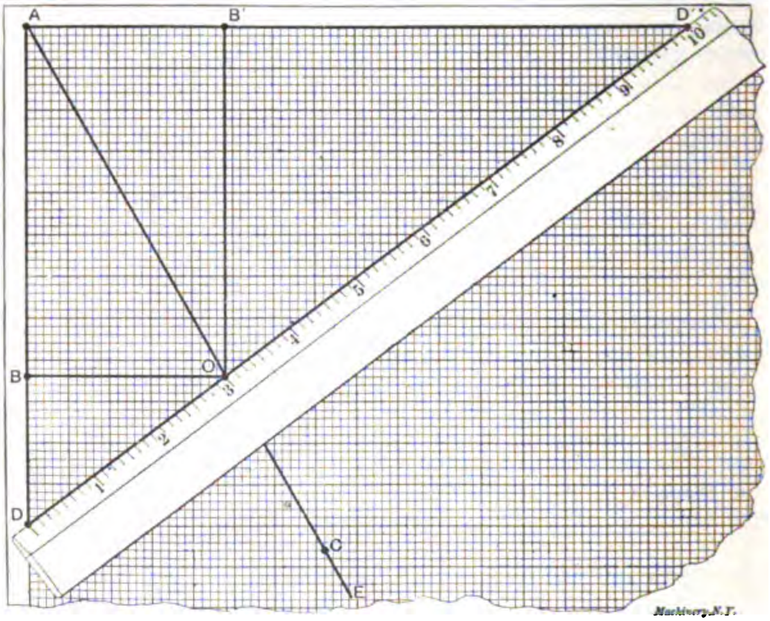

Fig. 9. Preliminary Solution with Rule and Cross-section Paper.

located, the line *A E*. Line *A E* is then the ratio line, that is, a line so drawn that the measurements taken from any point on it to the margin lines will be to each other in the same ratio as the required ratio between the driving and driven gear. Now, by shifting the ruler on the margin lines, always being careful that they cut off the required distance of 4½ inches on the graduations, it is found that when the rule is laid as shown in position No. 1, cutting the ratio line at *O′*, the distance from the point of intersection to corner *A* is at its maximum. For the minimum value, the tooth angle is the limiting feature. For a gear of this kind, 30 degrees is, perhaps, about as small as would be advisable, so when the ruler is inclined at an angle of about 30 degrees with margin line *A G′*, and occupies position No. 2 as shown, it will cut line *A E* at *O″*, and the distance cut off from the point of intersection to corner *A* will be at its minimum value. The ruler must then be located at some intermediate position between No. 1 and No. 2.

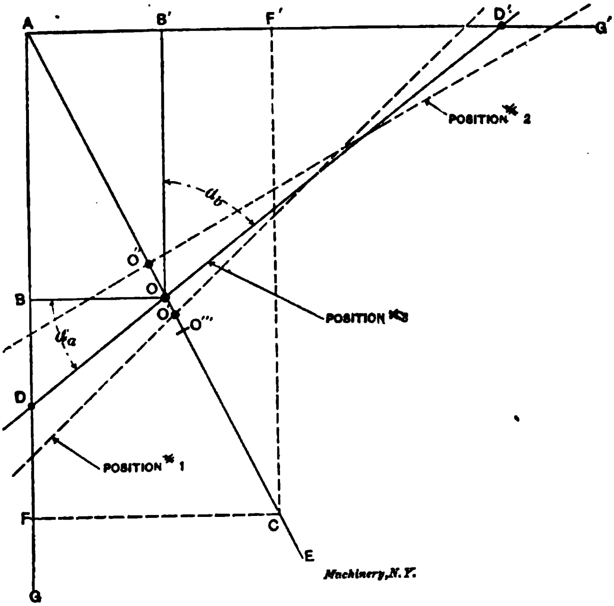

Fig. 10. Preliminary Graphical Solution for Problem No. 1.

Supposing, for example, 14 teeth in gear *a* and 28 teeth in gear *b* be tried. According to Rule 2, the equivalent diameter of gear *a* will then be 14 ÷ 12, or 1.1666 inch; the equivalent diameter of *b* will be 28 ÷ 12, or 2.3333 inches. Returning to the diagram to locate the point of intersection, it will be found that point *O‴* is so located that lines drawn from it to *A G* and *A G′* will be equal to 1.1666 inch and 2.3333 inches respectively, but this is beyond point *O′*, which was found to be the outermost point possible to intersect with a 4½-inch line, *D D'*. This shows that the conditions are impossible of fulfillment.

Trying next 12 teeth and 24 teeth, respectively, for the two gears, the equivalent diameters by Rule 2 will be 1 inch and 2 inches. Point *O* is now so located that *O B* equals 1 inch and *O B'* equals 2 inches. Seeing that this falls as required between *O'* and *O''*, stick a pin in at this point to rest the straight edge against, and shift the straight edge about until it is located in such an angular position that the margin lines *A G* and *A G'* cut off 4½ inches, or twice the required distance between the shafts, on the graduations. This gives the preliminary solution to the problem. Measuring as carefully as possible, *D O*, the pitch diameter of gear *a*, is found to be about 1.265 inch diameter, and *D' O*, the pitch diameter of gear *b*, about 3.235 inches. Angle *B O D*, the tooth angle of gear *a*, measures about 37° 50'. Angle *B' O D'*, the tooth angle of gear *b*, would then be 52° 10' according to Rule 1. To determine angle *B O D* more accurately than is feasible by a graphical process, use the following rule:

Rule 3. *The tooth angle of gear a in a pair of mating helical gears, a and b, whose axes are 90° apart, must be so selected that the equivalent diameter of gear b plus the product of the tangent of the tooth angle of gear a by the equivalent diameter of gear a, will be equal to the product of twice the center distance by the sine of the tooth angle of gear a.*

That is to say, in this case, $O B' + (O B \times \text{the tangent of angle } B O D) = D D' \times \text{the sine of angle } B O D$. Perform the operations indicated, using the dimensions which were derived from the diagram, to see whether the equality expressed in this equation holds true. Substituting the numerical values:

$$2 + (1 \times 0.77661) = 4.5 \times 0.61337,$$
$$2.77661 = 2.76016,$$

a result which is evidently inaccurate.

The solution of the problem now requires that other values for angle *B O D*, slightly greater or less than 37° 50', be tried until one is found that will bring the desired equality. It will be found finally that if the value of 38° 20' be used as the tooth angle of gear *a*, the angle is as nearly right as one could wish. Working out Rule 3 for this value:

$$2 + (1 \times 0.79070) = 4.5 \times 0.62024,$$
$$2.79070 = 2.79108.$$

This gives a difference of only 0.00038 between the two sides of the equation. The final value of the tooth angle of gear *a* is thus settled as being equal to 38° 20'. Applying Rule 1 to find the tooth angle of gear *b* we have: 90° — 38° 20' = 51° 40'. The next rule relates to finding the pitch diameter of the gears.

Rule 4. *The pitch diameter of a helical gear equals the equivalent diameter divided by the cosine of the tooth angle; or the equivalent diameter multiplied by the secant of the tooth angle.*

If a table of secants is at hand, it will be somewhat easier to use the second method suggested by the rule, since multiplying is usually easier than dividing. Using in this case, however, the table of cosines, and performing the operation indicated by Rule 4, we have for the pitch diameter of gear *a*:

$$1 \div 0.78442 = 1.2748, \text{ or } 1.275 \text{ inch, nearly;}$$

and for the pitch diameter of gear *b*:

$$2 \div 0.62024 = 3.2245, \text{ or } 3.225 \text{ inch, nearly.}$$

To check up the calculations thus far, the pitch diameter of the two gears thus found may be added together. The sum should equal twice the center distance, thus:

$$1.275 + 3.225 = 4.500,$$

which proves the calculations for the angle.

Rule 5. *The outside diameter of a helical gear equals the pitch diameter plus the quotient of* 2 *divided by the diametral pitch of the cutter used.*

Applying this rule to gear *a:*

$$1.2748 + (2 \div 12) = 1.2748 + 0.1666 = 1.4414 = 1.441 \text{ inch, nearly.}$$

For gear *b:*

$$3.2245 + (2 \div 12) = 3.2245 + 0.1666 = 3.3911 = 3.391 \text{ inches, nearly.}$$

In cutting spur gears of any given pitch, different shapes of cutters are used, depending upon the number of teeth in the gear to be cut. For instance, according to the Brown & Sharpe system for involute gears, eight different shapes are used for a gear from 12 teeth to a rack. The fact that a certain cutter is suited for cutting a 12-tooth spur gear is no sign that it is suitable for cutting a 12-tooth helical gear, since the fact that the teeth are cut on an angle alters their shape considerably. To find out the number of teeth for which the cutter should be selected, use the following rule:

Rule 6. *The number of teeth for which the cutter should be selected to cut a helical gear is found by dividing the number of teeth in the gear by the cube of the cosine of the tooth angle.*

Applying this rule to gear *a:*

$$12 \div 0.784^3 = 12 \div 0.4818 = 25-,$$

and for gear *b:*

$$24 \div 0.620^3 = 24 \div 0.2383 = 100+,$$

giving, according to the Brown & Sharpe catalogue, cutter No. 5 for gear *a* and cutter No. 2 for gear *b*.

In gearing up the head of the milling machine to cut these gears it is necessary to know the lead of the helix or "spiral" required to give the tooth the proper angle. To find this, use Rule 7.

Rule 7. *The lead of the helix or "spiral" of a helical gear is equal to the product of the cotangent of the tooth angle by the pitch diameter by* 3.14.

In solving problems by this rule, as for Rule 6, it will be sufficient to use trigonometrical functions to three significant places only, this being accurate enough for all practical purposes. Solving by Rule 7 to find the lead to set up the gearing for in cutting *a:*

$1.275 \times 1.265 \times 3.14 = 5.065$, or 5 1/16 inches, nearly;

for gear *b:*

$3.225 \times 0.791 \times 3.14 = 8.010$, or 8 3/32 inches, nearly.

The lead of the helix must be, in general, the adjustable quantity in any spiral gear calculation. If special cutters are to be made, the lead of the helix may be determined arbitrarily from those given in the milling machine table; this will, however, probably necessitate a cutter of fractional pitch. On the other hand, by using stock cutters and varying the center distance slightly, we might find a combination which would give us for one gear a lead found in the milling machine table, but it would only be chance that would make the lead for the helix in the mating gear also of standard length. It is then generally better to calculate the milling machine change gears according to the usual methods to suit odd leads, rather than to adapt the other conditions to suit an even lead. It will be found in practice that the lead of the helix may be varied somewhat from that calculated without seriously affecting the efficiency of the gears.

The remaining rules relating to the proportions of the teeth do not vary from those for spur gears and are here set down for the sake of completeness only.

Rule 8. *The thickness of the tooth of a standard gear at the pitch line is equal to* 1.5708 *divided by the diametral pitch of the cutter.*

For gears *a* and *b* of our problem this gives:

$$1.5708 \div 12 = 0.1309 \text{ inch.}$$

Rule 9: *The addendum of a standard gear is equal to* 1 *divided by the diametral pitch of the cutter.*

For gears *a* and *b* this will give:

$$1 \div 12 = 0.0833 \text{ inch.}$$

Rule 10. *The whole depth of the tooth of a standard gear is equal to* 2.1571 *divided by the diametral pitch of the cutter.*

This gives for gears *a* and *b:*

$$2.1571 \div 12 = 0.1797 \text{ inch.}$$

This completes all the calculations required to give the essential data for making our first pair of helical gears. To illustrate the variety of conditions for which these problems may be solved, the other cases will be worked out somewhat differently. In the case just considered no allowance was made for possible conditions which might have limited the dimensions of the gears, and the problem was solved for what might be considered general good practice. Gear *a*, however, might have been too small to put on the shaft on which it was intended to go, while gear *b* might have been too large to enter the space available for it. If, as we may assume, these gears are intended to drive the camshaft of a gas engine, the solution would probably be unsatisfactory. Case No. 2 will therefore be solved for a center distance of 3⅝ inches as required, but the two gears will be made of about equal diameter. Fig. 11 shows the preliminary graphical solution of this problem, the reference letters in all cases being the same as in Fig. 10. With a 10-pitch cutter, if this suited the judgment of the designer, 15 teeth in gear *a* and 30 teeth in gear *b* would require that the point of intersection on the ratio line *A E* be located at *O* where *B O* equals the equivalent diameter of gear *a*, which equals 1½ inch, while *B′ O* equals the equivalent diameter of gear *b*, or 3 inches, both calculated in accordance with Rule 2. The required condition now is that *D O* be approximated to *D′ O*; that is to say, that the pitch diameters of the two gears be about equal. After continued trial it will be found impossible to locate *O*, using a cutter of standard diametral pitch, so that *D O* and *D′ O* shall be equal, and at the same time have *D D′* equal to twice the required center distance, which is 2 × 3⅜ inches or 6¾ inches. If this center distance could be varied slightly without harm, *B D* could be taken as equal to *A B*; then it would be found that a line drawn from *D* through *O* to *D′*, though giving a somewhat

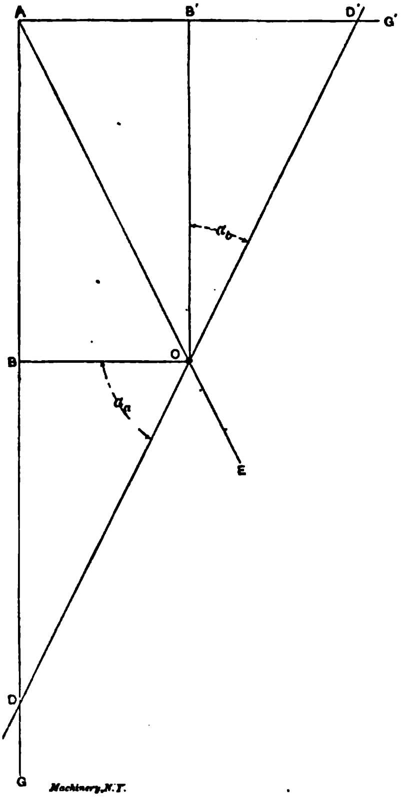

Fig. 11. Solution of Problem No. 2 for Equal Diameters.

shortened center distance, would make two gears of exactly the same pitch diameter.

Drawing line *D O D′*, however, as first described to suit the conditions of the problem, and measuring it for a preliminary solution the following results are obtained: The tooth angle of gear $a =$ angle $B\,O\,D = 63°\,45'$; and the tooth angle of gear $B =$ angle $B'\,O\,D' = 90° - 63°\,45' = 26°\,15'$, according to Rule 1. Performing the operations indicated in Rule 3 to correct these angles, it is found that when the tooth angle of gear *a* is 63° 54′, and that for gear *b* is 26° 6′, the equation of Rule 3 becomes

$$3 + (15 \times 2.04125) = 6.75 \times 0.89803$$
$$6.06187 = 6.06170$$

which is near enough for all practical purposes. The other dimensions are easily obtained as before by using the remaining rules.

To still further illustrate the flexibility of the helical gear problem, the third case, for a center distance of 4 inches, will be solved in a third way. It is shown in MacCord's Kinematics that to give the least amount of sliding friction between the teeth of a pair of mating helical gears, the angles should be so proportioned that, in our diagrams, line *D D′* will be approximately at right angles to ratio line *A E*. On the other hand, to give the least end thrust against the bearings, line *D D′* should make an angle of 45° with the margin lines *A G* and *A G′*, in the case of gears with axes at an angle of 90°, as are the ones being considered. The first example worked out in detail was solved in accordance with "good practice," and line *D D′* was located about one-half way between the two positions just described, thus giving in some measure the advantage of a comparative absence of sliding friction, combined with as small degree of end thrust as is practicable. To illustrate some of the peculiarities of the problem, Case 3 will now be solved to give the minimum amount of sliding friction, neglecting entirely the end thrust, which is considered to be taken up by ball thrust bearings or some equally efficient device. On trial it will be found that, with the same number of teeth in the gear and with the same pitch as in Case 2, giving in Fig. 12, *B O*, the equivalent diameter of gear *a*, a value of 1½ inch, and *B′ O*, the equivalent diameter of gear *b*, a value of 3 inches as in Fig. 11, line *D D′* which is equal to twice the center distance, or 8 inches, can then lie at an angle of about 90° with *A E*, thus meeting the condition required as to sliding friction. Thus this diagram, while relating to gears having the same pitch and number of teeth as Fig. 11, yet has an entirely different appearance, and gives different tooth angles and center distances, solving the problem as it does for the least sliding friction instead of for equal diameters of gears.

Measuring the diagram as accurately as may be, the following results are obtained: Tooth angle of gear $a = B\,O\,D = 28°$; tooth angle of gear $b =$ angle $B'\,O\,D' = 90° - 28° = 62°$. This is the preliminary solution. After accurately working it out by the process before described, we have as a final solution, tooth angle of gear $a = 28°\,28'$;

tooth angle of gear $b = 61° 32'$. From this the remaining data can be calculated.

For designers who feel themselves skillful enough to solve such problems as these graphically without reference to calculations, the diagram may be used for the final solution. The variation between the results obtained graphically and those obtained in the more accurate mathematical solution is a measure of the skill of the draftsman as a graphical mathematician. The method is simple enough to be readily copied in a note book or carried in the head. If the graphical method is to be used entirely, it will be best not to trust to the cross-section paper, which may not be accurately ruled; instead skeleton diagrams like those shown in Figs. 10, 11, and 12 may be drawn. For rough solutions however, to be afterward mathematically corrected, as in the examples considered in this article, good cross-section paper is accurate enough. It permits of solving a problem without drawing a line. Point *O* may be located by reading the graduations; a pin inserted here may be used as a stop for the rule, from which the diameter and center distance are read directly; dividing *A D*, read from the paper, by *D D'*, read from the rule, will give the sine of the tooth angle of the gear *a*.

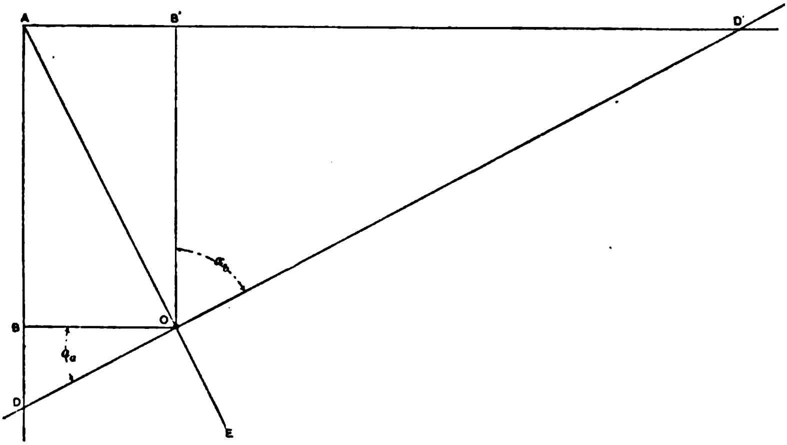

Fig. 12. Solution of Problem No. 8 for Minimum Sliding Friction.

Formulas for Spiral Gearing.

For sensible people, who prefer their rules to be embodied in formulas, the appended list has been prepared, using the following reference letters, which agree in general with the nomenclature of the Brown & Sharpe gear books.

N_a = No. of teeth in gear *a*.
N_b = No. of teeth in gear *b*.
R = Velocity ratio $= N_b \div N_a$.
P'' = Normal diametral pitch or pitch of cutter.
E = Equivalent diameter (explained above).

D = Pitch diameter.
C = Center distance.
B = Blank or outside diameter.
T = No. of teeth for which cutter is selected.
L = Lead of spiral.
γ = Angle of axes.
a = Angle of tooth with axis.
t = Thickness of tooth on pitch line.
S = Addendum.
$D'' + f$ = Whole depth of tooth.

Where subscript letters $_a$ and $_b$ are used, reference is made to gears a and b, as for instance, "N_a" and "N_b," where the letter N refers to the number of teeth in gears a and b, respectively, of a pair of gears a and b.

$$\gamma = a_a + a_b. \tag{29}$$

$$E = \frac{N}{P''}. \tag{30}$$

$$E_b + (E_a \times \tan a_a) = 2\,C \times \sin a_a. \tag{31}$$

$$D = \frac{E}{\cos a} = E \times \sec a. \tag{32}$$

$$B = D + \frac{2}{P''} \tag{33}$$

$$T = \frac{N}{(\cos a)^3} \tag{34}$$

$$L = \cot a \times D \times \pi. \tag{35}$$

$$t = \frac{1.5708}{P''}. \tag{36}$$

$$S = \frac{1}{P''}. \tag{37}$$

$$D'' + f = \frac{2.1571}{P''}. \tag{38}$$

CHAPTER III.

DIAGRAMS FOR DESIGNING SPIRAL GEARS.

Great difficulties are usually experienced in designing spiral gears, and these difficulties are greatly accentuated when one has to design them for two shafts whose center distance cannot be altered to suit the gears, and also when the angle between the shafts is not a right angle, and the speed ratio is not equal. The general practice is to work out the gears by lengthy mathematics, and should the answer not come out as desired, then a new trial is made, varying either one or the other factor, until the angles and diameters are correct. This method of "cut and try" entails a great deal of work and waste of time. The following method, together with the diagrams used with it, will remove some of the difficulties, and enable one to arrive at the data required in a very short time. The method adopted is graphical, but the results may be checked by simple figuring.

As the pitch diameter, spiral angle, and circular pitch are interdependent, they cannot be considered as a starting point in solving the problem, because they are not known. The starting point, therefore, must be the speed ratio, and some idea of the strength required, together with the center distance. These factors, as a rule, can easily be ascertained. As it is common usage to employ ordinary spur gear cutters for regular diametral pitch to cut spiral gears with, the normal pitch, or distance from one tooth to the next measured at right angles to the tooth, must be the same as the pitch of a spur gear for which the cutter to be used is intended; therefore the corresponding diametral pitch and the speed ratio must be the initial data, all others being obtained afterwards.

Three diagrams are given for the graphical solution of spiral gears. The diagram in Fig. 13 shows the relation between the quotient of number of teeth ÷ diametral pitch, spiral angles, and pitch diameters. The diagram in Fig. 14 shows the relation between the diametral pitch, the number of teeth, and the quotient of the number of teeth ÷ diametral pitch. Finally, the diagram in Fig. 15 shows the relation between the pitch diameter, the spiral angle, and the lead of the helix. We will now proceed to give some typical examples illustrating the use of the diagrams.

Example 1. Given a gear having 24 teeth, 6 diametral pitch, and a spiral angle of 40 degrees. Find the pitch diameter.

First obtain the value of the ratio, number of teeth ÷ diametral pitch, which, in this case, can be obtained without referring to diagram Fig. 14, being simply $24 \div 6 = 4$. Locate 4 on the horizontal line in diagram Fig. 13, and project vertically until the line from figure 4 intersects the line for 40 degrees spiral angle. Then follow the circular arc from this point, either to the right or downward, reading off 5.22 on the corresponding scale, this being the pitch diameter. Should the diameter be required accurately, we can figure it by the formula:

$$\text{Pitch diameter} = \frac{\text{No. of teeth}}{\text{Diametral pitch}} \times \frac{1}{\text{cos spiral angle}}$$

$$= 4 \times \frac{1}{\text{cos 40 deg.}} = 5.222 \text{ inches.}$$

This also gives a check of the result obtained by means of the diagram. The lead of the helix is now obtained from Fig. 15, by projecting the pitch diameter 5.22 horizontally to the radial line for the spiral angle, and then, following the vertical line to the lead scale at the bottom of the diagram, we find, in this case, a lead of 19.6 inches. Of course, the outside diameter of the blank would be 5.222 + 2 × 1/6 = 5.555 inches, which is the pitch diameter + 2 times the addendum.

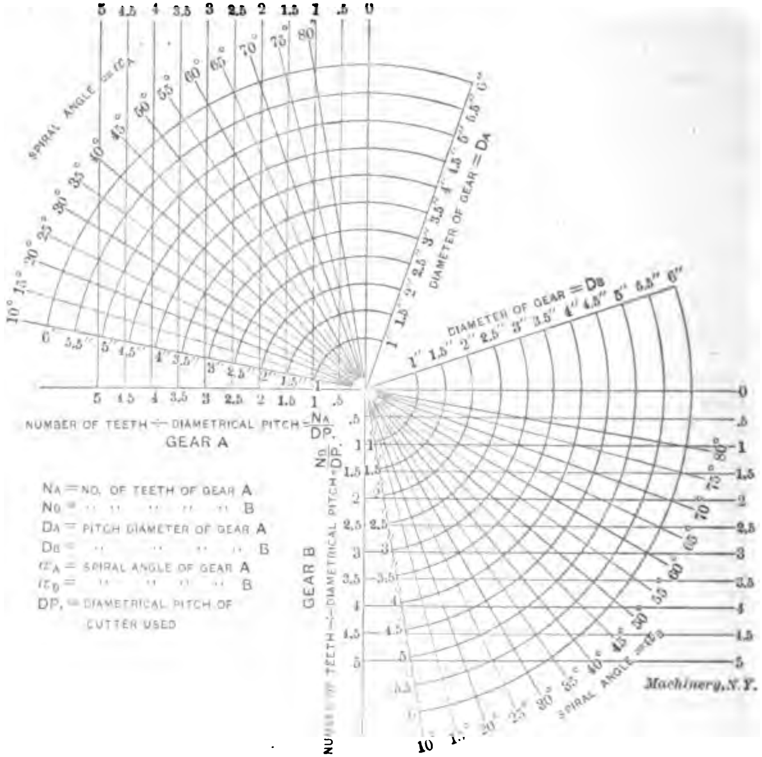

Fig. 13. Diagram of Relation between Number of Teeth, Diametral Pitch, Spiral Angles, and Pitch Diameters.

Example 2. Required two gears which are to be equal in all respects, the diametral pitch being 8, and the centers to be approximately 4 inches apart.

As the centers are not fixed, the gears in this case may be made with 45 degrees spiral angle, and the center distance may be slightly adjusted to suit the pitch diameters. Referring to Fig. 13, follow the circular arc from diameter of gear = 4 inches, until it intersects the radial line for 45 degrees spiral angle; then follow the vertical line down to the scale of the ratio between the number of teeth and diametral pitch, which is found to be 2.82. Then, from Fig. 14, we find that with this ratio and 8 diametral pitch, the number of teeth is not a whole number, but the nearest number is 23, giving a ratio of 2.875 instead of 2.82, which, by reversing the process and referring to diagram Fig. 13, gives a pitch diameter of 4.07 inches. These results may be checked as follows:

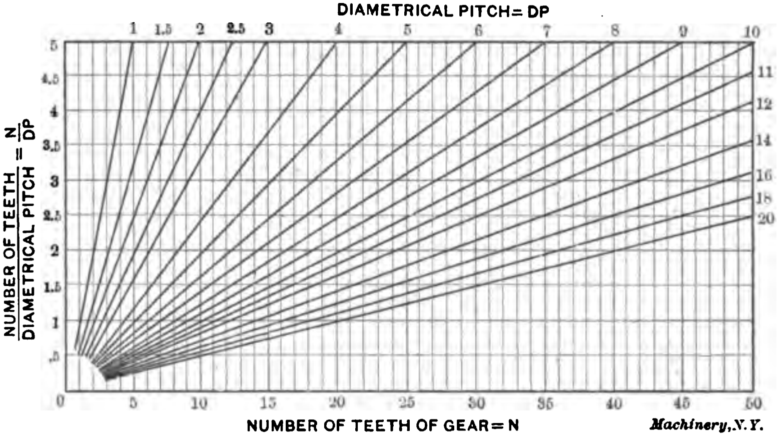

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Fig. 14. Relation between Diametral Pitch, Number of Teeth, and Quotient of Number of Teeth divided by Diametral Pitch.

$$\text{Pitch diameter} = \frac{\text{No. of teeth}}{\text{Diametral pitch}} \times \frac{1}{\cos 45 \text{ deg.}}$$

$$= 2.875 \times \frac{1}{0.707} = 4.07 \text{ inches.}$$

The outside diameter is $4.07 + 2 \times 0.125 = 4.32$. The lead, as obtained from diagram Fig. 15, in the same way as in Example 1, is 12.79 inches.

Example 3. Required a pair of spiral gears having a normal pitch corresponding to 10 diametral pitch, having a given center distance of 2½ inches approximately, the sum of the spiral angles being 90 degrees, and the speed ratio equal to 5 to 1.

In this case both portions of diagram Fig. 13 are used, the upper part being employed for one gear and the lower part for the other, the easiest way being to get a strip of paper with two lines marked on its edge 5 inches (twice the center distance) apart, drawn to the same scale as the diagram. Move this strip of paper on the diagram (so that the edge of the strip passes through the center), as indicated at *A*, Fig. 16, until the lines marked coincide with points where the ratios of number of teeth ÷ diametral pitch equal 5 ÷ 1, and then determine from Fig. 14 that these diameters also give whole numbers of teeth with 10 diametral pitch. We find that 0.5 and 2.5 at 78 degrees and 12 degrees are two such positions, and also 0.6 and 3.0 at 70 degrees and 20 degrees. If we use the latter values, we will have 6 teeth and 30 teeth at 70 and 20 degrees angle respectively. The exact diameters can now be determined, as in our previous problem, and are 1.75 and 3.19 inches, respectively, the outside diameters being 0.2 inch larger, or 1.95 and 3.39 inches, respectively. This gives the center distances 2.47. These values can now be figured from the formulas as before, and the leads obtained.

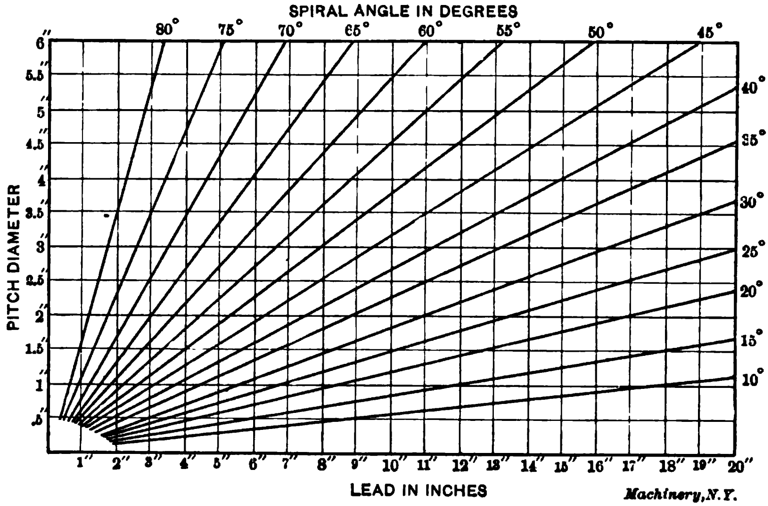

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Fig. 15. Relation between Pitch Diameter, Spiral Angle, and Lead of Helix.

Example 4. Required a pair of spiral gears, having a fixed center distance of 4.5 inches, running at equal speeds, the diametral pitch being 7. The method of procedure is similar to that of the last example, using a strip of paper having a distance of 9 inches marked on the edge in the proper scale, as indicated at *B* in Fig. 16. At about 40 degrees spiral angle we find in Fig. 13 the ratio of number of teeth to diametral pitch to equal 3.14. This ratio must be adjusted on diagram 14, as previously shown, so as to enable one to get a whole number of teeth with 7 diametral pitch, this number being in this case 22. The ratio is then 3.143, and following from this in Fig. 13 to the 40-degree line, one obtains a pitch diameter of about 4.1 inches for one gear, and at 50 degrees about 4.9 inches for the other.

The spiral angles should now be carefully checked mathematically as follows:

$$\text{Cos spiral angle (first gear)} = 3.143 \times \frac{1}{4.1} = 0.766;\ \text{spiral angle} = 40\text{ deg.}$$

$$\text{Cos spiral angle (second gear)} = 3.143 \times \frac{1}{4.9} = 0.642;$$

$$\text{spiral angle} = 50\text{ deg., nearly.}$$

Now obtain the leads from diagram Fig. 15 in the same way as before, giving the leads of the gears 15.4 and 12.9 inches, respectively.

Example 5. Required a pair of spiral gears, the axes of which are at an angle of 120 degrees; center distance 4.125; the ratios between number of teeth and diametral pitch should be to each other as 2 to 3, and the diametral pitch equals 5.

We require first of all two numbers representing the ratios of number of teeth to diametral pitch, these two numbers bearing the ratio to each other of 2 to 3, and giving a whole number of teeth with 5 diametral pitch. These two numbers, when projected onto two spiral angle lines in a diagram made up as in Fig. 13, the sum of the angles of which equals 120 or 60 degrees, give two diameters whose sum equals the center distances multiplied by 2, or 8.25. In this case we cannot use both parts of the diagram Fig. 13, as it is made up for shafts at 90 degrees angle, and for this reason we must take the two readings from the same part of the diagram. The ratios 3 and 4.5 at 30 degrees give corresponding diameters of 3.5 and 5.2, the sum being 8.7. The ratios 2.8 and 4.2 giving 14 and 21 teeth at 25 and 35 degrees, respectively, have diameters of 3.1 and 5.15 (equals 8.25). From this we see that we must use 14 and 21 teeth and the ratios 2.8 and 4.2. The diameters and spiral angles can now be obtained graphically and more accurately in this manner:

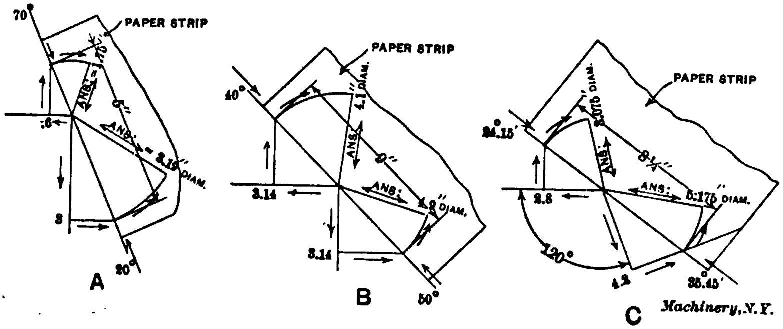

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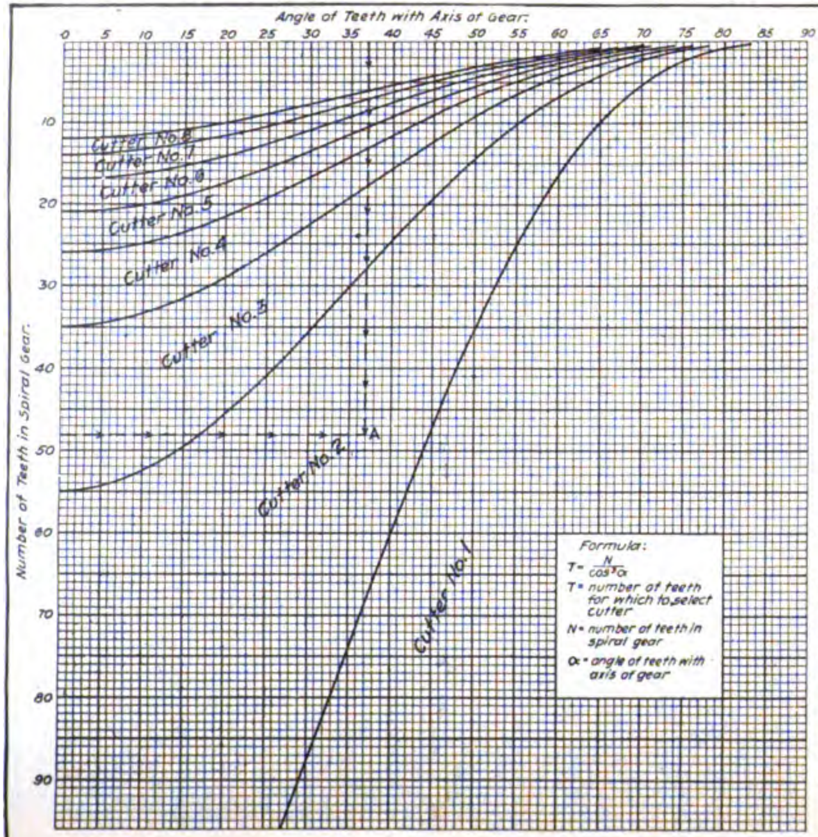
Fig. 16. Separate Diagrams for the Solution of some of the Problems Presented.

Draw two radial lines, as shown at *C* in Fig. 16, at 120 degrees angle on a separate piece of paper, and lay off on these to same scale 2.8 and 4.2. From these points draw lines at right angles to the radial lines. It is now necessary to find the position of a line 8.25 inches long,



Formula:

$T = \frac{N}{\cos^3 \alpha}$

T = number of teeth for which to select cutter

N = number of teeth in spiral gear

α = angle of teeth with axis of gear

The cutters used for milling spiral or helical gears are standard spur gear cutters, the number of a cutter and its pitch for a given case being defined by the angle (with axis) and normal pitch. This diagram gives the numbers of the cutters only, the pitch having been previously determined.

The selection of the cutter is fixed by the formula given in the lower right-hand corner of the diagram. The delimiting curves thereon were plotted by the formula, the area between the curves being the field of intersection of the combinations of angles and numbers of teeth covered by each designated cutter number.

For example, suppose the angle of the teeth of a gear is 37 degrees with its axis, and the number of teeth is 48. The point A, at which the horizontal line (representing the tooth number), and the vertical line (representing the angle) intersect, falls within the area marked "Cutter No. 2" Therefore, a No. 2 cutter is required to cut a 48-tooth spiral gear having the teeth at an angle of 37 degrees with its axis.

Fig. 17. Diagram for Determining Cutter to Use for Milling Spiral Gears.

terminating upon these lines, and passing through the center. A strip of paper is used in the same manner as before, and upon careful measuring of the respective distances from the center to the lines, one obtains the distances 3.075 and 5.175 inches, which represent the respective diameters, the sum being 8.25. The spiral angle is obtained by measuring or calculating as follows:

$$\text{Cos spiral angle of first gear} = 2.8 \times \frac{1}{3.075} = 0.910;$$
$$\text{spiral angle} = 24 \text{ deg. } 15 \text{ min.}$$

$$\text{Cos spiral angle of second gear} = 4.2 \times \frac{1}{5.175} = 0.812;$$
$$\text{spiral angle} = 35 \text{ deg. } 45 \text{ min.}$$

The above examples will show the careful student the manner of working out each kind of gear required, and if the directions are properly followed, this method will be found to be a great time-saver. It may be mentioned that it is advisable to keep the spiral angle as nearly equal in the two gears as possible in order to obtain the greatest efficiency of transmission. It should be noted that when diagrams of this type are to be used for practical calculation of spiral gears, they should be laid out in a much larger scale than is possible to show in these pages, and it would be advisable to lay out radial lines in Fig. 13 for every degree, and vertical and horizontal lines for every tenth of an inch, and circular arcs for equally fine subdivisions. The same is true of the diagrams in Figs. 14 and 15. In Fig. 14, horizontal lines should be laid out for every tenth of an inch, and vertical lines should be laid out for all whole numbers of teeth. In Fig. 15, the horizontal lines should be laid out for every tenth of an inch, vertical lines for at least every 0.2 of an inch, and radial lines for every degree. This diagram should also be laid out so that leads over 20 inches may be read off, as well as those below this figure.

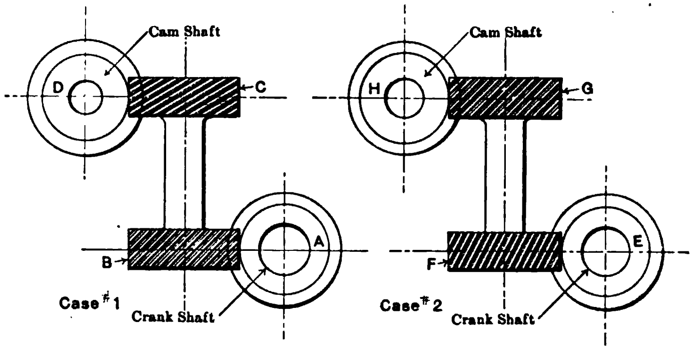
In Fig. 17 is given a diagram for determining the cutter to use when milling the teeth of spiral gears. The instructions for the use of the diagram are given directly on the chart itself, so that no other explanation is necessary. This diagram was contributed to MACHINERY by Elmer G. Eberhardt, and appeared in the September, 1907, issue.

CHAPTER IV.

COMPARISON OF EFFICIENCY OF SPIRAL GEARS.

Suppose a problem, accompanied by the data shown in Fig. 18, were presented as follows: "Given two different sets of spiral gears for gas engines. In each case the cam shaft runs at half the speed of the crank-shaft; to be decided which is the better arrangement for efficiency and wearing qualities, taking into consideration the nature of the work the drive has to perform, *viz.*, a single cylinder gas engine working on the 'Otto' cycle."

The solution of this problem involves a little work along the line of resolution of forces and the calculation of efficiency; it is entirely elementary, but interesting nevertheless, as a practical illustration of the working of well-known principles in mechanics. For the sake, then, of their value as illustrations of the principles involved, these calculations are here given in detail.



	Angle of Teeth with Axis	Diameter of Pitch Circle	Number of Teeth
A	45°	4"	20
B	45°	4"	20
C	63°–26′	4"	10
D	26°–34′	4"	20

	Angle of Teeth with Axis	Diameter of Pitch Circle	Number of Teeth
E	63°–26′	4"	10
F	26°–34′	4"	20
G	45°	4"	20
H	45°	4"	20

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Fig. 18. The Two Arrangements to be Investigated.

The problem requires us to find which of the two arrangements, that in Case 1 or Case 2, is superior in efficiency and wearing qualities. It may be roughly stated that, other things being equal, the more efficient of two mechanisms is the more durable. We will consider this to be true in this case, so will examine the two arrangements for efficiency in the transmission of power. The power losses in the various journals we cannot estimate, because we do not know enough about the arrangement and design of the bearing surfaces. We can easily make an estimate for the power lost in the thrust bearings, and we may also get a comparative idea, at least, of the power lost in the rubbing of the teeth on each other, so to these losses, which are the principal ones, we will confine ourselves.

When two bodies are sliding under pressure, the power lost is equal to the continued product of the normal pressure between the surfaces, the linear velocity of the rubbing, and the coefficient of friction. To estimate the power lost at the various bearing points we are to consider, we have then to estimate these three factors for each case.

We will first estimate the relative bearing pressures at the different places where friction is met with in Case 1. To be logical, we will commence our calculations at the driven end of the train of gears, since the forces in the mechanism are due to the resistance offered by the driven members. Fig. 19 is another view of Case 1 as shown in Fig. 18. Gears *A* and *B* make contact on line *Y Z*, which represents the direction of the teeth at the point of contact; *W X* represents the position of the teeth of gears *C* and *D* in contact.

As gear *C* revolves in the direction shown, its teeth, set at the angle of the line *W X*, have a wedging action on those of gear *D* which revolves them in the direction shown. The action and the forces involved can best be understood by referring to Fig. 20. Here *C* is a slide moving upward. Its beveled edge, representing the tooth surface of gear *C*, forces to the left on the beveled edge on slide *D*, which represents the tooth surface of gear *D*. If slide *D* offers a resistance to this movement, of a magnitude represented by the length of line F_5 in the parallelogram of forces shown, slide *C* will evidently have to exert a force equal to F_3 to overcome this resistance. The resulting normal pressure on the inclined bearing surface of contact will evidently be F_4. The end thrust or pressure against its abutment of slide *D* will be F_3, while that of slide *C* against its abutment will be F_5.

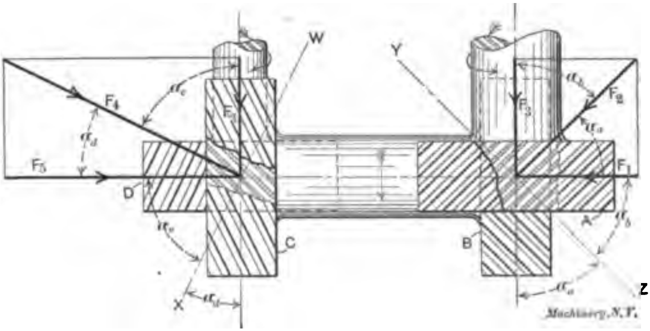

Fig. 19. Force Diagrams for Case 1.

Understanding the method of applying the parallelogram of forces in Fig. 20, we may transfer the construction to gears *C* and *D* in Fig. 19. Having F_5 given, we can find F_4 and F_3 as there shown. F_3 is the tangential pressure at the pitch line, required to be given by gear *C* to move the mechanism against the resistance F_5 offered by gear *D*. Since gears *B* and *C* have the same diameter, F_3 must likewise be the tangential pressure applied at the pitch line to gear *B*. Constructing a second parallelogram of forces for gears *A* and *B*, as shown, we find that F_2 is the normal pressure between the faces of the teeth in contact, and F_1 is the tangential force which has to be brought to bear at the pitch line of gear *A* to move the mechanism. Consider that F_5 equals unity (since we require comparative results only) and measure the other forces to this scale. (This can be done fully as well by calculation as by measurement. An elementary knowledge of trigonometry will give us the following results:

$F_5 = 1$
$F_4 = F_5 \div \sin \alpha_c = 1 \div 0.894 = 1.118$
$F_3 = F_5 \times \tan \alpha_d = 1 \times 0.500 = 0.500$
$F_2 = F_3 \div \sin \alpha_a = 0.500 \div 0.707 = 0.707$
$F_1 = F_2 \times \tan \alpha_b = 0.500 \times 1.000 = 0.500.$

We have next to find the rubbing velocities of the various bearing points. Fig. 21 will assist us in this. Here we have the same slides *C* and *D*, representing gears *C* and *D* in Fig. 19 or Fig. 22. If we consider that slide *C* is moved upward at a uniform velocity, in a unit of time it will traverse a distance equal to V_3, moving from position *g h* to *g′ h′*. This evidently forces slide *D* to the left at a uniform velocity, moving it in a unit of time from *e f* to *e′ f′*, a distance measured by dimension V_5. The beveled surface of slide *D* has meanwhile slipped on that of slide *C* so that corners *f* and *h*, which were in contact, have reached positions *f′* and *h′*, a distance measured by dimension V_4. It is evident then that V_3, V_4, and V_5 may be taken as measures of relative velocities of the parts in question.

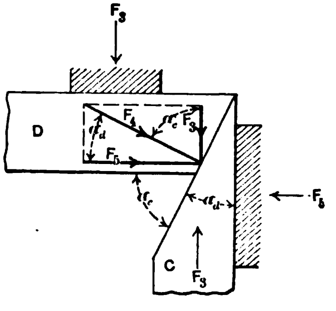

Fig. 20. Illustration of Principle Involved in Fig. 19.

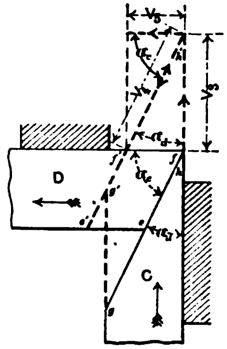

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Fig. 21. Illustration of Principle Involved in Fig. 22.

Since the mechanism shown in Fig. 21 represents, in principle, conditions existing between gear *C* and *D* in Fig. 22, we may transfer the velocity diagram of Fig. 21 to Fig. 22, where V_5 rerpesents the pitch velocity of gear *D*, V_4 the rate of rubbing at the pitch line between gears *C* and *D*, and V_3 the circumferential velocity at the pitch line of gear *C*. The circumferential velocity at the pitch line of gear *B* is evidently the same as that of gear *C*, since they are of the same diameter and move together. V_3 being thus known, a similar velocity diagram may be drawn for gears *A* and *B*, in which V_1 equals the velocity at the pitch line of gear *A*, and V_2 equals the velocity of sliding between the teeth of gears *A* and *B*.

We may, if we wish, measure these lines to the scale $V_1 = 1$ to obtain the relative velocities desired, or, better, we may derive formulas from these velocities, thus making unnecessary the drawing of diagrams for subsequent examples of this kind. By a simple use of trigonometrical functions, after carefully examining the diagrams, it is plain that the following relations hold true:

$$V_1 = 1$$
$$V_2 = V_1 \div \sin \alpha_a = 1 \div 0.707 = 1.414$$
$$V_3 = V_1 \times \tan \alpha_b = 1 \times 1.000 = 1.000$$
$$V_4 = V_3 \div \sin \alpha_c = 1 \div 0.894 = 1.118$$
$$V_5 = V_3 \times \tan \alpha_d = 1 \times 0.500 = 0.500$$

The power lost in any bearing is equal to the continued product of the total pressure on that bearing, the velocity of sliding, and the coefficient of friction. We will first find the power lost in end thrust. Since our calculation is being made for comparison only, and not for positive results, we will consider the coefficient of friction as being equal to 1. We will make the assumption that the mean diameter of the end thrust bearings of the various shafts is equal to half the pitch diameter of the gears. The mean velocity of rubbing will then be half the velocity of the gears at the pitch line. For the loss of power in the thrust bearing of shaft *A* we have:

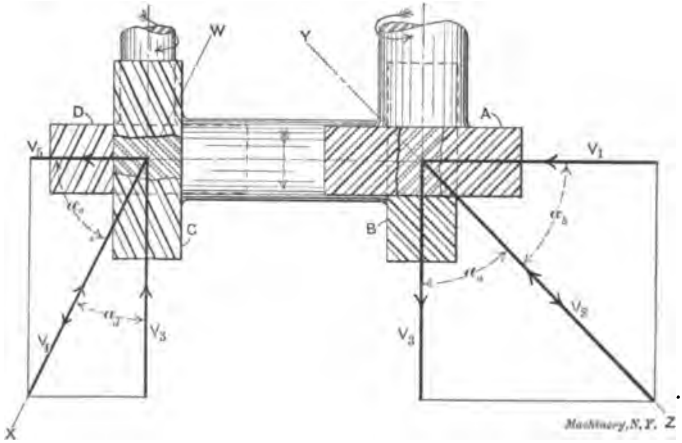

Fig. 22. Velocity Diagrams for Case 1.

$$F_3 \times \frac{V_1}{2} \times 1 = 0.500 \times 0.500 \times 1 = 0.250.$$

The end thrust on the intermediate shaft is that due to the difference between the opposing forces F_1 and F_5 in Fig. 19. For lost work in the end thrust of the intermediate shaft we then have:

$$(F_3 - F_1) \times \frac{V_3}{2} \times 1 = (1 - 0.500) \times 0.500 \times 1 = 0.250.$$

The loss in the thrust bearing of shaft D equals

$$F_3 \times \frac{V_5}{2} \times 1 = 0\ 500 \times 0.250 \times 1 = 0.125.$$

Adding these three losses together we get a total value of 0.625 as the power loss in end thrust.

For the power loss in tooth friction, we had better use a somewhat higher coefficient; perhaps 1.5 would be about right. The velocity of sliding between gears A and B is V_2, the normal pressure of the surfaces of contact is F_2. We have then for the lost power at this point:

$$F_2 \times V_2 \times 1.5 = 0.707 \times 1.414 \times 1.5 = 1.500.$$

Similarly, the work lost between gears C and D equals

$$F_4 \times V_4 \times 1.5 = 1.118 \times 1.118 \times 1.5 = 1.875.$$

The total loss due to tooth friction is then equal to the sum of these two or 3.375, which, added to the loss in the thrust bearings, gives us $3.375 + 0.625 = 4.0$, the total loss with this form of gearing.

It will not be necessary to draw new diagrams, like those in Figs. 19 and 22, for the second case, since we may use the formulas already derived for obtaining the various forces and velocities, making, however, the following substitutions. This change is in accordance with the data in Case 2.

Change α_a to $\alpha_e = 68° 26'$ Change α_c to $\alpha_g = 45°$
" α_b to $\alpha_f = 26° 34'$ " α_d to $\alpha_h = 45°$

Solving these formulas for velocities, we obtain the following quantities:

$V_1 = 1$
$V_2 = V_1 \div \sin\ \alpha_e = 1 \div 0.894 = 1.118$
$V_3 = V_1 \times \tan \alpha_f = 1 \times 0.500 = 0.500$
$V_4 = V_3 \div \sin\ \alpha_g = 0.500 \div 0.707 = 0.707$
$V_5 = V_3 \times \tan \alpha_h = 0.500 \times 1.000 = 0.500$

and for pressures we have the following:

$F_5 = 1$
$F_4 = F_5 \div \sin\ \alpha_g = 1 \div 0.707 = 1\ 414$
$F_3 = F_5 \times \tan \alpha_h = 1 \times 1.000 = 1.000$
$F_2 = F_3 \div \sin\ \alpha_e = 1.000 \div 0.894 = 1.118$
$F_1 = F_3 \times \tan \alpha_f = 1.000 \times 0.500 = 0\ 500$

The work lost with the thrust bearing on shaft E equals

$$F_3 \times \frac{V_1}{2} \times 1 = 1 \times 0.5 \times 1 = 0.5.$$

That lost in the intermediate shaft equals

$$(F_5 - F_1) \times \frac{V_3}{2} \times 1 = (1 - 0.5) \times 0.25 \times 1 = 0.125.$$

The loss in power due to end thrust in shaft H equals

$$F_3 \times \frac{V_5}{2} \times 1 = 1 \times 0.25 \times 1 = 0.25$$

These three losses added together equal 0.875.

The loss of power due to tooth friction between E and F, assuming a coefficient of friction of 1.5 as before, equals

$$F_2 \times V_2 \times 1.5 = 1.118 \times 1.118 \times 1.5 = 1.875.$$

Friction loss between G and H equals

$$F_4 \times V_4 \times 1.5 = 1.414 \times 0.707 \times 1.5 = 1.5.$$

The tooth friction loss in the tooth surfaces then equals $1.875 + 1.500 = 3.375$. For Case 2 the total lost work due to tooth friction and end thrust friction equals $3.375 + 0.875 = 4.250$. The difference between this quantity and the 4.000 obtained for Case 1 is scarcely large enough to be of any practical importance. There is but one consideration, in fact, we can think of for preferring one construction to the other. The 45-degree gears have teeth of slightly smaller size than those of the other pair in each case, and they are therefore somewhat weaker. In Case 1, these teeth are subjected to a normal pressure F_2 of 0.707. In Case 2 they are subjected to a normal pressure F_4 of 1.414, twice as great. In Case 1, then, the strongest teeth are bearing the greatest strain, which is as it should be.

CHAPTER V.

SETTING THE TABLE WHEN MILLING SPIRAL GEARS.

In cutting a spiral gear in a milling machine as ordinarily arranged, it is necessary to set the table to the helix angle in order that the sides of the cutter may not interfere, or drag in the cut. But the helix angle varies with the depth, being greatest at the top of the tooth, less at the pitch line, and still less at the bottom of the cut. In fact, if the cut were deep enough to reach all the way to the center of the piece being operated on, the helix angle would become zero, or parallel to the center line. If the general run of mechanics were asked what would be the proper angle at which to set the table, they would say that the helix angle at the pitch line would be the one to determine the setting. This setting has the effect of making the width of the cut exactly right at the pitch line, but it does so at the expense of undercutting and weakening the teeth. For quite some time the writer has thought that the helix angle at or near the bottom of the cut should be the one to set the table to in order to get a strong tooth, and to convince other mechanics of the correctness of this view the following experiments were made:

The piece G, Fig. 23, is a cast iron taper stem with a flange cast on, and fits in the dividing head of the milling machine. H is a brass chuck that was made for another job, and is fastened to G with four screws. *K* is a piece of 1⅝-inch round brass, with the ends faced true, while *L* is a piece of scrap brass with two faces machined at an angle of 30 degrees, one of these faces being tapped for the screw *M*, and also containing the two dowel pins *N*. *H*, *K*, and *L* are sweated together with soft solder.

The six pieces of sheet brass, *A*, *B*, *C*, *D*, *E*, and *F*, Fig. 24, were drilled so they would fit onto the dowel pins on the 30-degree face of the improvised chuck, *F* being shown in place in the line cut, Fig. 23. These six pieces were placed in succession on this chuck and the curved edges of all turned to a diameter of 1.23 inch. The object was to make a spiral cut in each of these six pieces, varying the setting of the table angle, and also the shape of the cutter, and to compare the shape of the cut with that of the cutter that made it.

The lead used was 5.33 inches to one turn, and the depth of cut ¼ inch, both these elements being alike in all six cases. The pieces of sheet brass were intended to stand at right angles with the cut, but, of course, this was impossible, as the helix angle varied with the depth, so they were set to stand at right angles with the helix at half depth. Assuming this helix angle to be 30 degrees, we can find the diameter of the imaginary cylinder whose surface is at half the depth of the cut by multiplying the lead, 5.33 inches, by the tangent of 30 degrees, and dividing by 3.1416, which gives 0.98 inch. Adding 0.25 inch to this, we get 1.23 inch for the outside diameter, and also, by subtracting 0.25 from 0.98, we get 0.73 inch for the diameter at the bottom of the cut. Knowing the outside diameter to be 1.23 inch, we multiply this by 3.1416 and divide by 5.33 to get the tangent of the helix angle at the top, which we find to be 35 degrees 56 minutes. In a similar manner we multiply the bottom diameter, 0.73, by 3.1416 and divide by 5.33 to get the tangent of the helix angle at bottom of cut, which we find to be 23 degrees 17 minutes.

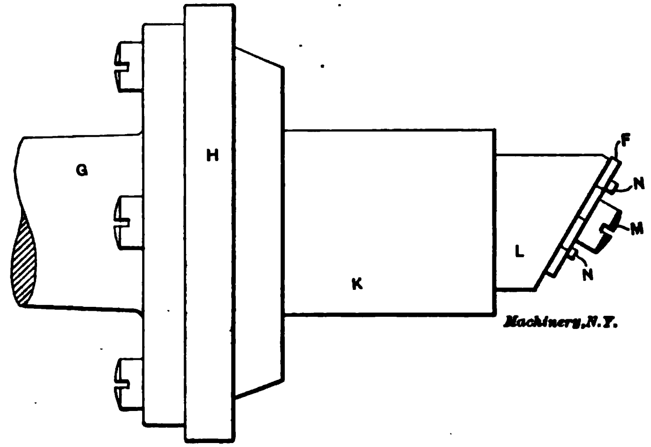

Fig. 23. Chuck for Mounting the Pieces shown in Fig. 24.

The cutter used in this job was a fly-cutter, the holder being shown in the half-tone illustration Fig. 25, while its blades, *R* and *S*, are shown in the line illustration Fig. 24. *R* was used to cut *A*, *B*, and *C*, while *S* was used to cut *D*, *E*, and *F*. The table setting was nearly 36 degrees for *A* and *D*, 30 degrees for *B* and *E*, and about 23¼ degrees for *C* and *F*.

The shapes of the cuts show that the width of the cutter is accurately reproduced only at the particular depth where the helix angle is the same as the table setting, this point being shown by the arrow heads at the sides of the various cuts. The shapes of the cuts also show that the departure from the true form of the cutter due to faulty table setting is less in the case of the more flaring cutter *S* than in the case of cutter *R*, whose sides come nearer to being parallel.

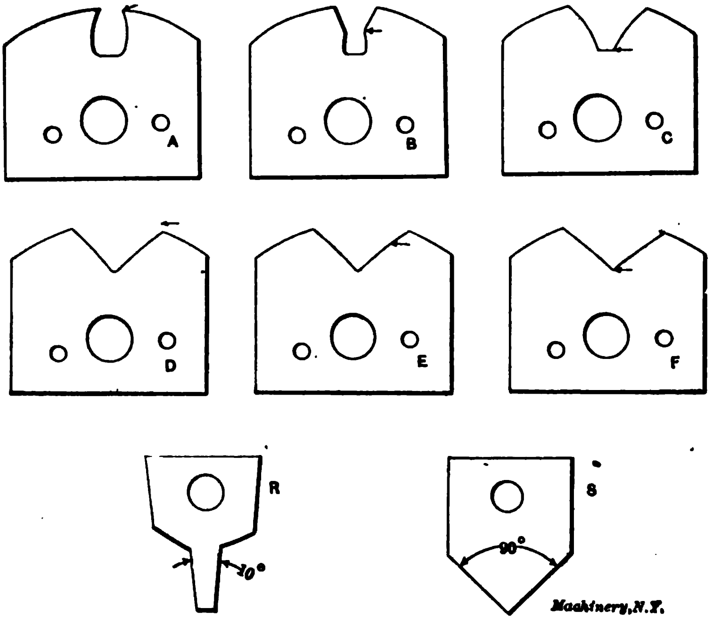

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Fig. 24. Pieces Milled on the Chuck, Fig. 23, with Fly-cutters R and S, showing the Effect of varying the setting of the Milling Machine Table.

This demonstrates that the table setting for a spiral gear should be the same as the helix angle at or near the bottom of the cut, because at this point the sides of the cutter come closer to parallelism, while at the top of the cut they are more flaring, and the table setting not being correct for the helix angle at this point would produce a comparatively slight error. This also suggests a slight modification in selecting a cutter to do the job, as the tops of the teeth would be rounded off somewhat more than in the case of a spur gear cut with the same cutter. Therefore, the writer suggests that it would be well to select a cutter for a greater number of teeth than the spiral gear formula

$$T = \frac{N}{\cos^3 a} \text{ calls for.}$$

Setting the table for a less angle than that of the pitch line helix also has the effect of slightly increasing the width of the cut at the pitch line, but not to the extent that a comparison of C and R would seem to indicate, as in the experiments here described the depth of the cut was purposely made a very large percentage of the diameter in order to accentuate the errors due to faulty setting of the table, and if the table is to be set correct for the bottom of the cut, it might be well to consider the normal circular pitch as slightly greater than that rightfully belonging to the cutter in use, and size the blank accordingly. These experiments were entirely of a qualitative nature, and were only intended to guide the judgment of the designer and the man who puts the design in material form in cold metal.

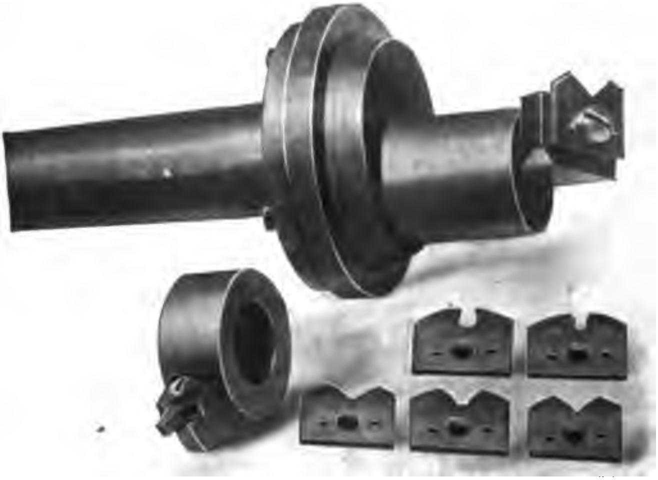

Fig. 25. Chuck, Fly-cutter, and Pieces Milled in Experiments Described.

Those not thoroughly familiar with universal milling machine use should carefully distinguish between the table angle and the helix angle produced by the gearing of the dividing head. The dividing head is geared to produce the required helix angle, measured on the pitch line the same as usual, of course. What is advocated, in order to reduce interference, is simply setting the table to some helix angle between the pitch line and the dedendum or root circle rather than to the helix angle indicated by the pitch line. A point worth attention, also, is that the interference of a cutter increases with increase of diameter. Small cutters, therefore, tend to reproduce their outlines more accurately than large cutters, other things being equal.

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